ASTRONOMY 2400. PHYSICS OF STARS

Assignment 3. **Due Date: April 3, 2012.**

1. Calculate the mean free path for λ = 5000 Å photons in Earth's atmosphere if it has the opacity of the solar photosphere, which was assumed to be κ_{5000} = 0.264 cm^2 g^{-1} in a textbook example. Assume that the average density of Earth's atmosphere is 1.2×10^{-3} g cm^{-3}.

2. Suppose that the Sun was 100% carbon (coal, for instance) and that burning it can extract 3 eV per carbon nucleus. Given an inexhaustible supply of oxygen from outside for burning the coal, how long could burning carbon maintain the Sun's current luminosity?

3. Calculate the amount of energy released or absorbed (in MeV) in the following reactions:

(a) $^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg} + \gamma$

(b) $^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{16}\mathrm{O} + 2\,{}^{4}\mathrm{He}$

(c) $^{19}\mathrm{F} + {}^{1}\mathrm{H} \rightarrow {}^{16}\mathrm{O} + {}^{4}\mathrm{He}$

The mass of ^{12}C is 12.000000 u by definition, and the masses of ^{16}O, ^{19}F, and ^{24}Mg are 15.994915 u, 18.99840 u, and 23.98504 u, respectively. Identify each of the reactions as being exothermic or endothermic.

4. Complete the reaction sequences given below. For each reaction justify the choice of nucleons and leptons involved, according to arguments of conservation of electric charge, number of nucleons, and number of leptons.

(a) $^{27}\mathrm{Si} \rightarrow {}^{?}\mathrm{Al} + e^{+} + ?$

(b) $^{?}\mathrm{Al} + {}^{1}\mathrm{H} \rightarrow {}^{24}\mathrm{Mg} + {}^{4}?$

(c) $^{35}\mathrm{Cl} + {}^{1}\mathrm{H} \rightarrow {}^{36}\mathrm{Ar} + ?$

5. (a) At what rate is the mass of the Sun decreasing as a consequence of nuclear reactions in its core? Express the result as a rate of mass loss in solar masses per year.

(b) What is the rate of mass loss for the Sun arising from the solar wind, in units of solar masses per year? Is the value obtained in part (a) larger or smaller than the rate of mass loss arising from the solar wind?

(c) Assume that the rates calculated in parts (a) and (b) remain constant over the main-sequence lifetime of ~10^{10} years for the Sun. Calculate the total loss of mass from the Sun during its main-sequence lifetime arising from nuclear reactions and the solar wind. What proportion of the total mass of the Sun is expected to be lost over that time period as a result of both processes?

ASTR 2400

Assignment 3.

1. The mean free path for particles or photons is given by:

$$l = \frac{1}{n\sigma_\lambda}, \quad \text{or} \quad l = \frac{1}{\kappa_\lambda \rho}, \quad \text{where}$$

n is the particle density, σ_λ is the collisional cross-section, κ_λ = opacity at wavelength λ, and ρ is the density.

For air we are given $\kappa_{5000} = 0.264\ cm^2/gm$ and an average density of Earth's atmosphere of $\rho = 1.2 \times 10^{-3}\ gm/cm^3$.

$$\therefore\ l = \frac{1}{\kappa_\lambda \rho} = \frac{1}{(0.264\ cm^2/gm)(1.2 \times 10^{-3}\ gm/cm^3)}$$

$$= 3.1566 \times 10^3\ cm$$

$$= 31.57\ \text{meters}$$

i.e. a $\lambda = 5000\,\text{Å}$ photon should be able to travel ~32 meters before scattering off an atmospheric molecule.

2. A carbon nucleus has a mass of 12.000000 u, where u is the atomic mass unit $= 1.66054 \times 10^{-24}$ gm

Each conversion of carbon (to carbon monoxide or carbon dioxide) through interactions with oxygen extracts 3 eV of energy.

$M_\odot = 1.989 \times 10^{33}$ gm.

∴ No. of carbon nuclei in the Sun can be estimated here as:

$$\text{No. (C)} = \frac{\text{mass of Sun}}{\text{mass of C nuclei}}$$

$$= \frac{1.989 \times 10^{33} \text{ gm}}{12.000000 \times 1.66054 \times 10^{-24} \text{ gm}}$$

$$= 9.9817 \times 10^{55}$$

each of which is capable of generating 3 eV of energy.

