A CURRENT APPLICATION OF THE METHODS OF SECULAR AND STATISTICAL PARALLAX

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ABSTRACT. The methods of secular and statistical parallax for homogeneous groups of Galactic stars are applied in a practical (classroom) exercise to establish the luminosity of bright B3 V stars. The solar motion of 20 km s^{-1} relative to group stars exceeds their random velocities of ± 10 km s^{-1}, a condition adopted for preference of secular parallax to statistical parallax. The group parallax of $\langle \pi_{\rm ups} \rangle = 5.81 \pm 0.83$ mas and derived luminosity $\langle M_V \rangle = -0.98 \pm 0.33$ for B3 V stars from upsilon components of proper motion should be close to the true value. The weighted mean *Hipparcos* parallax of $\langle \pi_{\rm Hip} \rangle = 5.75 \pm 0.27$ mas for the same sample, and implied luminosity of $\langle M_V \rangle = -1.00 \pm 0.15$, confirm the secular parallax solution. Both solutions are close to $M_V = -0.83$ for ZAMS stars of the same type, implying that Malmquist bias in the selection of stars mainly accounts for the presence of unresolved binaries, slightly evolved objects, and rapidly rotating stars in the sample.

Key words: methods: statistical — parallaxes — stars: fundamental parameters.

1. Introduction

The method of statistical parallax was important to Galactic astronomers of the last century, but seems of minor interest today, most current Galactic research involving instead the astrophysics of galaxies like the Milky Way. Yet there are still occasions where the technique can be involved in current problems in Galactic astronomy, so the author continues to instruct students in the methodology for courses in Galactic astronomy taught at his home institution. The problem outlined here is used as a learning exercise for application of the technique, but proves to be of more than just pedagogical interest since it addresses current questions regarding luminosity calibrations.

2. Statistical and Secular Parallaxes

The technique involves the use of proper motions and radial velocities to establish the space motion of a group of stars relative to the Sun, and is explained very well by Mihalas & Routly (1968), Mihalas & Binney (1981), and Scheffler & Elsässer (1988). The basic premise is that the Sun's motion relative to nearby stars in the Galaxy creates a baseline of observation, similar in nature to that created by the Earth's annual orbital motion around the Sun, from which one can triangulate the distances to stars from their temporal angular displacements, namely their proper motions across the sky. The angular motions of nearby stars resulting from their different distances from the Sun are often masked by individual space motions and observational uncertainties, but it is possible to evaluate an average for an entire homogeneous group once the Sun's motion relative to the group is measured (in km s^{-1}). The individual space motions of group stars are not known *a priori*, which is why the technique devolves to a statistical approach that yields a mean parallax for the group from their proper motions and radial velocities.

The first step in any study involving statistical parallax is to isolate a group of stars with a common set of parameters and then establish the Sun's motion relative to the group. Radial velocities can be used for the latter purpose, as outlined by Mihalas & Routly (1968) and Mihalas & Binney (1981), provided the stars in the group are randomly distributed on the sky. Failing that, they may be sufficiently randomly distributed in Galactic longitude to establish a space motion for the group reliably, at least for motion in the Galactic plane.

There are two components of any star's proper motion: one reflecting the Sun's motion relative to the star, denoted as the upsilon υ component, and the other perpendicular to it reflecting solely the star's space motion, denoted as the tau τ component. The former is dominated by the Sun's motion relative to the group of stars, while the latter is presumably random for a large enough sample. The two components can be calculated from the direction of the solar motion relative to the group, the solar apex, denoted by right ascension A and declination D, and the observed proper motion components, μ_α in right ascension, and μ_δ in declination. Three equations are needed to solve for the position angle ψ and angular distance λ of each star from the direction to the solar apex since there are ambiguities in the sine function for angles of 0°–180°. The upsilon and tau components of proper motion for each star then follow from the equations of spherical and Cartesian geometry:

$$\upsilon = \mu_\alpha \cos\delta \sin\psi - \mu_\delta \cos\psi$$

$$\tau = \mu_\alpha \cos\delta \cos\psi + \mu_\delta \sin\psi.$$

The secular parallax is then calculated from a knowledge of the Sun's motion $v_\odot$ relative to the group:

$$\langle\pi\rangle = \frac{4.74\langle\upsilon \sin\lambda\rangle}{v_\odot\langle\sin^2\lambda\rangle}$$

where the triangular brackets on both sides of the equation represent straight averages. The statistical parallax is calculated differently using:

$$\langle\pi\rangle = \frac{4.74\langle|\tau|\rangle}{\langle|v_R + v_\odot \sin\lambda|\rangle}$$

involving the use of absolute values for tau and radial velocity difference. Secular parallaxes are predicted to work best when the solar motion dominates the group random velocities, and statistical parallaxes otherwise.

