

ASTRONOMY 1100. INTRODUCTION TO ASTROPHYSICS

Assignment 2.**Due Date: February 26, 2014.**

1. Suppose that you discover an alien planetary system in which a planet circles the parent star of the system once every 4 years at an average distance from the star of 3 A.U. Calculate the mass of the star relative to the mass of our Sun. [Hint: It is safe to assume that the planet's mass is negligibly small in comparison to the mass of the star.]
2. The bright star Altair in the constellation of Aquila (the Eagle) has a surface temperature of 7400 K. By comparison, the surface temperature of the Sun is 5779 K. Relative to the Sun, how much *more* energy is emitted per second from each square metre of Altair's surface? Show how you reached your answer.
3. During the 1970s and 1980s, balloon-borne instruments detected 511 keV photons coming from the direction of the centre of our Galaxy. (*k* means *kilo*, or thousand, so $1 \text{ keV} = 10^3 \text{ eV}$.) What is the wavelength of such photons? In what part of the electromagnetic spectrum do they originate?
4. You are given a traffic ticket for going through a red light ($\lambda = 6500 \text{ \AA}$). You tell the judge that because you were approaching the light, the Doppler effect caused a blue shift that made the light appear green ($\lambda = 5000 \text{ \AA}$). How fast would you have had to be moving for that to be true? Would the judge be justified in giving you a speeding ticket? Explain.
5. Suppose your Newtonian reflecting telescope has an objective mirror 25 cm in diameter with a focal length of 2 m. What magnification do you get with eyepieces having focal lengths of: (a) 9 mm, (b) 20 mm, and (c) 55 mm? Show how you arrived at your answers. What is the telescope's angular resolution?
6. How large in diameter is the image of the Sun (angular diameter = 30.6 arcminutes) at the focus of a telescope that has an objective with a focal length of 4.5 metres? How large would the image appear (*i.e.* What would be its magnification?) if it were viewed with an eyepiece having a focal length of 12.5 mm? Remember that there are 206265 arcseconds per radian.
7. Suppose that there is a planet in the solar system with a semi-major axis of $a = 77.2 \text{ A.U.}$, as predicted by the Titius-Bode relation. Calculate how long would it take such a planet to orbit the Sun. Suppose that the planet is an icy object with an albedo $A = 0.50$, and calculate its corresponding surface temperature. Would such a planet be expected to have an atmosphere? Why or why not?

ASTR 1100

Assignment 2.

1. According to the Newtonian version of Kepler's 3rd Law,

$$(m_1 + m_2) = a^3/P^2, \text{ for } m_1, m_2 \text{ in } M_\odot, a \text{ in A.U., } P \text{ in yrs.}$$

Here we have $m_1 = M_*$, mass of the star

$m_2 = m_p \ll M_*$, mass of the alien planet

$a = 3 \text{ A.U.}$

$P = 4 \text{ yrs.}$

$$\begin{aligned} \therefore (M_* + m_p) &\approx M_* = 3^3/4^2 M_\odot \\ &= 27/16 M_\odot \\ &= 1.6875 M_\odot \end{aligned}$$

Therefore the alien planet orbits a star that has a mass 1.69 times larger than that of our own Sun.

2. The flux from a star is given by $F = \sigma T^4$, where σ is the Stefan-Boltzmann constant and T is the temperature (in Kelvin).

Altair has $T = 7400\text{ K}$, and the Sun has $T_\odot = 5779\text{ K}$.

$$\therefore \frac{\text{Flux (Altair)}}{\text{Flux (Sun)}} = \frac{\sigma T(\text{Altair})}{\sigma T_\odot^4} = \left(\frac{7400}{5779}\right)^4$$
$$= (1.280498356)^4$$
$$= 2.69$$

In other words Altair generates 2.7 times more radiant energy per square meter at its surface than is generated by the Sun.

3. The light energy of Photons is given by $E = \frac{hc}{\lambda}$,

where h = Planck's constant $= 6.6261 \times 10^{-34} \text{ Js}$

c = speed of light $= 2.9979 \times 10^8 \text{ m/s}$

λ = wavelength, in meters.

But here $E = 511 \text{ keV}$

$$= 5.11 \times 10^5 \text{ eV}$$

$$\text{where } 1 \text{ eV} = 1.6021765 \times 10^{-19} \text{ J.}$$

∴ The wavelength of the photons is given by;

$$\lambda = \frac{hc}{E} = \frac{(6.6261 \times 10^{-34} \text{ Js})(2.9979 \times 10^8 \text{ m/s})}{(5.11 \times 10^5 \text{ eV})(1.6021765 \times 10^{-19} \text{ J/eV})}$$

$$= 2.426296493 \times 10^{-12} \text{ m } (2.43 \times 10^{-3} \text{ nm})$$

$$= 2.43 \times 10^{-10} \text{ cm}$$

$$= 2.43 \times 10^{-2} \text{ Å}$$

i.e. The photons have wavelengths of 0.0243 Å (0.00243 nm)
placing them in the gamma-ray region of the spectrum

