ASTRONOMY 2400 PHYSICS OF STARS

Take-home Mid-Term Test, February 2012. Name INSTRUCTOR

1. Short answer questions. Fill in the blanks or provide brief answers:

   a. Primary minimum for an eclipsing binary system corresponds to an eclipse of the HOTTER (larger, smaller, hotter, cooler) star in the system. (Select the correct response.)

   b. In its simplest form, the mass-luminosity relation tells us that the luminosity of any star in solar units is proportional to its mass in solar units roughly to the 4TH power.

   c. The orbital inclination *i* of a binary system is measured in what sense: relative to our line of sight, relative to the plane of the sky, relative to the line of nodes, in some other fashion, etc.?

      RELATIVE TO THE PLANE OF THE SKY

   d. What is a gray stellar atmosphere? A GRAY ATMOSPHERE IS ONE IN WHICH THE OPACITY OF THE GAS, κ_ν, IS INDEPENDENT OF WAVELENGTH, I.E. IT HAS THE SAME, CONSTANT VALUE OVER ALL WAVELENGTHS.

   e. Why are partition functions for atoms and ions always numerically close to the statistical weight for the ground state of the same atoms and ions?

      THE PARTITION FUNCTION EXPRESSES HOW VARIOUS ATOMIC ENERGY LEVELS ARE POPULATED, AND SINCE MOST ATOMS AND IONS EXIST MOSTLY IN THE GROUND STATE (LOWEST ENERGY STATE), $u(T) \simeq g$ (GROUND STATE)

   f. What does the fact that the triplet lines of neutral helium (He I) reach their greatest strength in dwarf B2 V stars tell us about B2 V stars?

      IT TELLS US THAT B2 V STARS HAVE $T \sim 17{,}000$ K, WHICH IS WHERE THE SAHA AND BOLTZMANN EQUATIONS TELL US THAT THE HELIUM TRIPLET LINES ARE AT MAXIMUM STRENGTH.

   g. Where in stellar atmospheres are spectral lines formed?

      IN THE HIGHEST LEVELS OF THE ATMOSPHERE, JUST BEFORE THE RADIATION ESCAPES INTO SPACE

   h. The lines of neutral iron (Fe I) are strong in G-type stars because their atmospheres contain lots of iron, more iron than any other element. True or false? Explain.

      FALSE, THE LINES ARE STRONG BECAUSE THE TEMPERATURES ARE OPTIMUM FOR THE FORMATION OF Fe I LINES. HYDROGEN IS MUCH MORE ABUNDANT

   i. LTE stands for LOCAL THERMODYNAMIC EQUILIBRIUM.

   j. The spectroscopic designation B2 Iae tells us that a star is:

      A HOT STAR, B2 OF THE SPECTRAL TYPE B SEQUENCE, A LUMINOUS, CLASS Ia, SUPERGIANT, AND ALSO AN EMISSION-LINE STAR.

2. A visual binary system consists of two identical dwarf stars orbiting each other with a period of 48.44 years. The absolute parallax of the system is π = 0.192 arcsecond and the deprojected length of the system's semi-major axis is a = 3.68 arcsecond.

   a. What are the masses of the individual stars in the system?

ACCORDING TO KEPLER'S 3RD LAW, $(m_1 + m_2) = a^3/P^2$

HERE $a = \frac{a('')}{\pi('')}$

$\therefore (m_1 + m_2)$ in $m_\odot$ $= \left(\frac{3.''68}{0.''192}\right)^3 / (48.44)^2$

$= 3.00\ m_\odot$

SINCE THE STARS ARE IDENTICAL, $m_1 = m_2$

$\therefore 2m_1 = 3.00\ m_\odot$

$\therefore m_1 = m_2 = \frac{3.00}{2} m_\odot = 1.50\ m_\odot$

   b. What are the likely spectral types of the two stars in the system?

THE STARS ARE KNOWN TO BE DWARFS, SO ARE OF LUMINOSITY CLASS V.

