ASTRONOMY 2400 PHYSICS OF STARS

Take-home Mid-Term Test, February 2012. Name ________________________________

1. Short answer questions. Fill in the blanks or provide brief answers:

   a. Primary minimum for an eclipsing binary system corresponds to an eclipse of the ________________________________ (larger, smaller, hotter, cooler) star in the system. (Select the correct response.)

   b. In its simplest form, the mass-luminosity relation tells us that the luminosity of any star in solar units is proportional to its mass in solar units roughly to the ____________ power.

   c. The orbital inclination i of a binary system is measured in what sense: relative to our line of sight, relative to the plane of the sky, relative to the line of nodes, in some other fashion, etc.?

   d. What is a gray stellar atmosphere?

   e. Why are partition functions for atoms and ions always numerically close to the statistical weight for the ground state of the same atoms and ions?

   f. What does the fact that the triplet lines of neutral helium (He I) reach their greatest strength in dwarf B2 V stars tell us about B2 V stars?

   g. Where in stellar atmospheres are spectral lines formed?

   h. The lines of neutral iron (Fe I) are strong in G-type stars because their atmospheres contain lots of iron, more iron than any other element. True or false? Explain.

   i. LTE stands for ____ .

   j. The spectroscopic designation B2 Iae tells us that a star is:

2. A visual binary system consists of two identical dwarf stars orbiting each other with a period of 48.44 years. The absolute parallax of the system is π = 0.192 arcsecond and the deprojected length of the system's semi-major axis is a = 3.68 arcsecond.

   a. What are the masses of the individual stars in the system?

   b. What are the likely spectral types of the two stars in the system?

   b. What are the approximate surface temperatures of the two stars in the system?

3. An optical depth of $\tau_\lambda = 10$ corresponds to a very deep level in stellar atmospheres. Calculate how many times a typical photon has been scattered before it escapes a star from such a level?

4. At what temperature is the $n = 3$ level of hydrogen, $\chi_{ex} = 12.08$ eV, as well populated as the ground state of hydrogen? (Note: $g_3 = 18$, $g_1 = 2$ for hydrogen.)

5. According to the Eddington approximation for a gray atmosphere, from what optical depth in a stellar atmosphere do the emitted photons originate from gas at a temperature that is exactly twice the star's effective temperature?

6. Demonstrate that limb darkening in stars can often be approximated closely by an equation of the form:

$$I(\theta) = I_o(1 - x + x\cos\theta)$$

where x is the limb-darkening coefficient. What value of x works best for the visual limb-darkening in the Sun? Explain the phenomenon of limb-darkening in terms of the radiance of hot gases.

Formulae

Boltzmann's Law:

$\log \frac{N_m}{N_n} = -\theta \chi_{mn} + \log \frac{g_m}{g_n}$, or $\log \frac{N_m}{N} = -\theta \chi_m + \log \frac{g_m}{u(T)}$, where N_m = number of atoms in level m, N_n = number of atoms in level n, g_m = statistical weight of level m, g_n = statistical weight of level n, χ_{mn} = excitation energy of level m with respect to level n, $\theta = 5040/T$, $u(T)$ is the partition function, k = Boltzmann's constant $=1.38065 \times 10^{-23}$ J K^{-1} = 8.6167×10^{-5} eV K^{-1}, and T = temperature in Kelvins. For hydrogen, $g_n = 2n^2$.

Saha Ionization Equation:

$\log \frac{N^{n+1}}{N^n} = 2.5 \log T - \theta I_n - \log P_e - 0.4771 + \log \left[\frac{2 u_{n+1}(T)}{u_n(T)} \right]$, where I_n = ionization potential from the nth state, N^{n+1} = number of atoms in the (n+1)th ionization state, N^n = number of atoms in the nth ionization state, and the electron pressure is given by $P_e = N_e kT$ (in dynes cm^{-2}). A simplified form of the Saha Equation is: $\frac{N^{n+1}}{N^n} P_e = \Phi(T)$.

Kepler's Third Law: $(M_1 + M_2)$ (in $M_\odot$) $= a^3/P^2$, for a in A.U. and P in years. The semi-major axis for visual binaries is given by: $a(\text{A.U.}) = \frac{a(\text{arcsec})}{\pi(\text{arcsec})}$.

Stellar luminosity: $L = 4\pi R^2 \sigma T_{eff}^4$, R = stellar radius, T_{eff} = effective temperature.

Magnitude relationship: $m_1 - m_2 = -2.5 \log \left(\frac{b_1}{b_2} \right)$

Stellar Masses: 32 $M_\odot$ (O5), 14 $M_\odot$ (B0), 2 $M_\odot$ (A0), 1.5 $M_\odot$ (F0), 1.0 $M_\odot$ (G2), 0.8 $M_\odot$ (K0), 0.4 $M_\odot$ (M0).

Temperature Distribution:
Plane parallel gray stellar atmosphere in LTE in the Eddington approximation:
$T^4 = \frac{3}{4} T_{eff}^4 \left(\tau_v + \frac{2}{3} \right)$, where τ_v is the vertical optical depth.

Optical depth: $\tau_\lambda = \sqrt{N}$, where N is the number of scatters.