4. The non-relativistic version of the Doppler equation is:

$$\frac{N_R}{c} = \frac{\lambda - \lambda_0}{\lambda_0}$$

Here $\lambda = 5000 \text{ \AA}$

$\lambda_0 = 6500 \text{ \AA}$

$$\therefore N_R = \left(\frac{5000 \text{ \AA} - 6500 \text{ \AA}}{6500 \text{ \AA}} \right) c$$

$$= -0.23 c$$

But $c = 2.9978 \times 10^8 \text{ m/s}$

$$\therefore N_R = -0.23 (2.9978 \times 10^8 \text{ m/s})$$

$$= -6.918 \times 10^7 \text{ m/s}$$

$$= -6.918 \times 10^7 \text{ m/s} \left(\frac{1}{1000 \text{ km/m}} \right) \left(\frac{60 \times 60}{1 \text{ hr}} \right) \text{ s/hr}$$

$$= -2.49 \times 10^5 \text{ km/hr}$$

Yes, the judge would be justified in giving you a speeding ticket.

In fact, given that speeding tickets are graduated according to the excess over the speed limit, the resulting fine may very well be astronomical.

5. The telescope has an objective mirror 25 cm in diameter, and a focal length of 2000 mm (= 2 m).

a), The magnification obtained with an eyepiece of focal

length 9 mm is: Mag = $\frac{\text{f.l. (objective)}}{\text{f.l. (eyepiece)}}$

$$= \frac{2000 \text{ mm}}{9 \text{ mm}} = 222.2 \times$$

b) The magnification for a 20 mm eyepiece is:

$$\text{Mag.} = \frac{\text{f.l. (objective)}}{\text{f.l. (eyepiece)}} = \frac{2000 \text{ mm}}{20 \text{ mm}} = 100 \times$$

c) The magnification for a 55 mm eyepiece is:

$$\text{Mag} = \frac{\text{f.l. (objective)}}{\text{f.l. (eyepiece)}} = \frac{2000 \text{ mm}}{55 \text{ mm}} = 36.4 \times$$

d) The telescope's angular resolution is given by:

$$\theta(\text{II}) = 2.5 \times 10^5 \lambda / D,$$

Here $D = 25 \text{ cm}$

$$\lambda = 5500 \text{ Å} = 5.5 \times 10^{-5} \text{ cm} \quad \text{for optical light.}$$

$$\therefore \theta(\text{II}) = \frac{(2.5 \times 10^5)(5.5 \times 10^{-5})}{25 \text{ cm}} \text{ cm}$$

$$\approx 0.55 \text{ arcsecond.}$$

The telescope has an angular resolution of $0.55''$, which is less than the seeing limitations of $1''\text{-}2''$.

6. The image scale of a telescope is given by:

$$S = \frac{206265 \text{ (")}}{f \text{ (mm)}}$$

For an image of the Sun through this telescope, $f = 4.5 \text{ m}$,
the image size is given by:

$$S = \frac{206265 \text{ "}}{4500 \text{ mm}} = 45.837 \text{ "/mm}$$

So the Sun, which has an angular diameter of
30.6 arcminutes = 30.6×60 arcseconds,

$$\text{the image diameter of } X = \frac{\theta}{S} = \frac{30.6 \times 60 \text{ "}}{45.837 \text{ "/mm}}$$

$$= 40.06 \text{ mm}$$

Its magnification if viewed through an eyepiece of 12.5 mm

would be: $\text{Mag} = \frac{\text{f.l. (objective)}}{\text{f.l. (eyepiece)}}$

$$= \frac{4500 \text{ mm}}{12.5 \text{ mm}}$$

$$= 360 \times \text{(360 times)}$$

7. According to Kepler's 3RD Law, $a^3 = P^2$ for solar system orbits, where a is in A.U., P is in years.

For a planet with a semi-major axis of $a = 77.2$ A.U., its orbital period is given by:

$$P(\text{yr}) = a^{3/2} \quad \text{for } a \text{ in A.U.}$$

$$= (77.2)^{3/2}$$

$$= 678.3 \text{ years}$$

If the planet is an icy object with $\alpha = 0.50$, its surface temperature can be calculated from:

$$T_{\text{bb}} = \frac{(1-\alpha)^{0.25} 279 \text{ K}}{r(\text{A.U.})^{0.5}} = \frac{(1-0.5)^{0.25} 279 \text{ K}}{(77.2)^{0.5}}$$

$$= 26.7 \text{ K}$$

$$\text{or } T_{\text{ss}} = \frac{(1-\alpha)^{0.25} 330 \text{ K}}{r(\text{A.U.})^{0.5}} = \frac{(1-0.5)^{0.25} 330 \text{ K}}{(77.2)^{0.5}}$$

$$= 31.6 \text{ K} \quad \text{for the substellar point of a slowly-rotating object}$$

At temperatures of 27–32 K all known common gases in the solar system are frozen out as ices on the surface of the planet, i.e. water ice, ammonia (NH_3) ice, methane (CH_4) ice, carbon dioxide (CO_2) ice,

H_2O at 273 K O_2 at 54 K CH_4 at 90 K

CO_2 at 195 K NH_3 at 195 K

Only hydrogen (H , $f_p = 14 \text{ K}$) and helium (He , $f_p = 1 \text{ K}$) might exist as gases, so the planet would not be expected to have an atmosphere.