1.5 $m_\odot$ STARS CORRESPOND TO SPECTRAL TYPE F0 (HANDOUT).

$\therefore$ THE LIKELY SPECTRAL TYPE OF BOTH STARS IS F0 V

   b. What are the approximate surface temperatures of the two stars in the system?

EFFECTIVE TEMPERATURE FOR F0 V STARS $\simeq$ 6900 K ACCORDING TO INSTRUCTOR CALIBRATION (ON-LINE),

7300 K ACCORDING TO CARROLL AND OSTLIE

LIKE SURFACE TEMPERATURE ~ 7000 K OR COOLER

3. An optical depth of $\tau_\lambda = 10$ corresponds to a very deep level in stellar atmospheres. Calculate how many times a typical photon has been scattered before it escapes a star from such a level?

BY THE DEFINITION OF OPTICAL DEPTH, $\tau_\lambda = \sqrt{N}$,

WHERE N IS THE NUMBER OF SCATTERS.

FOR $\tau_\lambda = 10$, $\sqrt{N} = 10$.

$\therefore N = 10^2 = 100$ SCATTERS, WHICH IS HOW MANY TIMES A PHOTON MUST SCATTER OFF GAS ATOMS BEFORE IT ESCAPES A STAR FROM AN OPTICAL DEPTH OF $\tau_\lambda = 10$.

4. At what temperature is the $n = 3$ level of hydrogen, $\chi_{ex} = 12.08$ eV, as well populated as the ground state of hydrogen? (Note: $g_3 = 18$, $g_1 = 2$ for hydrogen.)

ACCORDING TO THE BOLTZMANN FORMULATION:

$$\log \frac{N_3}{N_1} = -\theta \chi_{31} + \log \frac{g_3}{g_1}$$ FOR HYDROGEN, WHERE

$\theta = 5040/T$.

HERE, $\chi_{31} = 12.08$ eV, $g_3 = 18$, $g_1 = 2$, T IS UNKNOWN

i.e. $$\log \frac{N_3}{N_1} = \log 1 = -\frac{5040}{T} \times (12.08) + \log\left(\frac{18}{2}\right)$$

$$\therefore 0 = -\frac{60883.2}{T} + \log 9 = -\frac{60883.2}{T} + 0.9542425$$

$$\therefore T = \frac{60883.2}{0.9542425} = 63,803 \text{ K}$$

THAT IS THE TEMPERATURE AT WHICH THE 1ST AND 3RD ENERGY LEVELS OF HYDROGEN ARE EQUALLY WELL POPULATED.

5. According to the Eddington approximation for a gray atmosphere, from what optical depth in a stellar atmosphere do the emitted photons originate from gas at a temperature that is exactly twice the star's effective temperature?

ACCORDING TO THE EDDINGTON APPROXIMATION:

$$T^4 = \frac{3}{4} T_{eff}^4 \left(\tau_\nu + \frac{2}{3}\right).$$

WE ARE ASKED TO FIND τ_ν FOR $T = 2\,T_{eff}$

$$\therefore (2T_{eff})^4 = \frac{3}{4} T_{eff}^4 (\tau_\nu + 2/3) = 16\, T_{eff}^4$$

$$\therefore \tau_\nu + \frac{2}{3} = \frac{16\, T_{eff}^4}{3/4\, T_{eff}^4} = \frac{16 \times 4}{3} = 21\tfrac{1}{3}$$

$$\therefore \tau_\nu = 21\tfrac{1}{3} - 2/3 = 20\tfrac{2}{3} \simeq 20.7$$

A PHOTON FROM A STELLAR ATMOSPHERE ORIGINATING FROM GAS AT $T = 2T_{eff}$ ORIGINATES AT $\tau = 20.7$.

6. Demonstrate that limb darkening in stars can often be approximated closely by an equation of the form:

$$I(\theta) = I_0(1 - x + x\cos\theta)$$

where x is the limb-darkening coefficient. What value of x works best for the visual limb-darkening in the Sun? Explain the phenomenon of limb-darkening in terms of the radiance of hot gases.