∴ Total energy available $= 3 \times 9.9817 \times 10^{55}$ eV

$= 2.9945 \times 10^{56}$ eV

But $1 \text{ eV} = 1.60217646 \times 10^{-12}$ ergs

∴ Total energy available $= 2.9945 \times 10^{56} \text{ eV} \times 1.60217646 \times 10^{-12} \frac{\text{ergs}}{\text{eV}}$

$= 4.7977 \times 10^{44}$ ergs. $= E_{carbon}$

The Sun's luminosity is $L_\odot = 3.851 \times 10^{33}$ ergs/s

∴ Solar lifetime for burning carbon is t_c, where

$$t_c = \frac{E_c}{L_\odot} = \frac{4.7977 \times 10^{44} \text{ ergs}}{3.851 \times 10^{33} \text{ ergs/s}}$$

$$= \frac{1.2458 \times 10^{11} \text{ s}}{3.15576 \times 10^{7} \text{ s/year}}$$

$= 3948$ years

This seems impossible given that the historical record exceeds 4000 years.

3. The amount of energy released or absorbed can be calculated from examination of the masses of the participating nuclei

a) For $^{12}C + ^{12}C \rightarrow ^{24}Mg + \gamma$,

LHS Mass = 12.000000 u + 12.000000 u = 24.000000 u

RHS Mass = 23.98504 u, where u = 931.49432 MeV.

LHS − RHS Mass = 24.000000 u − 23.98504 u

= 0.01496 u × 931.49432 MeV/u

= 13.935 MeV with the mass difference transformed to energy generated by the reaction, i.e. exothermic.

b) For $^{12}C + ^{12}C \rightarrow ^{16}O + 2\,^{4}He$,

LHS Mass = 12.000000 u + 12.000000 u = 24.000000 u

RHS Mass = 15.994915 u + 2 (4.002603 u) = 24.000121 u

LHS − RHS Mass = 24.000000 u − 24.000121 u

= −0.000121 u × 931.49432 MeV/u

= −0.113 MeV with the mass difference accounted for by energy lost in the reaction, i.e. endothermic

c) For $^{19}F + ^{1}H \rightarrow ^{16}O + ^{4}He$

LHS Mass = 18.99840 u + 1.007825 u = 20.006225 u

RHS Mass = 15.994915 u + 4.002603 u = 19.997518 u

LHS − RHS Mass = 20.006225 − 19.997518 u

= 0.008707 u × 931.49432 MeV/u

= 8.111 MeV with the mass difference accounted for by energy generated in the reaction, i.e. exothermic

4. For nuclear reactions there must be conservation of (i) number of nucleons, (ii) number of leptons, (iii) electronic/nuclear charge, and (iv) particle spin.

a) For $^{27}Si \rightarrow {}^{?}Al + e^{+} + ?$

^{27}Si has 14 protons, 13 neutrons, $^{?}Al$ has 13 protons

Nucleons in $= 14 + 13 = 27$, Nucleons out $= 13 + ?$

$\therefore ? = 27 - 13 = 14$, i.e. $^{?}Al$ is ^{27}Al

Leptons in $= 0$, Leptons out $= 1 + ?$

$\therefore ? = 0 - 1 = -1$, so the missing particle is an antilepton.

Charge in $= +14$, Charge out $= +13 + 1 = +14$.

The missing particle, an antilepton, has no charge.

Spin in $= 14(\frac{1}{2}\hbar) + 13(\frac{1}{2}\hbar) = 27(\frac{1}{2}\hbar)$

Spin out $= 13(\frac{1}{2}\hbar) + 14(\frac{1}{2}\hbar) + (\frac{1}{2}\hbar) + ? = 27(\frac{1}{2}\hbar) + \frac{1}{2}\hbar + ?$

$\therefore ? = 27(\frac{1}{2}\hbar) - 27(\frac{1}{2}\hbar) - \frac{1}{2}\hbar = -\frac{1}{2}\hbar$.

The missing particle must be an antilepton with no charge and spin $\frac{1}{2}\hbar$, i.e. an antineutrino, (an electron antineutrino)

i.e. $^{27}Si \rightarrow {}^{27}Al + e^{+} + \bar{\nu}_e$

b) For $^{?}Al + {}^{1}H \rightarrow {}^{24}Mg + {}^{4}?$ Al has 13 protons, Mg has 12 protons

Nucleons in $= 13 + ? + 1 = 14 + ?$, Nucleons out $= 24 + 4 = 28$

So $? = 28 - 14 = 14$ and the isotope of Al is ^{27}Al

Leptons in $= 0$, Leptons out $= ?$, missing particle is not a lepton

Charge in $= +13 + 1 = +14$, Charge out $= +12 + ?$,

Another +2 of charge is needed, so missing particle must contain 2 protons, i.e. $^{4}? = {}^{4}He$.

