INSTRUCTOR

ASTRONOMY 1100. INTRODUCTION TO ASTROPHYSICS

Assignment 1. **Due Date: January 29, 2014.**

1. Suppose that you live at latitude 30°N. What is the elevation of the Sun above the southern horizon (its altitude) at local noon on the date of the Winter Solstice? Explain your reasoning.

2. Where on Earth must you be located to see the south celestial pole at the zenith? Determine the maximum possible elevation of the Sun above the horizon (*i.e.* its maximum altitude) at that location. On what date is the maximum altitude of the Sun reached?

3. The star Regulus (α Leonis) is located at $10^h\ 08^m$ of right ascension and +12° declination. Determine on what date (to within a few days) Regulus will be on the meridian at local apparent midnight.

4. Suppose that an umbral eclipse of the Moon occurs on the night of December 22, the Winter Solstice. Determine approximate values for the right ascension and declination of the Moon at mid-eclipse.

5. Suppose that the Moon revolved about the Earth in the same orbit as at present, but in the opposite direction. Would the synodic month be longer or shorter than the sidereal month in such a situation? Explain your reasoning. How long would the synodic month be if such a situation existed?

6. One trajectory that can be used to send spacecraft from the Earth to Venus is an ellipse that has aphelion at the Earth and perihelion at Venus. The spacecraft is launched from Earth and coasts along such an ellipse until it reaches Venus, on the other side of the Sun from the Earth. There a rocket is fired either to put the spacecraft into orbit around Venus or to cause it to land on the surface of Venus.

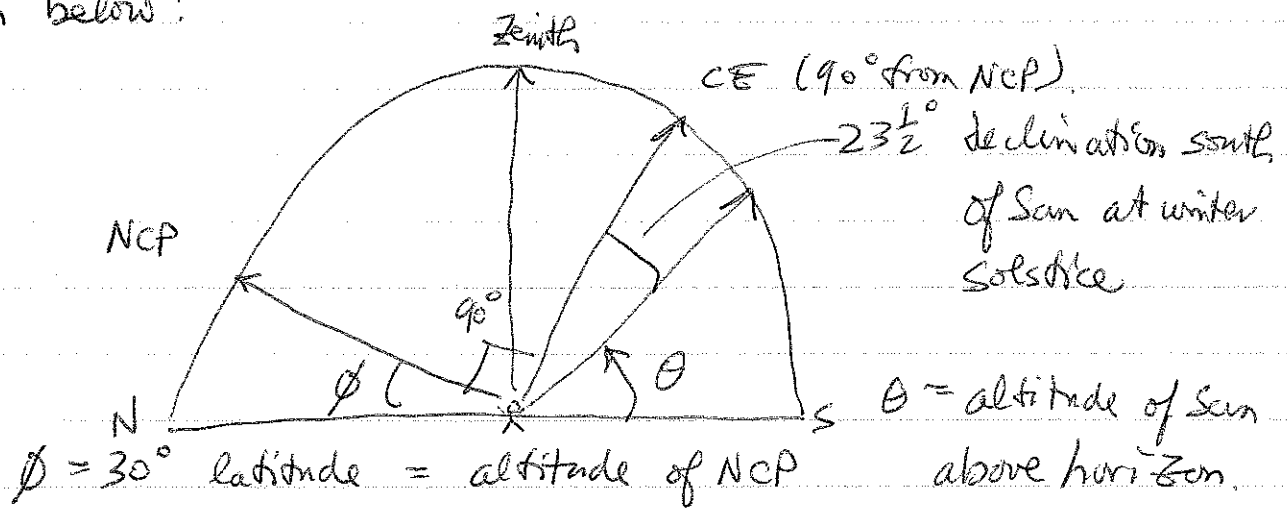
a. Calculate the semi-major axis of the ellipse corresponding to such a trajectory. Draw a picture showing such an elliptical orbit along with circles to represent (approximately) the orbits of Earth and Venus. Remember that the semi-major axis is *half* the length of the long axis of the ellipse. [The semi-major axes for Earth and Venus are 1.000 A.U. and 0.723 A.U., respectively.]

b. Calculate how long it will take in days for the spacecraft to make such a one-way trip from Earth to Venus.

ASTR 1100

Assignment 1.