ACCORDING TO THE EDDINGTON APPROXIMATION AND ITS $T(\tau_\nu)$ APPROXIMATION, THE SOLAR LIMB DARKENING SHOULD BE DESCRIBED BY

$$\frac{I(\theta)}{I_0} = 0.4 + 0.6\cos\theta,$$

WHERE I_0 IS THE INTENSITY AT DISK CENTRE.

THIS CAN BE ARRANGED TO:

$$\begin{aligned} I(\theta) &= I_0\,(0.4 + 0.6\cos\theta) \\ &= I_0\left[(1 - 0.6) + 0.6\cos\theta\right] \\ &= I_0\left[(1 - x) + x\cos\theta\right] \end{aligned}$$

IN GENERIC TERMS.

A VALUE OF $x = 0.6$ WORKS REASONABLY WELL FOR THE SUN, BUT A VALUE OF $x = 0.55$ IS BETTER

LIMB DARKENING OCCURS BECAUSE THE LINE OF SIGHT TO THE SUN AT DISK CENTRE PENETRATES TO DEEPER LAYER OF ITS ATMOSPHERE THAN NEAR THE LIMB. DEEPER LAYERS ARE HOTTER AND MORE RADIANT (RADIANCE ~ T^4), SO APPEAR BRIGHTER THAN THE COOLER UPPER LAYERS

Formulae

Boltzmann's Law:

$\log\frac{N_m}{N_n} = -\theta\chi_{mn} + \log\frac{g_m}{g_n}$, or $\log\frac{N_m}{N} = -\theta\chi_m + \log\frac{g_m}{u(T)}$, where N_m = number of atoms in level m, N_n = number of atoms in level n, g_m = statistical weight of level m, g_n = statistical weight of level n, χ_{mn} = excitation energy of level m with respect to level n, $\theta = 5040/T$, $u(T)$ is the partition function, k = Boltzmann's constant $=1.38065 \times 10^{-23}$ J K^{-1} $= 8.6167 \times 10^{-5}$ eV K^{-1}, and T = temperature in Kelvins. For hydrogen, $g_n = 2n^2$.

Saha Ionization Equation:

$$\log\frac{N^{n+1}}{N^n} = 2.5\log T - \theta I_n - \log P_e - 0.4771 + \log\left[\frac{2u_{n+1}(T)}{u_n(T)}\right]$$

, where I_n = ionization potential from the n$^{\text{th}}$ state, N^{n+1} = number of atoms in the (n+1)$^{\text{th}}$ ionization state, N^n = number of atoms in the n$^{\text{th}}$ ionization state, and the electron pressure is given by $P_e = N_e kT$ (in dynes cm^{-2}). A simplified form of the Saha Equation is: $\frac{N^{n+1}}{N^n}P_e = \Phi(T)$.

Kepler's Third Law: $(M_1 + M_2)$ (in $M_\odot$) $= a^3/P^2$, for a in A.U. and P in years. The semi-major axis for visual binaries is given by: $a(\text{A.U.}) = \frac{a(\text{arcsec})}{\pi(\text{arcsec})}$.

Stellar luminosity: $L = 4\pi R^2\sigma T_{\text{eff}}^4$, R = stellar radius, T_{eff} = effective temperature.

Magnitude relationship: $m_1 - m_2 = -2.5\log\left(\frac{b_1}{b_2}\right)$

Stellar Masses: 32 $M_\odot$ (O5), 14 $M_\odot$ (B0), 2 $M_\odot$ (A0), 1.5 $M_\odot$ (F0), 1.0 $M_\odot$ (G2), 0.8 $M_\odot$ (K0), 0.4 $M_\odot$ (M0).

Temperature Distribution:
Plane parallel gray stellar atmosphere in LTE in the Eddington approximation:

$T^4 = \frac{3}{4}T_{\text{eff}}^4\left(\tau_v + \frac{2}{3}\right)$, where τ_v is the vertical optical depth.

Optical depth: $\tau_\lambda = \sqrt{N}$, where N is the number of scatters.