Spin in $= 13(\frac{1}{2}\hbar) + 14(\frac{1}{2}\hbar) + \frac{1}{2}\hbar = 28(\frac{1}{2}\hbar)$

Spin out $= 12(\frac{1}{2}\hbar) + 12(\frac{1}{2}\hbar) + 2(\frac{1}{2}\hbar) + 2(\frac{1}{2}\hbar) = 28(\frac{1}{2}\hbar)$

Spin is conserved in the reaction indicated.

i.e. $^{27}Al + {}^{1}H \rightarrow {}^{24}Mg + {}^{4}He$

c) For $^{35}Cl + {}^{1}H \rightarrow {}^{36}Ar + ?$

^{35}Cl has 17 protons, 18 neutrons, ^{36}Ar has 18 protons, 18 neutrons

Nucleons in = 17 + 18 + 1 = 36

Nucleons out = 18 + 18 + ? = 36 + ?

Therefore the missing particle is not a nucleon since nucleon number is already conserved.

Leptons in = 0, Leptons out = 0 + ?

? cannot be a lepton since there are no leptons involved in the reaction.

Charge in = +17 +1 = +18, Charge out = +18 + ?

Charge is balanced so ? cannot have charge.

Spin in $= 17(\frac{1}{2}\hbar) + 18(\frac{1}{2}\hbar) + \frac{1}{2}\hbar = 36(\frac{1}{2}\hbar)$

Spin out $= 18(\frac{1}{2}\hbar) + 18(\frac{1}{2}\hbar) + ? = 36(\frac{1}{2}\hbar) + ?$

? cannot have spin, i.e. spin = 0

The only particle with no spin, no charge, and that is not a nucleon or lepton is the photon.

i.e. $? = \gamma$

and $^{35}Cl + {}^{1}H \rightarrow {}^{36}Ar + \gamma$

5. a) The proton-proton chain in the Sun's core converts 4 protons into 1 ^{4}He nucleus with the resulting loss of $0.028677\ u = 26.71$ MeV of energy.

$1\ u = 1.660540 \times 10^{-24}$ gm. $= 931.49432$ MeV

$1\ M_\odot = 1.989 \times 10^{33}$ gm.

Energy generated by the Sun / year $= L_\odot$ (ergs/s) $\times 3.15576 \times 10^{7}$ s/yr

$= 3.851 \times 10^{33} \times 3.15576 \times 10^{7}$ ergs/yr

$= 1.2153 \times 10^{41}$ ergs/yr

$1\ eV = 1.60217646 \times 10^{-12}$ ergs

$$\therefore E_\odot/\text{year} = \frac{1.2153 \times 10^{41}\ \text{ergs/yr}}{1.60217646 \times 10^{-12}\ \text{ergs/eV}} = 7.5852 \times 10^{52}\ \text{eV/year}$$

$$= \frac{7.5852 \times 10^{52}\ \text{eV/year}}{931.49432 \times 10^{6}\ \text{eV/u}} = 8.143 \times 10^{43}\ u\ /\text{year}$$

$$= 8.143 \times 10^{43}\ u/\text{year} \times 1.66054 \times 10^{-24}\ \text{gm/u}$$

$$= \frac{1.3522 \times 10^{20}\ \text{gm/year}}{1.989 \times 10^{33}\ \text{gm}/M_\odot}$$

$$= 6.8 \times 10^{-14}\ M_\odot/\text{year}$$

b) The solar wind accounts for a mass loss rate of $3 \times 10^{-14}\ M_\odot$/yr which is smaller than the rate of mass loss through nuclear reactions.

c) The total rate of mass loss is $6.8 \times 10^{-14}\ M_\odot/\text{year} + 3 \times 10^{-14}\ M_\odot/\text{year}$ $= 9.8 \times 10^{-14}\ M_\odot/\text{year} \sim 10^{-13}\ M_\odot/\text{year}$.

Over a time span of 10^{10} years, the Sun must lose $10^{10}\ \text{yr} \times 10^{-13}\ M_\odot/\text{yr} = 10^{-3}\ M_\odot$ from nuclear reactions and the solar wind, i.e. only ~0.1% of its mass.