

ASTRONOMY 1100. INTRODUCTION TO ASTROPHYSICS

Assignment 3.**Due Date: March 19, 2014.**

1. A solar flare is observed near the centre of the Sun's disk in white light imaging, and from spectroscopic observations it is determined that the ionized material in the ejecta is heading toward the Earth at a speed of 800 km s^{-1} . Calculate how long it will take for the flare ejecta to reach the Earth, *i.e.* how long it will be before the effects of the flare are evident in fluctuations of the Earth's geomagnetic field.
2. If the current rate of hydrogen burning in the Sun remains constant, what fraction of the Sun's mass will be converted into helium over the next 5×10^9 years? By how much will the total mass of the Sun have been reduced over the same time interval?
3. The star Aldebaran (α Tauri) is a K5 III giant. According to measurements obtained from the *HIPPARCOS* mission, Aldebaran has a trigonometric parallax of $\pi = 0''.05009 \pm 0''.00158$. Determine the distance to Aldebaran in parsecs. Also determine the uncertainty in that value (in parsecs).
4. Aldebaran (Question 3) has an observed photoelectric visual magnitude of $V = +0.87$. Determine the star's absolute visual magnitude, M_V . Is the value typical of a K giant? [Hint: See Figure 19–13 of the textbook.]
5. The *HIPPARCOS* Mission also provided data on the proper motion of Aldebaran (Question 3), and lists its proper motion as $+0.06278$ arcsecond per year in right ascension and -0.18936 arcsecond per year in declination. Determine the net proper motion of Aldebaran, and use the resulting value with the answer to Question 3 to establish the tangential velocity of the star. Aldebaran also has a radial velocity of $+54 \text{ km s}^{-1}$. What is the star's total space velocity with respect to the Sun?
6. As viewed from Earth, the Sun has a visual magnitude of $V = -26.75$. What will the Sun's brightness be, in magnitude units, as seen from the surface of one of Saturn's satellites? Saturn's mean distance from the Sun is 9.54 A.U.
7. At one stage during its birth, the protosun had a luminosity of $1000 L_\odot$ and a surface temperature of about 1000 K. What was its radius at that time? Express your answer in terms of the present-day solar radius, in kilometres, and in astronomical units (A.U.).
8. During the helium-burning stage of stellar evolution, three helium nuclei ${}^4\text{He}$, each of mass 4.002603 atomic mass units, are converted into a single carbon nucleus ${}^{12}\text{C}$, of mass 12.000000 atomic mass units, *i.e.* $3 {}^4\text{He} \rightarrow {}^{12}\text{C}$, where one atomic mass unit equals $1.66054 \times 10^{-27} \text{ kg}$. Calculate the amount of energy released in one such set of reactions. During the helium burning stages for high-mass stars, a star's luminosity is typically on the order of $1500 L_\odot$ or more. How long would it take for a typical star of this type to use up $1 M_\odot$ of helium fuel?

ASTR 1100

Assignment 3:

1. Speed is given by: $V = d/t$.

Here we know $d = 1$. A.U.

$$= 1.496 \times 10^{11} \text{ m}$$

and $V = 800 \text{ km/s}$

$$= 8 \times 10^5 \text{ m/s}$$

\therefore Time taken to reach Earth from the Sun is given by:

$$t = \frac{d}{V} = \frac{1.496 \times 10^{11} \text{ m}}{8 \times 10^5 \text{ m/s}}$$

$$= 1.870 \times 10^5 \text{ s}$$

$$= \frac{1.870 \times 10^5 \text{ s}}{60 \text{ s/min}}$$

$$= 3116.666 \text{ min}$$

$$= \frac{3116.666 \text{ min}}{60 \text{ min/hr}}$$

$$= 51.944 \text{ hours}$$

$$= \frac{51.944 \text{ hr}}{24 \text{ hr/day}}$$

$$= 2.164 \text{ days}$$

2. The rate at which the Sun emits radiation is $L_0 = 3.851 \times 10^{26} \text{ W}$
 $= 3.851 \times 10^{26} \text{ J/s}$

