ASTRONOMY 2400 PHYSICS OF STARS

Final Take-home Examination, April 2012. Name ______________________________

Answer all questions on the test pages, using backs of pages for overflow. All work must be your own. Answers should be complete but concise.

1. Short answer questions. Fill in the blanks:

   a. Nearly all of the binary systems used to establish the mass-luminosity relation are eclipsing systems. Very few are visual binary systems. The reason why so few visual binary systems can be used for that purpose is because ______________________________

   ____.

   b. The "helium flash" is what happens when ______________________________

   ____

   ____.

   c. What is meant by the term effective temperature? ______________________________

   ____

   d. A star whose spectrum exhibits absorption lines of singly-ionized helium (He II) would be assigned a spectral type of ______________________________ .

   e. Most of the photons that we receive from the Sun originate from an optical depth (τ_λ) of ____ in the photosphere.

   f. On the linear portion of the curve of growth the equivalent width of a spectral line is proportional to ____ .

   g. The *Rosseland mean* refers to the ______________________________ in stellar atmospheres and interiors.

   h. The *Kelvin-Helmholtz time scale* refers to ______________________________

   ____.

   i. A polytropic equation of state, one where $P = K\rho^\gamma$, is used in modelling stellar interiors to describe the gas in regions where ______________________________ (heat transport mechanism) dominates.

   j. A common characteristic of the cores of all main-sequence stars is that they __________

   ____

   ____.

2. a. Explain the origin of the Balmer lines of hydrogen in stellar spectra, and why they appear to reach their maximum strength in stars of spectral type A0. Specifically, since upper electronic levels in atoms become more heavily populated with increasing temperature, why do the Balmer lines not appear at maximum strength in the hottest known stars?

b. Supergiant stars classified spectroscopically as A0 Ia actually have lower effective temperatures than dwarfs stars classified spectroscopically as A0 V. Explain how such a situation can exist. Isn't the spectral type of a star, in this case "A0," directly related to a star's effective temperature?

3. a. Draw schematically the general curve of growth for the Sun. Label the two axes and indicate the parameters plotted as well as the direction in which the quantities increase. Label the three main sections of the curve, and indicate numerically the nature of the functional dependence in each portion.

b. Redraw the general curve of growth for a star like the Sun that you illustrated in part (a). Now illustrate schematically how the curve changes for a star that has the same effective temperature as the Sun but a surface gravity several orders of magnitude smaller, *i.e.* a supergiant with much lower gas pressure and much larger microturbulence.

c. Why is it not possible to use the general curve of growth for the Sun to derive the abundance of helium in the solar atmosphere?

4. a. Calculate the mean molecular weight μ_i of a fully ionized gas that consists of the following proportion of elements by mass: 73% hydrogen, 25% helium, and 2% heavy elements.

b. Derive an expression for the hydrogen-burning lifetime of a star in years as a function of its mass in solar units. [Assume that the hydrogen-burning lifetime for the Sun is 10^{10} years.]

c. What is the approximate main-sequence lifetime for the star Sirius, which has a spectral type of A1 V?

5. a. Complete the reaction sequence given below and justify the choice of missing nucleon or lepton according to the principles of conservation of electric charge, conservation of total nucleons, conservation of spin, and conservation of lepton number.

$$^{7}\mathrm{Be} + e^{-} \rightarrow {}^{7}\mathrm{Li} + ?$$

b. The reaction sequence illustrated in part (a) is part of a chain of reactions by which protons are converted into ^{4}He nuclei to generate the energy required to sustain the internal heating and pressure in the Sun. Some of the reactions in the chain produce neutrinos, which have been searched for since 1970 by various solar neutrino experiments. Such experiments have led to what is called the "solar neutrino deficit." What is the nature of the deficit?

6. a. Construct a theoretical H-R diagram to illustrate schematically the location of the main sequence for hydrogen-burning model stars of solar metallicity. Next indicate in the same diagram the main sequence for hydrogen-burning model stars composed of a significantly lower proportion of heavy elements.

b. Is it possible to use main-sequence fitting, where one fits the observed brightnesses and colours of cluster stars on the main sequence to a standard relationship derived for solar metallicity stars, for stars in globular clusters? Explain.

7. a. Derive an expression for the central temperature of a star T_C in terms of its mass and radius.

b. Use your relationship from part (a) to calculate the expected temperature of the gas at the centre of the Sun ($M_\odot = 1.989 \times 10^{33}$ g, $R_\odot = 6.9598 \times 10^{10}$ cm).

