ASTR 3500.1

Assignment 2.

1. Tangential velocity can be expressed as:

$$v_T \text{ (km/s)} = 4.74 \, \mu \, (''/\text{yr}) \, d \, (\text{pc})$$

Here we adopt $\mu = 0.02 \, ''/\text{yr}$ and $d = 100{,}000 \text{ pc}$ (100 kpc)

$\therefore v_T = 4.74 \, (0.02)(10^5)$ km/s
$= 9480$ km/s   for the speed of a point at the edge of the galaxy.

By comparison, the characteristic rotation speed of stars at the outer edge of the Milky Way is ~ 245-250 km/s, almost 40 times smaller.

OR

Using $d = 100{,}000 \text{ pc} \times \frac{1}{2} = 50{,}000 \text{ pc}$ as the radius of the galaxy for an assumed diameter of 100,000 pc (a bit large for a spiral), we find:

$v_T \text{ (km/s)} = 4.74 \, (0.''02/\text{yr}) \, (50{,}000 \text{ pc})$
$= 4740$ km/s   for stars on the edge of M101.

Again, by comparison, characteristic rotation speeds for stars in the outer regions of the Milky Way are only ~ 250 km/s, about 20 times smaller.

2. NGC 2639 has $v_{rot}(max) = 324$ km/s, and the Tully-Fisher relation for M_B is: $M_B = -9.95 \log v_{rot}(max) + 3.15$ for Sa galaxies.

$= -9.95 \log(324) + 3.15 = -24.979923 + 3.15$

$= -21.83$

Since $B_0 = 12.22$ for NGC 2639, $\therefore B - M_B = 5 \log d - 5$

$\therefore d = 10^{0.2(B - M_B + 5)} = 10^{0.2(12.22 + 21.83 + 5)}$

$= 10^{0.2(39.05)} = 10^{7.81}$ pc

$= 64.57$ Mpc

A brightness level of 25-B mag./arcsec^2 should be reached at:

$\log R_{25} = -0.249 M_B - 4.00$ kpc

$= -0.249(-21.83) - 4.00 = -5.436 - 4.00$

$= 1.4356508$

$\therefore R_{25} = 10^{1.4356508} = 27.268$ kpc (27.3 kpc)

The mass interior to 27.3 kpc can be established using the same technique applied to the Milky Way, i.e. via Kepler's 3RD Law.

$M_{NGC\,2639} = a^3/P^2$, where $a = 27.268$ kpc $\times 2.06265 \times 10^8$ AU/kpc

and $P = \frac{2\pi r}{v} = \frac{2\pi \times 27.268 \times 2.06265 \times 10^8 \times 1.496 \times 10^8}{324 \times 365.2564 \times 24 \times 60 \times 60}$ yrs

$= 5.1705 \times 10^8$ yr.

$\therefore M_{NGC\,2639} = (27.268 \times 2.0625 \times 10^8)^3 / (5.1705 \times 10^8)^2 \; M_\odot$

$= 6.6553 \times 10^{11} M_\odot$ $(6.7 \times 10^{11} M_\odot)$

$M_B(\odot) = 4.82 + 0.63 = 5.45$

$\therefore \log L(NGC\,2639)/L_\odot = -0.4(M_B - 5.45) = -0.4(-21.83 - 5.45)$

$= -0.4(-27.28) = 10.912$

$\therefore L(NGC\,2639) = 10^{10.912} L_\odot$

$= 8.1658 \times 10^{10} L_\odot$ $(8.2 \times 10^{10} L_\odot)$

The resulting mass-to-light ratio for the Sa galaxy NGC 2639 is: $M/L = 6.6553 \times 10^{11} M_\odot / 8.1658 \times 10^{10} L_\odot$

$= 8.15 \; M_\odot / L_\odot$

3. The colour indices for standard spiral galaxies in Table 25.1 are:

Sa. $\langle B-V \rangle = 0.75$
Sb. $\langle B-V \rangle = 0.64$
Sc. $\langle B-V \rangle = 0.52$

If these values are correlated with the average brightness of main-sequence stars then the corresponding spectral types are:

