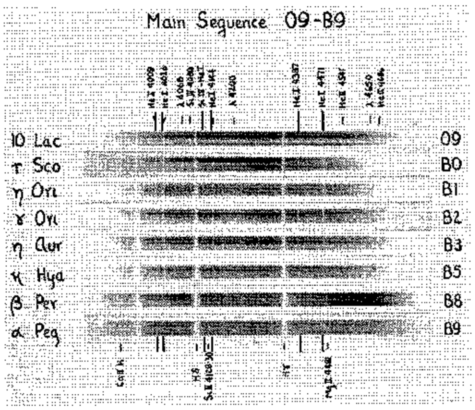
## ASTRONOMY 2400. PHYSICS OF STARS

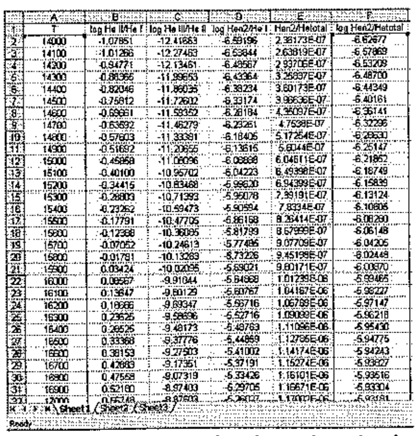
**Laboratory Exercise 2. Spectroscopic Temperatures of Stars    Due Date: February 9, 2012.**

Consider the following example of the use of the Saha and Boltzmann equations to infer surface temperatures of stars. The prominent absorption lines of neutral helium in photographic stellar spectra of B-type stars (denoted by bracketing lines in the spectra below) originate from the $n = 2\ ^3$P (triplet P) level of He I. When one examines spectra of main-sequence B-type stars, it can be noted that the lines reach maximum strength in main-sequence stars of spectral type B2. What is the corresponding temperature of main-sequence B2 stars? Input data needed are:
Excitation potential of $n = 2\ ^3$P level relative to ground state = 20.96 eV.
$u(T) = 1$ for He I. $u(T) = 2$ for He II. $u(T) = 1$ for He III.
Ionization potential for He I = 24.58 eV. Ionization potential for He II = 54.40 eV.
Statistical weights of He I energy levels: Ground State ($^1$S), $g = 1$, $n = 2$ level ($^3$P), $g = 9$.
$\log P_e$ (dynes cm$^{-1}$) = $-3.5 + 1.5 \log T$ (for hot main-sequence stars).




$$\frac{N(\text{He } 2^3\text{P})}{N(\text{He total})} = \frac{N(\text{He } 2^3\text{P})}{N(\text{He I}) + N(\text{He II}) + N(\text{He III})}$$
$$= \frac{N(\text{He } 2^3\text{P})/N(\text{He I})}{\frac{N(\text{He I})}{N(\text{He I})} + \frac{N(\text{He II})}{N(\text{He I})} + \frac{N(\text{He III})}{N(\text{He I})}}$$
$$= \frac{N(\text{He } 2^3\text{P})/N(\text{He I})}{1 + \frac{N(\text{He II})}{N(\text{He I})} + \frac{N(\text{He III})}{N(\text{He II})}\frac{N(\text{He II})}{N(\text{He I})}}$$

The solution involves setting up the appropriate line ratio using the Boltzmann and Saha relations as above right, then solving them numerically. It is less time consuming to set up a scheme in Excel or some other software routine to solve the equations for a variety of input temperatures. Typical results are shown below:



| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | T | log He II/He I | log He III/He II | log Hen2/He I | Hen2/Hetotal | log Hen2/Hetotal |
| 2 | 14000 | -1.07851 | -12.41663 | -6.59196 | 2.36173E-07 | -6.62677 |
| 3 | 14100 | -1.01266 | -12.27463 | -6.53844 | 2.63819E-07 | -6.57869 |
| 4 | 14200 | -0.94771 | -12.13461 | -6.48567 | 2.93706E-07 | -6.53209 |
| 5 | 14300 | -0.88365 | -11.99653 | -6.43364 | 3.25837E-07 | -6.48700 |
| 6 | 14400 | -0.82046 | -11.86035 | -6.38234 | 3.60173E-07 | -6.44349 |
| 7 | 14500 | -0.75812 | -11.72602 | -6.33174 | 3.96636E-07 | -6.40161 |
| 8 | 14600 | -0.69661 | -11.59352 | -6.28184 | 4.35097E-07 | -6.36141 |
| 9 | 14700 | -0.63592 | -11.46279 | -6.23261 | 4.7538E-07 | -6.32296 |
| 10 | 14800 | -0.57603 | -11.33381 | -6.18405 | 5.17254E-07 | -6.28630 |
| 11 | 14900 | -0.51692 | -11.20655 | -6.13615 | 5.6044E-07 | -6.25147 |
| 12 | 15000 | -0.45858 | -11.08096 | -6.08888 | 6.04611E-07 | -6.21852 |
| 13 | 15100 | -0.40100 | -10.95702 | -6.04223 | 6.49398E-07 | -6.18749 |
| 14 | 15200 | -0.34415 | -10.83468 | -5.99620 | 6.94399E-07 | -6.15839 |
| 15 | 15300 | -0.28803 | -10.71393 | -5.95078 | 7.39191E-07 | -6.13124 |
| 16 | 15400 | -0.23262 | -10.59473 | -5.90594 | 7.8334E-07 | -6.10606 |
| 17 | 15500 | -0.17791 | -10.47705 | -5.86168 | 8.26414E-07 | -6.08280 |
| 18 | 15600 | -0.12388 | -10.36085 | -5.81799 | 8.67999E-07 | -6.06148 |
| 19 | 15700 | -0.07052 | -10.24613 | -5.77486 | 9.07709E-07 | -6.04205 |
| 20 | 15800 | -0.01781 | -10.13283 | -5.73226 | 9.45199E-07 | -6.02449 |
| 21 | 15900 | 0.03424 | -10.02095 | -5.69021 | 9.80171E-07 | -6.00870 |
| 22 | 16000 | 0.08567 | -9.91044 | -5.64868 | 1.01239E-06 | -5.99465 |
| 23 | 16100 | 0.13647 | -9.80129 | -5.60767 | 1.04167E-06 | -5.98227 |
| 24 | 16200 | 0.18666 | -9.69347 | -5.56716 | 1.06769E-06 | -5.97147 |
| 25 | 16300 | 0.23625 | -9.58696 | -5.52716 | 1.09099E-06 | -5.96218 |
| 26 | 16400 | 0.28525 | -9.48173 | -5.48763 | 1.11096E-06 | -5.95430 |
| 27 | 16500 | 0.33368 | -9.37776 | -5.44859 | 1.12795E-06 | -5.94775 |
| 28 | 16600 | 0.38153 | -9.27503 | -5.41002 | 1.14174E-06 | -5.94243 |
| 29 | 16700 | 0.42883 | -9.17351 | -5.37191 | 1.15274E-06 | -5.93827 |
| 30 | 16800 | 0.47558 | -9.07319 | -5.33426 | 1.16101E-06 | -5.93516 |
| 31 | 16900 | 0.52180 | -8.97403 | -5.29705 | 1.16671E-06 | -5.93304 |
| 32 | 17000 | 0.56748 | -8.87603 | -5.26027 | 1.17007E-06 | -5.93181 |