8. One method of modelling stellar interiors is to rewrite the equations of continuity, hydrostatic equilibrium, and energy generation in terms of the mass interior to a specific radius rather than in terms of radius in the stellar interior. Show that the equations expressed in such a manner must be:

$$\frac{dr}{dM_r} = \frac{1}{4\pi r^2 \rho} \qquad \frac{dP}{dM_r} = -\frac{GM_r}{4\pi r^4} \qquad \frac{dL_r}{dM_r} = \varepsilon$$

Formulae

Boltzmann's Law:

$\log \frac{N_m}{N_n} = -\theta \chi_{mn} + \log \frac{g_m}{g_n}$, or $\log \frac{N_m}{N} = -\theta \chi_m + \log \frac{g_m}{u(T)}$, where N_m = number of atoms in level m, N_n = number of atoms in level n, g_m = statistical weight of level m, g_n = statistical weight of level n, χ_{mn} = excitation energy of level m with respect to level n, $\theta = 5040.1/T$, $u(T)$ is the partition function, k = Boltzmann's constant $= 1.38065 \times 10^{-23}$ J K^{-1} $= 8.6167 \times 10^{-5}$ eV K^{-1}, and T = temperature in Kelvins. For hydrogen, $g_n = 2n^2$.

Saha Ionization Equation:

$\log \frac{N^{n+1}}{N^n} = 2.5 \log T - \theta I_n - \log P_e - 0.4772 + \log \left[\frac{2 u_{n+1}(T)}{u_n(T)} \right]$, where I_n = ionization potential from the nth state, N^{n+1} = number of atoms in the (n+1)th ionization state, N^n = number of atoms in the nth ionization state, and the electron pressure is given by $P_e = N_e kT$ (in dynes cm^{-2}). A simplified form of the Saha Equation is: $\frac{N^{n+1}}{N^n} P_e = \Phi(T)$.

Kepler's Third Law: $(M_1 + M_2)$ (in $M_\odot$) $= a^3/P^2$, for a in A.U. and P in years. The semi-major axis for visual binaries is given by: $a(\text{A.U.}) = \frac{a(\text{arcsec})}{\pi(\text{arcsec})}$.

Stellar luminosity: $L = 4\pi R^2 \sigma T_{\text{eff}}^4$, R = stellar radius, T_{eff} = effective temperature.

Magnitude relationship: $m_1 - m_2 = -2.5 \log \left(\frac{b_1}{b_2} \right)$

Stellar Masses: 32 $M_\odot$ (O5), 14 $M_\odot$ (B0), 2 $M_\odot$ (A0), 1.5 $M_\odot$ (F0), 1.0 $M_\odot$ (G2), 0.8 $M_\odot$ (K0), 0.4 $M_\odot$ (M0).

Temperature Distribution:
Plane parallel gray stellar atmosphere in LTE in the Eddington approximation:
$T^4 = \frac{3}{4} T_{\text{eff}}^4 \left(\tau_v + \frac{2}{3} \right)$, where τ_v is the vertical optical depth.

Optical depth: $\tau_\lambda = \sqrt{N}$, where N is the number of scatters.

Perfect Gas Law: $P_g = \frac{\rho k T}{\mu m_H}$, where T is the temperature, k is Boltzmann's constant = 1.380658×10^{-16} erg/K, r is the density, m_H is the mass of a hydrogen atom $= 1.673534 \times 10^{-24}$ g, and m is the mean molecular weight.

Mean Molecular Weight: $\mu_i = \dfrac{1}{2X + \frac{3}{4}Y + \frac{1}{2}Z}$, for completely ionised gas, where X is the mass fraction of hydrogen, Y is the mass fraction of helium, and Z is the mass fraction of heavy elements.

Stellar Interior Equations:

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2},$$ hydrostatic equilibrium

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho,$$ conservation of mass

$$\frac{dL_r}{dr} = 4\pi r^2 \rho\varepsilon,$$ energy generation

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\overline{\kappa\rho}}{T^3}\frac{L_r}{4\pi r^2},$$ temperature gradient (radiative transport)

$$\frac{dT}{dr} = -\left(\frac{\gamma-1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM(r)}{r^2},$$ temperature gradient (convective transport)

Simplified Opacity:

$\overline{\kappa} \propto \dfrac{\rho(1+X)}{T^{3.5}}$, but also depends on heavy element abundance, Z.

Simplified Nuclear Reaction Rates:

$\varepsilon_{pp} \propto X^2 \rho T^4$, proton-proton chain

$\varepsilon_{CNO} \propto Z_{CNO} X \rho T^{20}$, CNO cycle

$\varepsilon_{3\alpha} \propto Y^3 \rho^3 T^{41}$, triple alpha process

Atomic Numbers:

1	hydrogen (H)
2	helium (He)
3	lithium (Li)
4	beryllium (Be)
5	boron (B)
6	carbon (C)
7	nitrogen (N)'
8	oxygen (O)
9	fluorine (F)
10	neon (Ne)
11	sodium (Na)
12	magnesium (Mg)
13	aluminum (Al)
14	silicon (Si)