ASTR 2400

Assignment 1

1. According to the observational data:

$P = 310$ days $= 310 / 365.259635$ d/orbital year

$= 0.8487113$ Earth years.

$a = 389.4 \pm 1.2\ R_\odot = \dfrac{389.4 \pm 1.2 \times 696265 \text{ km}}{1.495978715 \times 10^8 \text{ km/AU}}$

$= 1.8124 \pm 0.0056$ AU.

By Kepler's 3RD Law, $(M_1 + M_2) = a^3/P^2$, for a in AU, P in yrs, M_1 and M_2 in $M_\odot$.

$\therefore (M_1 + M_2) = \dfrac{a^3}{P^2} = \dfrac{(1.8124 \text{ AU})^3}{(0.8487113 \text{ yr})^2}$

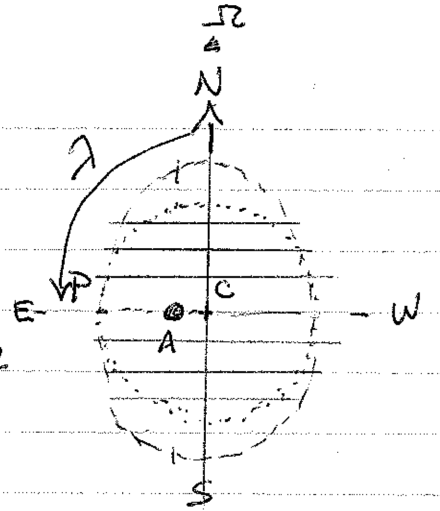
$= 8.2649873\ M_\odot\ (\pm 0.0766121\ M_\odot)$

Since $M_1 = M_2$

$\therefore M_1 = M_2 = \frac{1}{2}(M_1 + M_2) = \frac{1}{2}(8.2649873\ M_\odot)$

$= 4.13\ M_\odot\ (\pm 0.04\ M_\odot)$

2. For ASO 2313, the fact that the observations fit a circle simplifies the calculations. The orbital eccentricity is found from



$$e = \frac{CA}{CP} = \frac{ae\cos i}{a\cos i} = \frac{1.''0}{3.''5} = 0.2857142$$

Bisected chords running parallel to CAP are connected to produce the projected semi-minor axis, which is perpendicular (90°) to CAP. Extending it by the factor $k = 1/(1-e^2)^{1/2} = 1/[1-(0.2857)^2]^{1/2}$

$= 1/(0.9183673)^{1/2} = 1/0.9583$

$= 1.0434984$

produces the auxiliary ellipse of major axis length $\alpha = 1.0434984 \times 3.''5$

$= 3.''6522444$

$= a$

Semi-minor axis of the auxiliary ellipse has length $\beta = 3.''5$

$\therefore \cos i = \frac{\beta}{\alpha} = \frac{3.''5}{3.''6522444} = 0.9583148$, $\therefore i = \cos^{-1}(0.9583148)$

$= 16.°60155$

By geometry, $\Omega = 0°$, line of nodes runs NS, and $\lambda = 90°$.

$\therefore \tan \omega = \frac{\tan \lambda}{\cos i} = \frac{\tan 90°}{\cos 16.°60} = \infty$, $\therefore \omega = \tan^{-1} \infty = 90°$.

The time between successive periastron passages is $1998.481 - 1982.261 = 16.22$ yrs $= 1982.261 - 1966.041$, i.e. $P = 16.22$ years. The most recent epoch should be 1998.481, since the next is in 2014.701.

i.e. $\omega = 90°$, $i = 16.°60$, $P = 16.22$ years, $T = 1998.481$, $a = 3.''65$, $e = 0.286$

Also, $(M_1 + M_2)$ in $M_\odot = [a('') / \pi('')]^3 / P(\text{yr})^2$

$= [3.6522444/0.297]^3/(16.22)^2$

$= (12.297119)^3/(16.22)^2$

$= 7.0681938$

The combined mass of the stars in the system is $7.07\ M_\odot$

3. Since $i = 90°$ for the orbit, $\sin i = 1$ for all parameters of the stars.

$\therefore a_1 \sin i = a_1 = 21{,}600\ P(\text{days})\ 2K\ (1-e^2)^{1/2}/\pi$ km