Over a time span of 5×10^9 years, the total amount of radiation emitted is $3.851 \times 10^{26} \text{ J/s} \times 5 \times 10^9 \text{ yrs} \times 3.1556982 \times 10^7 \text{ s/yr}$
 $= 6.076 \times 10^{43} \text{ J}$

That amount of radiation corresponds to a mass equivalent of

$$\begin{aligned} m &= E/c^2 \\ &= \frac{6.076 \times 10^{43} \text{ J}}{(2.9979 \times 10^8 \text{ m/s})^2} \\ &\approx 6.761 \times 10^{26} \text{ kg} \\ &= \frac{6.761 \times 10^{26} \text{ kg}}{1.989 \times 10^{30} \text{ kg/M}_\odot} \\ &= 0.0003399 M_\odot \end{aligned}$$

The total mass of the sun will be reduced by
 $\sim 0.034\%$. (very little)

3. The distance to a star of parallax π is given by:

$$d = \frac{1}{\pi} \quad , \text{ where } d \text{ in pc, } \pi \text{ in arcsec.}$$

here $\pi = 0.05009 \pm 0.00158$.

$$\therefore d = \frac{1}{0.05009} \text{ pc} = 19.96 \text{ pc.}$$

And $\frac{\Delta d}{d} = \frac{\Delta \pi}{\pi}$

$$\therefore \Delta d = d \times \frac{\Delta \pi}{\pi} = 19.96406468 \text{ pc} \times \frac{\pm 0.00158}{0.05009}$$
$$= 0.629730928 \text{ pc}$$

$$\therefore d = 19.96 \pm 0.63 \text{ pc.}$$

4. Aldebaran has $V = +0.87$ and $M_V = V - 5 \log d + 5$.
Hence $d = 19.96$ pc.

$$\begin{aligned} M_V &= 0.87 - 5 \log(19.96) + 5 \\ &= 0.87 - 6.5008 + 5 \\ &= -0.63 \end{aligned}$$

That value is indeed typical of a K giant, where
 $M_V \approx 0.00$.

Bonus for uncertainty:

$$\text{From } m - M = 5 \log d - 5,$$

$$\Delta(m - M) = \Delta M = 5 \log_{10} e \frac{\Delta d}{d}$$

$$\log_{10} e = 0.4343$$

$$\frac{\Delta d}{d} = \frac{\Delta \pi}{\pi} = \frac{\pm 0.00158}{0.05009}$$

$$\begin{aligned} \therefore \Delta M &= 5 \times (0.4343) \times \left(\frac{\pm 0.00158}{0.05009} \right) \\ &= \pm 0.068495 \end{aligned}$$

$$\approx \pm 0.07$$

$$\text{i.e. } M_V(\text{Aldebaran}) = -0.63 \pm 0.07$$

5. The total proper motion of Aldebaran is given by:

$$\begin{aligned}\mu &= \sqrt{(\mu_x)^2 + (\mu_y)^2} \\ &= \sqrt{(0.06278)^2 + (-0.18936)^2} \text{ arcsec/yr.} \\ &= (0.039798538)^{1/2} \text{ arcsec/yr.} \\ &= 0.199495709 \text{ arcsec/yr.}\end{aligned}$$

But $v_T = 4.74 \text{ km/s}$, for v_T in km/s, μ in arcsec/yr, change

$$\begin{aligned}\therefore v_T &= 4.74 \times 0.199495709 \times 19.96406468 \text{ km/s} \\ &= 18.87821248 \text{ km/s} \\ &= 18.88 \text{ km/s.}\end{aligned}$$

Total space velocity is given by:

$$\begin{aligned}v_{\text{space}} &= \sqrt{(v_T^2 + v_R^2)}^{1/2} \\ &= \sqrt{(18.87821248)^2 + (54)^2}^{1/2} \\ &= (356,3869 + 2916)^{1/2} \\ &= (3272,386905)^{1/2} \\ &= 57,20478044 \text{ km/s}\end{aligned}$$

i.e. $v_{\text{space}} = 57 \text{ km/s}$ (to 2 sig figs.)

6. The magnitude equation is $m_1 - m_2 = -2.5 \log b_1/b_2$
 $= -5 \log d_2/d_1$,

since brightness varies as $1/d^2$, d = distance.