Weights assigned to individual stars in the method of secular parallax vary according to the sine of a star's angular distance from the solar apex for the group, in order to maximize the influence of those stars displaying the largest degree of angular displacement produced by the Sun's motion. An alternate version using the maximum likelihood technique was developed by Clube & Dawe (1980a,b). For classroom exercises the standard methodology provides the most direct approach to solving practical problems, and is the technique followed here.

3. A Practical Example: B3 V Stars

Class exercises are normally chosen to be completed within a time span not exceeding a week or two, which limits the quantity and type of data that can be analyzed. For the present purpose an exercise was developed to find the mean absolute magnitude of B3 V stars using the methods of secular and statistical parallax in conjunction with information available from *The Bright Star Catalogue* (Hoffleit & Jaschek 1982). There are 88 bright stars classified as B3 V that have coordinates, proper motions, and radial velocities summarized by Hoffleit & Jaschek (1982). For the exercise the author also calculated unreddened visual magnitudes V_0 by removing small amounts of reddening and extinction for affected stars. The exercise was first developed in an era prior to the general availability of high-speed computers, so ease of calculation was originally a concern. The availability of *The Hipparcos and Tycho Catalogues* (ESA 1997) in recent years has also provided an improved set of proper motions, as well as absolute parallaxes that allow one to test the results.

An essential step in deriving secular and statistical parallaxes is to establish the solar motion relative to the group, since the resulting values of A and D are necessary for calculating the angles λ and ψ for each star in the group, while $v_\odot$ is required for solving the parallaxes. Omission of that step greatly reduces the accuracy of the solutions, since one must rely on parameters derived for quite different stars to obtain comparable values. For the sample of B3 V stars considered here, solutions for the solar motion relative to the group are given in Table 1 for the epochs 1900.0 and 2000.0. Proper motion data for the former were taken from Hoffleit & Jaschek (1982), and for the latter from ESA (1997). The radial velocities are those cited by Hoffleit & Jaschek (1982). Differences between solutions in $v_\odot$ for the two epochs are the result of rounding errors in the calculations.

Table 1: Parallax Solutions for B3 V Stars.

Parameter	Epoch (1900)	Epoch (2000)
A (solar motion)	265°.384	266°.145
D (solar motion)	43°.112	43°.071
$v_\odot$ (km s^{-1})	20.0465	20.0466
$\langle V_0\rangle$	5.20 ± 0.12	5.20 ± 0.12
$\langle\pi_{\rm ups}\rangle$ (mas)	6.07 ± 0.91	5.81 ± 0.83
$\langle\pi_{\rm tau}\rangle$ (mas)	5.34 ± 0.58	4.42 ± 0.51
$\langle\pi_{\rm Hip}\rangle$ (mas)	···	5.20 ± 0.38
$\langle\pi_{\rm Hip}\rangle_{\rm wgt}$ (mas)	···	5.75 ± 0.27
$\langle M_V\rangle_{\rm ups}$ (B3 V)	-0.88 ± 0.34	-0.98 ± 0.33
$\langle M_V\rangle_{\rm tau}$ (B3 V)	-1.16 ± 0.26	-1.57 ± 0.27
$\langle M_V\rangle_{\rm Hip}$ (B3 V)	···	-1.22 ± 0.19
$\langle M_V\rangle_{\rm Hip\,wgt}$ (B3 V)	···	-1.00 ± 0.15

The calculation of mean secular and statistical parallaxes follows from the equations given previously, with solutions given in Table 1 for upsilon $\langle\pi_{\rm ups}\rangle$ and tau $\langle\pi_{\rm tau}\rangle$ components, as well as for revised *Hipparcos* parallaxes (van Leeuwen 2007). The desired mean absolute magnitude follows from the standard formula: $\langle M_V\rangle = \langle V_0\rangle + 5\log\langle\pi\rangle + 5$. In the case of revised *Hipparcos* parallaxes, mean and weighted mean parallaxes were calculated for the sample, with weights assigned according to the cited absolute uncertainty in the parallax (not the relative uncertainty), and the resulting absolute magnitude was calculated as both a straight average and a weighted average, with the uncertainty in the mean dereddened magnitude for the stars included. The observed scatter in the absolute magnitudes inferred from individual *Hipparcos* parallaxes is $\pm0^{\rm m}.77$, with values of M_V ranging from $+0.10$ to -4.05.