$= 21{,}600\,(3.96)(2\times 108)(1-e^2)^{1/2}/\pi$

$= 5.8810222\times 10^6$ km $(1-e^2)^{1/2}$

$a_2 \sin i = a_2 = 21{,}600\,(3.96)(2\times 111)\,(1-e^2)^{1/2}/\pi$ km

$= 6.0443839\times 10^6$ km $(1-e^2)^{1/2}$

We do not know e, but assume that it is zero, i.e. $e = 0.00$.

Then, a (orbit) $= a_1 + a_2$

$= 5.8810222\times 10^6$ km $+ 6.0443839\times 10^6$ km

$= 1.1925406\times 10^7$ km $/ 1.4960\times 10^8$ km/A.U.

$= 0.0797152$ A.U.

And $P = 3.96^d = 3.96^d / 365.259635\ {}^d/\text{yr} = 0.0108416$ year

$\therefore \mathcal{M}_1 + \mathcal{M}_2 = a^3/P^2$ for a in A.U., P in yr.

$= (0.0797152)^3/(0.0108416)^2$

$= 4.3096075\ \mathcal{M}_\odot$

But $\dfrac{\mathcal{M}_1}{\mathcal{M}_2} = \dfrac{K_2}{K_1} = \dfrac{111 \text{ km/s}}{108 \text{ km/s}} = 1.0277778$

$\therefore (1 + 1.0277778)\,\mathcal{M}_2 = 4.3096075\ \mathcal{M}_\odot$

$\therefore \mathcal{M}_2 = \dfrac{4.3096075}{2.0277778}\ \mathcal{M}_\odot = 2.1252859\ \mathcal{M}_\odot$

And $\mathcal{M}_1 = 1.0277778 \times 2.1252859\ \mathcal{M}_\odot = 2.1843217\ \mathcal{M}_\odot$

i.e. $\mathcal{M}_1 = 2.18\ \mathcal{M}_\odot$, $\mathcal{M}_2 = 2.13\ \mathcal{M}_\odot$

Also $(\mathcal{M}_1 + \mathcal{M}_2)\sin^3 i = 10.38\times 10^{-8}\, P\,(1-e^2)^{3/2}\,(K_1 + K_2)^3$

$= 10.38\times 10^{-8}(3.96)\,(1-e^2)^{3/2}\,(108 + 111)^3\ \mathcal{M}_\odot$

$\therefore \mathcal{M}_1 + \mathcal{M}_2 = 4.3174\ \mathcal{M}_\odot\ (1-e^2)^{3/2}$

Since this result is the same as the result obtained by assuming that $e = 0.00$, the orbital eccentricity must be zero.

4. By the mass-luminosity relation, use either:

$$\frac{L}{L_\odot} = \left(\frac{m}{m_\odot}\right)^4$$

or $\log L/L_\odot = 4.20 \sin(\log m/m_\odot - 0.281) + 1.174$

Here $m = 4\,m_\odot$.

By the simple 1st equation, $L/L_\odot = (m/m_\odot)^4$

$$= 4^4$$

$$= 256$$

i.e. the star is 256 times more luminous than the Sun.

By the 2ND equation, $\log \frac{L}{L_\odot} = 4.20 \sin\left(\log \frac{m}{m_\odot} - 0.281\right) + 1.174$

i.e. $\log \frac{L}{L_\odot} = 4.20 \sin(\log 4 - 0.281) + 1.174$

$$= 4.20 \sin(0.6020599 - 0.281) + 1.174$$

$$= 4.20 \sin(0.3210599 \text{ radians}) + 1.174$$

$$= 4.20 \sin(18^\circ.395382) + 1.174$$

$$= 4.20\,(0.3155725) + 1.174$$

$$= 1.3254048 + 1.174 = 2.4994048$$

$\therefore L/L_\odot = 10^{2.4994048} = 315.8$

i.e the star is 316 times more luminous than the Sun.
In essence, both relationships indicate that the star is ~300 times more luminous than the Sun.