Saturn has a mean distance from the Sun of 9.54 A.U., relative to Earth at 1.00 A.U.

$$\begin{aligned} V_0(\text{Earth}) - V_0(\text{Saturn}) &= -5 \log(9.54/1.00) \\ &= -5 \times 0.979548374 \\ &= -4.897741874 \\ &= -4.90 \end{aligned}$$

But $V_0(\text{Earth}) = -26.75$.

$$\begin{aligned} V_0(\text{Saturn}) &= V_0(\text{Earth} + 4.90) \\ &= -26.75 + 4.90 \\ &= -21.85 \end{aligned}$$

which is how bright the Sun will appear from the surface of one of Saturn's satellites.

7. The luminosity of a star is given by:

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

That is better expressed as a ratio:

$$\frac{L}{L_0} = \left(\frac{R}{R_0}\right)^2 \left(\frac{T_{\text{eff}}}{T_0}\right)^4$$

Here $L = 1000 L_0$, $T_{\text{eff}} = 1000 K$, and $T_0 = 5779 K$.

$$\therefore \left(\frac{1000 L_0}{L_0}\right) = \left(\frac{R}{R_0}\right)^2 \left(\frac{1000 K}{5779 K}\right)^4$$

$$\begin{aligned}\therefore \frac{R}{R_0} &= \left[\frac{1000 \times (5779)^4}{1000} \right]^{1/2} \\ &= (1.115348989 \times 10^6)^{1/2} \\ &= 1056.1\end{aligned}$$

At that time, the Sun's radius must have been
~1056 times larger than at present, i.e. $1056 R_0$

$$\begin{aligned}R &= 1056.1 R_0 \\ &= 7.35 \times 10^8 \text{ m} = 7.35 \times 10^8 \text{ km} \\ &= 4.91 \text{ A.U.} \quad (\text{outside Earth's orbit!})\end{aligned}$$

8. The combined mass of 3^4He nuclei is $4.002603 \times 3 = 12.007809$ amu.

The mass of ^{12}C is 12.000000 amu.

, The mass difference converted to energy is:

$$\frac{12.007809 - 12.000000}{12.007809} = \frac{0.007809 \text{ amu.}}{12.007809 \text{ amu.}} = 6.50 \times 10^{-4}$$

$$1 \text{ amu.} = 1.66054 \times 10^{-27} \text{ kg}$$

$$\text{One set of reactions consumes } 0.007809 \times 1.66054 \times 10^{-27} \text{ kg} \\ = 1.296715686 \times 10^{-29} \text{ kg.}$$

Energy released is $E = mc^2$

$$= (1.2967 \times 10^{-29}) \times (2.9979 \times 10^8)^2 \text{ J} \\ = 1.165410827 \times 10^{-12} \text{ J} = 1.165 \times 10^{-12} \text{ J}$$

For a star with $L = 1500 L_\odot$

$$= 1500 \times 3.851 \times 10^{26} \text{ J/s} \\ = 5.7765 \times 10^{29} \text{ J/s}$$

$1 M_\odot$ of He fuel is capable of releasing energy equivalent to a mass of $6.50 \times 10^{-4} M_\odot$ of matter.

$$\text{i.e. } E = mc^2 = (6.50 \times 10^{-4})(1.989 \times 10^{30} \text{ kg})(2.9979 \times 10^8 \text{ m/s})^2 \\ = 1.162520765 \times 10^{44} \text{ J}$$

The time taken to consume that amount of fuel at the specified rate

i.e.: $t = \text{amount of fuel} / \text{rate of consumption}$

$$= 1.162520765 \times 10^{44} \text{ J} / 5.7765 \times 10^{29} \text{ J/s}$$

$$= 2.0125 \times 10^{14} \text{ s}$$

$$= \underline{\underline{2.0125 \times 10^{14} \text{ s}}}$$

$$3.1556952 \times 10^7 \text{ s/yr}$$

$$= 6.377 \times 10^6 \text{ yrs.}$$

He-burning takes roughly 6 million years to consume $1 M_\odot$ of helium fuel.