1. The situation on the meridian for an observer at 30°N is shown below:



At the winter solstice the Sun is 23½° south of the celestial equator. By geometry:

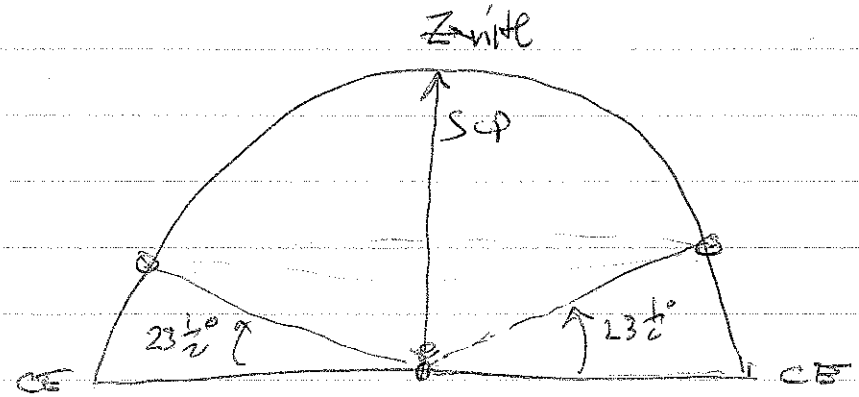
$$180° = \phi + 90° + 23\tfrac{1}{2}° + \theta$$

where $\phi = 30°$ for an observer at 30°N.

$$\therefore \theta = 180° - 90° - 23\tfrac{1}{2}° - 30°$$
$$= 36.5°$$

In other words the Sun's altitude above the southern horizon at mid-day (local noon) on the Winter solstice is 36½°.

2. For an observer to see the south celestial pole at their zenith (altitude 90°), then the altitude of the SCP of 90° must be equivalent to their latitude south, i.e. $\phi = 90°S$.
The observer must be located at the Earth's South Pole.
The Sun is located on the ecliptic and has a declination of 0° (vernal equinox), +23½° (summer solstice), 0° (autumnal equinox), and −23½° (winter solstice).
When declination (Sun) = 0°, it will appear on the horizon for an observer at the South Pole. Its maximum altitude (angle above horizon) occurs when the Sun is furthest south of the celestial equator, i.e. at the winter solstice, $\delta = -23\frac{1}{2}°$.
The situation at that time for an observer is shown below:



The maximum altitude of the Sun is therefore the absolute value of its declination at the winter solstice, i.e.
Altitude (Sun) = $|-23\frac{1}{2}°| = 23\frac{1}{2}°$,

above any point on the horizon, all day.

3. At midnight the local apparent solar time, LAST $= 0^h$.

i.e. LAST $=$ HA($\odot$) $+ 12^h = 0^h$

$\therefore$ HA($\odot$) $= 0^h - 12^h = 24^h - 12^h = 12^h$.

For Regulus (α Leo) to be on the meridian,

HA(Regulus) $= 0^h$.

But Sidereal Time $=$ RA of star on meridian $= 10^h\,08^m$.

$= $ RA($\star$) $+$ HA($\star$)

$= $ RA($\odot$) $+$ HA($\odot$)

But HA($\odot$) $= 12^h$.

$\therefore$ RA($\odot$) $=$ Sidereal Time $-$ HA($\odot$)

$= 10^h\,08^m - 12^h$

$= 34^h\,08^m - 12^h$

$= 22^h\,08^m$

But the Sun is at 0^h (24^h) RA on March 20, and moves through $\sim 2^h$ of RA per month.

So, the Sun should be at RA $= 22^h$ on February 20.

It moves through 8^m in $\sim 8/120 \times 61^d \simeq 4^d$

i.e. the Sun should be at RA $= 22^h\,08^m$ on Feb. 20 $+ 4^d$

$=$ Feb, 24.

Regulus will be on the meridian at local midnight on or about February 24.

4. There is an umbral eclipse of the Moon on the night of December 22, the winter solstice. At that time,
$RA(\odot) = 18^h$, $\delta(\odot) = -23\frac{1}{2}^\circ$.
But for an umbral eclipse of the Moon to occur, it must be 180° <u>away</u> <u>from</u> the Sun.
180° away from the Winter Solstice on the celestial sphere is the Summer Solstice, where $RA = 6^h$, $\delta = +23\frac{1}{2}^\circ$

The Moon's approximate RA and declination at mid-eclipse are therefore:

$RA(\text{Moon}) = 6^h$
$\text{Declination}(\text{Moon}) = +23\frac{1}{2}^\circ$.

5. If the Moon orbited the Earth in the same orbit as at present but in a reverse sense, its orbital period would still be 27.3215 days (see notes), but in a negative sense, i.e. $P_{rot} = -27.3215$ days.
Presumably its sense of rotation is the same as now, i.e. it always keeps one face pointed towards Earth.
A day on the Moon would then correspond to its synodic period, i.e. $P_{day} = S$ for Moon.