Information about the luminosities of B3 V stars has also been established from older parallaxes and membership in open clusters, as well as from zeroage main sequence (ZAMS) calibrations. For example, the ZAMS of Turner (1976) implies $M_V = -0.83$ for stars with the intrinsic colour of B3 V stars, $(B-V)_0 = -0.20$, and spectral type calibrations for B3 V stars imply $M_V = -1.5$ (Turner 1980) or $M_V = -1.7$ (Blaauw 1963; Keenan 1963).

The Sun's 20 km s^{-1} motion relative to the sample of B3 V stars exceeds the random motions of the stars relative to one another of ± 10 km s^{-1}, so the secular parallax $\langle\pi\rangle_{\rm ups}$ should be closer to the true parallax than the statistical parallax $\langle\pi\rangle_{\rm tau}$. Malmquist bias is expected to be important for the sample, given that *The Bright Star Catalogue* is magnitude limited, thereby sampling preferentially the most luminous stars of spectral type B3 V. In that case the derived luminosity from secular parallax should be greater than the true mean value for the class. Interestingly enough, the secular parallax produces a luminosity that is only $0^{\rm m}.15$ more luminous than the value expected for ZAMS stars, but $\sim 0^{\rm m}.6$ *less* luminous than standard literature values for class V dwarfs. If there are unresolved binaries, slightly evolved objects, or rapidly rotating stars in the sample, then the luminosity derived from the secular parallax is in excellent agreement with the ZAMS, but not class V, value. It appears that Malmquist bias in this instance actually accounts for the presence of unresolved binaries, slightly evolved objects, and rapidly rotating stars in the sample.

Spectral type-M_V calibrations make use of spectral classifcations of mixed quality in the literature. Stars identified spectroscopically as B3 V may therefore include subgiants and slightly evolved objects. That is not the case for spectral classifications in *The Bright Star Catalogue*, which are generally of high quality. The small discrepancy between the value of M_V derived for B3 V stars using secular parallax and those in published calibrations is therefore not unusual.

The weighted mean *Hipparcos* parallax for the sample, $\langle\pi_{\rm Hip}\rangle_{\rm wgt} = 5.75 \pm 0.27$ mas, confirms the secular parallax, $\langle\pi_{\rm ups}\rangle = 5.81 \pm 0.83$ mas. The mean luminosity for sample B3 V stars, $\langle M_V\rangle = -1.00 \pm 0.15$, therefore coincides closely with the secular parallax value, $\langle M_V\rangle = -0.98 \pm 0.33$. An unweighted average for *Hipparcos* parallaxes gives a value $\sim 0^{\rm m}.2$ more luminous, implying that the smallest parallaxes are associated with the largest cited uncertainties. That is not entirely self-evident, since some stars with the largest parallaxes in the sample actually have relatively large cited uncertainties.

An identical conclusion was found for Cepheid variables studied by *Hipparcos* (Turner 2010), namely Cepheids of small parallax are typically associated with large cited uncertainties. That is why researchers prefer stars with small relative parallax uncertainties for calibration purposes. For *Hipparcos* parallaxes, however, it appears that many stars with small parallax uncertainties also deviate significantly, i.e. by several σ, from the true parallax, in other words the cited precision may be overstated (Turner 2010). The methods of secular and statistical parallax may therefore continue to serve as useful tools for calibration purposes.

4. Discussion

Presented here is an application of secular and statistical parallax to the study of a homogeneous group of Galactic stars, in a manner useful for classroom demonstrations or research assignments. The sample considered here consists of 88 stars in *The Bright Star Catalogue* classified as B3 V. The solution generates a luminosity for the stars close to the value expected for a selection of ZAMS stars of that spectral type contaminated by unresolved binaries, slightly evolved objects, and rapidly rotating stars. The expected Malmquist bias applying to the sample selection appears to account implicitly for the latter effect. The result is confirmed by *Hipparcos* parallaxes for stars in the same sample, a rare instance where alternate solutions can be used to check the consistency of results from secular and statistical parallax.

The numerical simplicity of the method stands in contrast to more complicated versions, for example the maximum likelihood technique developed by Clube & Dawe (1980a,b). It may therefore be possible to extend the type of class exercise posed here to problems of greater astronomical interest, for example the luminosities of important distance calibrators such as the Cepheids discussed by Clube & Dawe (1980b).

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