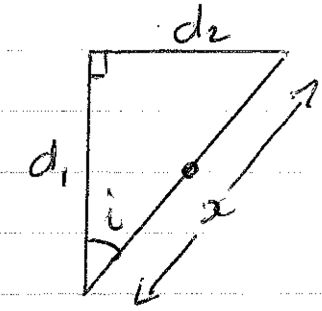
Assignment 3.



1. The ring has a long axis that measures 28.7 mm and a short axis measuring 20.0 mm. The short axis corresponds to d_1 in the diagram, the long axis to x.

$\therefore \cos i = \dfrac{d_1}{x} = \dfrac{20.0\text{ mm}}{28.7\text{ mm}} = 0.6968641$

$\therefore i = \cos^{-1}(0.6968641) = 45^\circ.82405$

$\therefore d_2 = x \sin i = x \sin(45^\circ.82405) = 0.7172031x$

But $d_2 = \dfrac{340^d \times 24^h/d \times 60^m/h \times 60^s/m \times 2.9979 \times 10^8 \text{ m/s}}{1.4960 \times 10^{11} \text{ m/AU} \times 206265 \text{ AU/pc}}$

$= 0.2853991$ pc.

$\therefore x = 0.2853991 \text{ pc} / 0.7172031 = 0.3979334$ pc

But x subtends an angle of $1''.66$. From the small angle equation we know that the distance r to SN 1987A can be found from: $x = \dfrac{\theta r}{206265} = \dfrac{(1.66)r}{206265}$

$\therefore r = \dfrac{206265\,(0.3979334\text{ pc})}{1.66} = 49,445.622$ pc

i.e. The distance to SN 1987A is ~ 49,446 pc.

The corresponding distance modulus is:

$$\begin{aligned} m - M &= 5 \log r - 5 \\ &= 5 \log(49445.622) - 5 \\ &= 5(4.6941278) - 5 \\ &= 23.47 - 5 = 18.47 \end{aligned}$$

This result agrees with current best estimates for the distance modulus of the LMC of 18.50 ± 0.10.

2. The mass of the Galaxy can be estimated using Kepler's 3RD Law in conjunction with the orbital speed (244 km/s) and distance (75 kpc) of a clump in the Magellanic Stream. The orbital period of the clump is given by:

$$P = \frac{2\pi R}{244 \text{ km/s}} = \frac{2\pi \times 75{,}000 \text{ pc} \times 206265 \text{ AU/pc} \times 1.496\times10^{8} \text{ km/AU}}{245 \text{ km/s} \times 3.1558 \times 10^{7} \text{ s/yr}}$$

$$= 1.8807 \times 10^{9} \text{ yr.}$$

The semi-major axis in A.U. is:

$$a = 75{,}000 \text{ pc} \times 206265 \text{ AU/pc} = 1.547 \times 10^{10} \text{ AU.}$$

Since the mass of the clump is negligible relative to the mass of the Galaxy, $m_c << M_G$, thus, in solar masses:

$$(m_c + M_G) \simeq M_G = \frac{a^3}{P^2} = \frac{(1.547\times10^{10})^3}{(1.8809\times10^{9})^2} M_\odot$$

$$= 1.0465 \times 10^{12} M_\odot$$

This result is about an order of magnitude larger than values obtained from stellar and gas orbits in the Galactic plane, possibly because the orbital speed is too large.

By the conservation of energy:

$$\frac{1}{2} m_c (v_r^2 + v_T^2) - \frac{G M_G m_c}{r_{initial}} = \frac{1}{2} m_c (v_r^2 + v_T^2) - \frac{G M_G m_c}{r_{final}}$$

But the transverse velocity of the cloud is unchanged while the radial velocity goes from 0 km/s to -245 km/s in falling from $r_{initial} = 100$ kpc to $r_{final} = 50$ kpc. Thus:

$$M_G = \frac{v_r^2}{2G} \frac{1}{(1/r_{final} - 1/r_{initial})} = \frac{(245 \text{ km/s})^2}{2G\,(1/50 - 1/100) \text{ kpc}^{-1}}$$

$$= \frac{(2.45\times10^{5})^2 (2.06265\times10^{8}) (1.4960\times10^{11})}{2(6.6726\times10^{-11})(1/50 - 1/100)} \text{ kg}$$

$$= 1.3879\times10^{42} \text{ kg} \,/\, 1.989\times10^{30} \text{ kg}/M_\odot$$

$$= 6.978 \times 10^{11} M_\odot$$

This value ($7\times10^{11} M_\odot$) is also larger than estimates obtained from stars in the Galaxy's outer disk