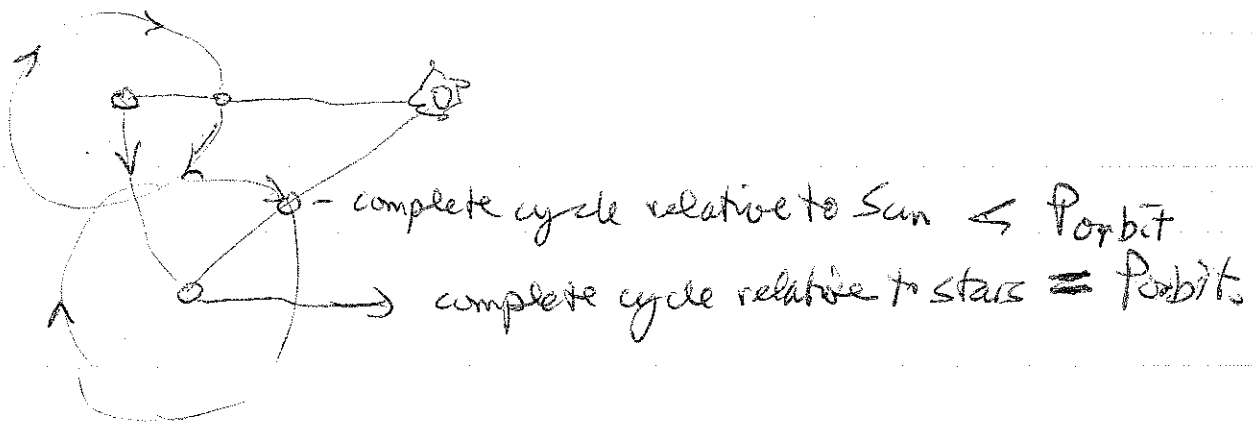
But $P_{day} = \dfrac{P_{rot} \cdot P_{sid}}{(P_{sid} - P_{rot})}$ and $P_{sid} = 365^d.2564$.

$$\therefore P_{day} = \frac{(-27^d.3215)(365^d.2564)}{(365.2564 - -27.3215)}$$

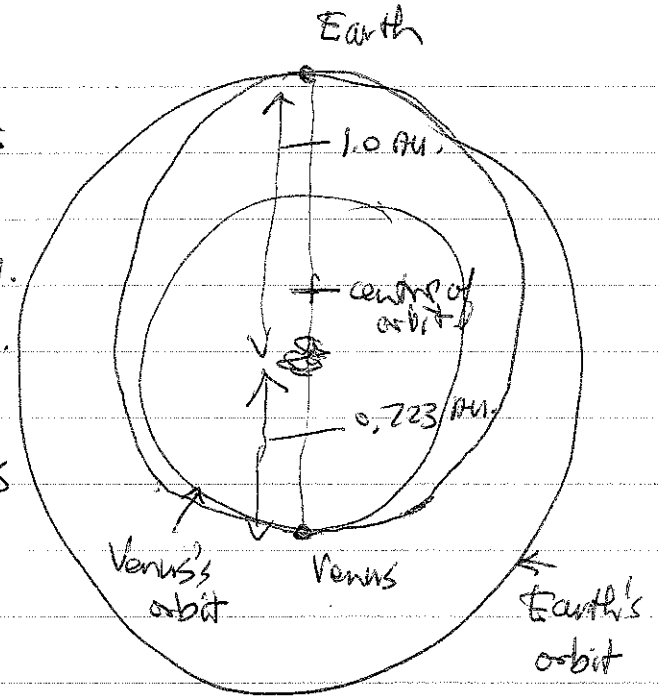
$$= \frac{(-27^d.3215)(365^d.2564)}{392^d.5779}$$

$$= -25^d.42$$

The synodic month would therefore be shorter than its present value ($29^d.5306$) and would go through a cycle of phases in reverse order to the present.
Synodic month $= 29^d.53$ (now), $= 25^d.42$ if orbit retrograde.



6. The situation for the spacecraft's trajectory is shown at right. Aphelion is at Earth, $r = 1.0$ AU. Perihelion is at Venus, $r = 0.723$ A.U.



The major axis of the spacecraft's orbit therefore has a length of $2a = 1.0$ A.U. $+ 0.723$ A.U.
$= 1.723$ A.U.

The semi-major axis, a, of the spacecraft's orbit is therefore, $a = \frac{1}{2}(2a) = \frac{1}{2}(1.723 \text{ A.U.})$
$= 0.8615$ A.U.

The orbital period is given by Kepler's 3RD Law, i.e.
$P^2 = a^3$
$\therefore P = a^{3/2} = (0.8615)^{3/2}$ year
$= 0.799618664$ year
$= 0.799618664 \times 365.^{d}256363$
$= 292.0658$ days

But the orbit is symmetrical about its major axis, it takes P (time) to go from Earth back to Earth, but only $\frac{1}{2}P$ (time) to go from Earth to Venus.
$\therefore$ Time (Earth $\rightarrow$ Venus) $= \frac{1}{2}P = \frac{1}{2}(292.^{d}0658)$
$= 146$ days $(146.^{d}033)$

, duration of one-way trip.