Sa. G8, $(B-V) = 0.74$
Sb. G2, $(B-V) = 0.63$
Sc. F8, $(B-V) = 0.52$

Although the majority of stars in spiral galaxies are dwarfs, the overall brightness may be more typical of giant stars, which dominate evolved populations. In that case the best matches might be:

Sa. G2 III, $(B-V) = 0.77$
Sb. G0 III, $(B-V) = 0.65$
Sc. F8 III, $(B-V) \simeq 0.54$

If the stars are of above-solar or below-solar metallicity, such a procedure is also invalid, although the average types are interestingly around the Sun's type. Yet the average star in such a galaxy is more likely to be a M dwarf or white dwarf.

The procedure is fine, but biases the results to hotter spectral types than the average star in such galaxies. Perhaps the influence of interstellar gas has been overlooked.

4. The specific frequency for globular clusters in a galaxy is given by: $S_N = N_{total}\, 10^{0.4(M_V+15)}$

Here $N_{total} \sim 350$ (estimated)

$M_V = -21.7$ for M31.

$\therefore S_N = 350\, 10^{0.4(-21.7+15)}$

$= 350\, 10^{0.4(-6.7)}$

$= 350\, 10^{-2.68}$

$= 0.73$, fairly typical of Sb galaxies

For NGC 3311, $N_{total} \simeq 17{,}000$, $M_V \simeq -22.4$

$\therefore S_N = 17{,}000\, 10^{0.4(-22.4+15)}$

$= 17{,}000\, 10^{0.4(-7.4)}$

$= 17{,}000\, 10^{-2.96}$

$= 18.64$, fairly typical of cD galaxies

The relative abundance of globular clusters per luminosity interval in cD galaxies far exceeds the values found in spirals. If cD galaxies are produced via mergers with spirals then apparently globular clusters must be formed in the process. Far more likely is that cD galaxies are formed through mergers with elliptical galaxies, which have far higher specific frequencies than spiral galaxies.

5. M87 is estimated to have a mass of $\mathcal{M} \simeq 3 \times 10^{13}\, \mathcal{M}_\odot$ lying within 300 kpc of its centre. The corresponding circular velocity at a distance of 300 kpc from its centre is given by:

$$v_{circ} = \left(\frac{G\,\mathcal{M}_{M87}}{R_{M87}}\right)^{1/2}$$

$$= \left(\frac{6.6726 \times 10^{-11} \times 3 \times 10^{13} \times 1.989 \times 10^{30}}{3 \times 10^5 \times 206265 \times 1.496 \times 10^{11}}\right)^{1/2} \text{m/s}$$

$$= (4.301 \times 10^{11})^{1/2} \text{ m/s} = 6.5582 \times 10^5 \text{ m/s}$$

$$= 655.82 \text{ km/s}$$

The corresponding period of revolution can be calculated from Kepler's 3RD Law, i.e.:

$$P(yr) = \left[\frac{a\,(AU)^3}{\mathcal{M}_{M87}\,(\mathcal{M}_\odot)}\right]^{1/2}$$

$$= \left[\frac{(3 \times 10^5 \times 206265)^3}{3 \times 10^{13}}\right]^{1/2}$$

$$= (7.898 \times 10^{18})^{1/2}$$

$$= 2.81 \times 10^9 \text{ years}$$

By way of comparison, the Milky Way galaxy is estimated to have an age of no more than 13-14 Myr, roughly 5 times larger.

The Virial temperature of material near the outer edge of M87 can be calculated from: $T_{virial} = \frac{\mu\, m_H\, \sigma^2}{3k}$

Here, $\mu \simeq 0.6$ (adopted), $\sigma = 6.5582 \times 10^5$ m/s, $m_H = 1.6726 \times 10^{-27}$ kg, $k = 1.3807 \times 10^{-23}$ J/K, so:

$$T_{virial} = \frac{0.6\,(1.6726 \times 10^{-27})(6.5582 \times 10^5)^2}{3\,(1.3807 \times 10^{-23})}$$

$$= 1.0421 \times 10^7 \text{ K}$$

$$= 10.4 \text{ million K}$$

But the outer regions of M87 cannot be in virial equilibrium from above.