3. The velocity dispersion for galaxies in the Virgo cluster is 666 km/s. The radius of the Virgo cluster is 1.5 Mpc (textbook gives diameter of 3 Mpc). The mass of the Virgo cluster can be established from the Virial Theorem using the equation:

$$M_{vc} = \langle R\,(pc)\rangle \left(\frac{\langle v^2 (km/s)\rangle^{1/2}}{4.637 \times 10^{-2}}\right)^2 M_\odot$$

$$= (1.5 \times 10^6) \left(\frac{666}{4.637 \times 10^{-2}}\right)^2 M_\odot$$

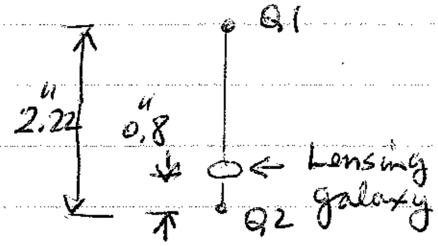
$$= 3.09 \times 10^{14} M_\odot \quad \text{roughly } 3000\ M_\odot$$

If the textbook relationship is used instead the solution becomes larger by a factor of 5/2,

i.e. $M_{vc} = \frac{5}{2} \times 3.09 \times 10^{14} M_\odot$

$$= 7.74 \times 10^{14} M_\odot$$

4. The situation for the quasar images is shown at right. Image Q1 is $2.22'' - 0.8'' = 1.42''$ from the lensing galaxy. Image Q2 is $0.8''$ away.



The distance of the quasar and lensing galaxy can be found from the relativistic Hubble Law, i.e.:

$$d = \frac{c}{H_0}\left[\frac{(1+z)^2-1}{(1+z)^2+1}\right] = 2.9979 \times 10^{3} h^{-1}\left[\frac{(1+z)^2-1}{(1+z)^2+1}\right] \text{Mpc}$$

Thus, the distance to the quasar Q0142-100 is:

$$d_Q = 2.9979 \times 10^{-3} h^{-1}\left[\frac{(1+2.727)^2-1}{(1+2.727)^2+1}\right] \text{Mpc}$$

$$= 2595.2\, h^{-1} \text{ Mpc}$$

And the distance to the lensing galaxy is:

$$d_{LG} = 2.9979 \times 10^{-3} h^{-1}\left[\frac{(1+0.493)^2-1}{(1+0.493)^2+1}\right] \text{Mpc}$$

$$= 1141.1\, h^{-1} \text{ Mpc}$$

The mass of the lensing galaxy can be found using the relation: $M_{LG} = \frac{-\theta_1\theta_2 c^2}{4G} \cdot \left(\frac{d_Q\, d_{LG}}{d_Q - d_{LG}}\right)$,

Here $\theta_1 = -1.42'' = -1.42/206265$ radians, $\theta_2 = 0.8'' = 0.8/206265$ radians

$$\therefore M_{LG} = \frac{(1.42)(0.8)(2.9979\times10^{8})^2(2595.2)(1141.1)\times10^{6}\times1.496\times10^{11}\times206265}{(206265)^2(4)(6.6726\times10^{-11})\ (2595.2-1141.1)}$$

$$= 5.6502\times10^{41} \text{ kg} / 1.989\times10^{30} \text{ kg}/M_\odot \quad h^{-1}$$

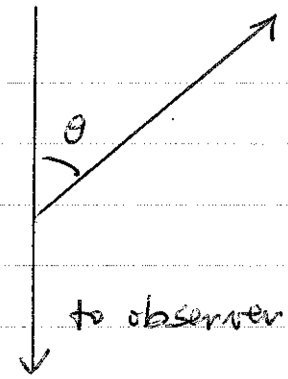
$$= 2.8407\times10^{11}\ M_\odot \quad h^{-1}$$

In other words, for $h = 0.71$,

$$M_{LG} = 2.8407\times10^{11}/0.71 \quad M_\odot$$

$$= 4.0\times10^{11}\ M_\odot$$, roughly twice the mass of the Milky Way.

5. Equation 4.31 in the textbook indicates that, for the geometry indicated at right,



$$\Delta t_{obs} = \frac{\Delta t_{rest}}{(1 - v^2/c^2)^{1/2}} \left[1 + (v/c)\cos\theta\right]$$

$$= \gamma\, \Delta t_{rest} \left[1 + (v/c)\cos\theta\right],$$

where $\gamma \equiv 1/(1 - v^2/c^2)^{1/2}$

For the case described here: $\theta = 180°$, $v \simeq c$

$$\therefore \Delta t_{obs} = \gamma\, \Delta t_{rest} \left(1 + \frac{v}{c}\cos 180°\right)$$

$$= \gamma\, \Delta t_{rest} \left(1 - v/c\right) \quad \text{since } \cos 180° = -1$$

$$= \gamma\, \Delta t_{rest} \frac{(1 - v/c)(1 + v/c)}{(1 + v/c)}$$

But $(1 - v/c)(1 + v/c) = (1 - v^2/c^2) = 1/\gamma^2$

And $(1 + v/c) \simeq 1 + 1 = 2$

$$\therefore \Delta t_{obs} = \gamma\, \Delta t_{rest} \left(\frac{1}{\gamma^2}\right) \cdot \left(\frac{1}{2}\right)$$

$$= \frac{\Delta t_{rest}}{2\gamma}, \text{ as required.}$$