Sheet1 / Sheet2 / Sheet3
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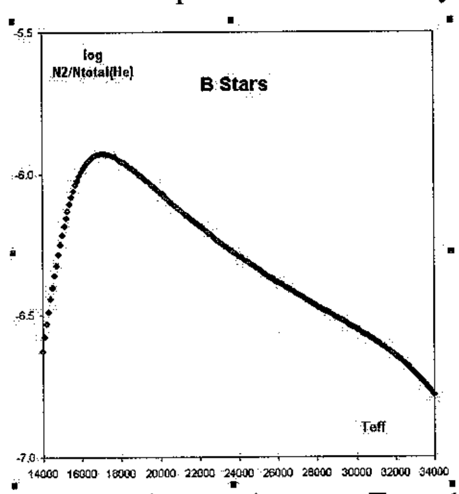
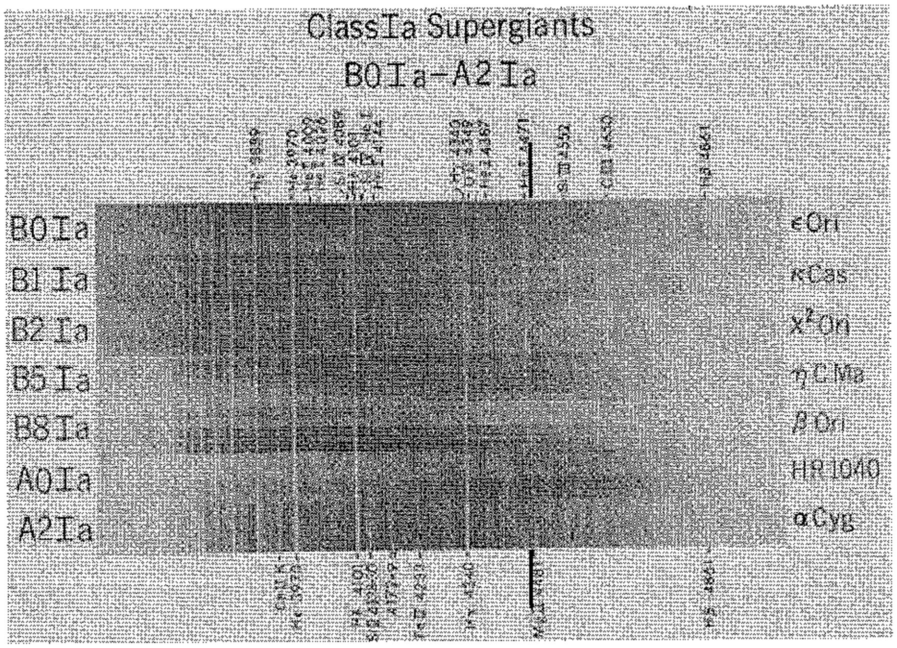



A graph of the results reveals that the desired ratio goes through a maximum at $T_{eff}$ = 17,100 K, which is therefore inferred to be the surface temperature of main-sequence B2 stars.

Now consider the following problem. Spectroscopic observations of B and A-type supergiants reveal that the prominent absorption line of singly-ionized magnesium (Mg II) at $\lambda$4481 Å (denoted by bracketing lines in the spectra below) peaks in strength for class Ia supergiants near spectral type A0, but you should judge that for yourself from the spectra. The line decreases sharply in strength for hotter stars, eventually disappearing near spectral type B2 Ia, and decreases slowly in strength for cooler stars. The $\lambda$4481 Å line of Mg II originates from the $n = 2\ ^2$D (doublet D) level, which has an energy of 8.859 eV relative to the ground state of Mg II. Use the Boltzmann and Saha relations to determine the corresponding temperature at which the Mg II lines reach maximum strength for such stars, *i.e.* establish the effective temperature $T_{\rm eff}$ for class Ia supergiants of spectral type established by you from the spectra below.




Include a graphical plot of $N_2/N_{\rm total}$ versus $T$ to confirm your results. Next refer to Appendix E of *Modern Astrophysics* by Carroll and Ostlie and compare your result with theirs (keep in mind that your result may be better!). What is the effective temperature of the stars in question?

Input data needed:
Excitation potential of $n = 2\ ^2$D level relative to ground state = 8.859 eV.
$u(T) = 1$ for Mg I. $u(T) = 2$ for Mg II. $u(T) = 1$ for Mg III. $u(T) = 2$ for Mg IV.
Ionization potential for Mg I = 7.645 eV. Ionization potential for Mg II = 15.032 eV.
Ionization potential for Mg III = 80.119 eV. Other ions are unimportant.
Statistical weights of Mg II energy levels: Ground State ($^2$S), $g = 2$, $n = 2$ level ($^2$D), $g = 4$.
$\log P_e$ (dynes cm$^{-1}$) = $-10.165 + 2.77 \log T$ (for hot supergiant stars).

Note that, while there are more ionization states for magnesium (Mg) than there are for helium (He), it is not necessary to consider more than the first four (Mg I, Mg II, Mg III, and Mg IV). You will need to set up the appropriate line ratio equation yourself. As above, the least time consuming method of solving the problem is to use computer software. See your instructor for assistance if you have never used Excel before.