ASTR 2400

Assignment 2.

1. The Maxwell-Boltzmann velocity distribution is described by:

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

The most probable speed, maximum velocity, is reached when:

$df(v)/dv = 0.$

But
$$\frac{df(v)}{dv} = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left[2ve^{-mv^2/2kT} - \frac{2vv^2m}{2kT}e^{-mv^2/2kT}\right]$$
$$= 8\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v e^{-mv^2/2kT}\left[1 - \frac{v^2m}{2kT}\right]$$

For $\frac{df(v)}{dv} = 0$ at maximum velocity, $1 - \frac{v^2 m}{2kT} = 0$

$$\therefore \frac{mv^2}{2kT} = 1$$

or $v^2 = \frac{2kT}{m}$

Thus, $v_{mp} = \sqrt{\frac{2kT}{m}}$

2. In order to work out the degree of ionization at the centre of the Sun, we use the Saha equation:

$$\log \frac{N^{n+1}}{N^n} = 2.5 \log T - \theta I_n - \log P_e - 0.4771 + \log \left[\frac{2 u_{n+1}(T)}{u_n(T)}\right],$$

where P_e is the electron pressure in dynes/cm^2.

We are given $N_e = 6.4 \times 10^{25}$ /cm^3 at the Sun's centre, and $P_e = N_e kT$, where $k = 1.38065 \times 10^{-16}$ erg/K in cgs units.

$$\therefore P_e = 6.4 \times 10^{25} /\text{cm}^3 \times 1.38065 \times 10^{-16} \text{ erg/K} \times 15.8 \times 10^6 \text{ K (for } T_c)$$

$$= 1.3961 \times 10^{17} \text{ dynes/cm}^2$$

Thus, for hydrogen at the Sun's core:

$$\log \frac{N^{ion}}{N^n} = 2.5 \log (15.8 \times 10^6) - \frac{5040}{15.8 \times 10^6}(13.595) - \log (1.3961 \times 10^{17}) - 0.4771 + \log \left(\frac{2 \times 1}{2}\right)$$

$$= 17.996643$$
$$- 0.0043366$$
$$- 17.144921$$
$$- 0.4771$$
$$+ 0.0000$$
$$= 0.3702854$$

i.e. $N(\text{ions})/N(\text{neutrals}) = 10^{0.3702854} = 2.34577$

In other words, according to the Saha equation, only 70% $(2.34577/(1+2.34577))$ of the hydrogen at the centre of the Sun is ionized. It is not fully ionized according to the Saha equation.

3. According to the given parameters, there are 6.4×10^{25} electrons per cm^3 at the centre of the Sun. For electrical neutrality there must also be 6.4×10^{25} protons per cm^3 at the centre of the Sun. In other words, there must be 6.4×10^{25} H atoms / cm^3 at the Sun's centre. Each hydrogen atom is therefore allowed to occupy a volume of $1/6.4 \times 10^{25}$ cm^3, at most.

The volume of a H atom is given by $\frac{4}{3}\pi r^3$, where r is its radius.

$$\therefore \frac{4}{3}\pi r^3 = \frac{1}{6.4 \times 10^{25}} \text{ cm}^3$$

$$\therefore r^3 = 3/(4\pi \times 6.4 \times 10^{25}) \text{ cm}^3$$

$$= 3.7302 \times 10^{-27} \text{ cm}^3$$

$$\text{or } r = (3.7302 \times 10^{-27})^{1/3} \text{ cm}$$

$$= 1.5509 \times 10^{-9} \text{ cm}$$

$$= 0.15509 \text{ Å} < 0.528 \text{ Å (Bohr orbital)}$$

In other words, hydrogen atoms are so closely packed at the centre of the Sun that the separation of their nuclei is less than the dimensions of the 1st Bohr orbital. That means that electrons cannot exist bound to protons at the Sun's core because the H nuclei are so closely packed. $\therefore$ At the centre of the Sun, the hydrogen must be fully ionized.

This is referred to as pressure ionization.

4. Spectral Classifications:

Object 52. Balmer lines Hγ (4340 Å), Hδ (4101 Å), and Hε (3970 Å) visible, as is Ca II K-line (3933 Å) ~ 1/2 strength of Balmer lines. Best match around A5 – A7. Depression blueward of Hγ suggests weak G-band, which begins around spectral type F0. Ca I (4226 Å) only weakly visible, so must be hotter than F0. Best match ~ A7. Balmer line wings ~50 Å wide, implying luminosity class V.

i.e. Sp.T. = A7 V

Object HD23158. Balmer lines Hγ (4340 Å) and Hδ (4101 Å) visible, but Ca II H & K (3933, 3968 Å) stronger. G-band visible, as is Ca I (4226 Å). Ca I is slightly weaker than the G-band, indicating a spectral type hotter than ~F5, say F2. The Ti II, Fe II blend at λ4172-8 is present but not strong, indicating a luminosity class of III – V, as also indicated by H-line wings, which are strong. The ratio Ca I (4226) / Ti II-Fe II (4172-8) indicates class V.

i.e. Sp.T = F2 V

Object 458. Strongest line is Ca I (4226 Å) and Ca II H & K (3933, 3968), also Fe I (4386 Å). This occurs in K stars, particularly at K5. Sr II (4077 Å) is weak relative to Fe I (4046 Å), implying a dwarf (class V). The lack of a CN break at λ~4200 and bluer also implies class V.

i.e. Sp.T. = K5 V

Object HD23630. Strongest features are Balmer lines of hydrogen, Hγ (4340 Å), Hδ (4101 Å), and Hε (3968 Å). Ca II K (3933 Å) is very weak. He I (4121 Å) is weaker than Si II (4128-30 Å) implying a type between B9 and A0. Weak Ca II K-line implies B9 - B9.5, as does Mg II (4481 Å). Line wings of H ~ 50-60 Å wide, implying a dwarf, luminosity class V.
i.e. Sp.T. = B9-A0 V.

Object, NGC 281-195. Strongest lines are hydrogen Balmer lines, which can be counted through Hβ, Hγ, Hδ, Hε, ... H14 (weak). He II lines at λ4542 Å and 4200 Å are not present, so star is B0 or cooler. He I and O II lines surrounding Hδ imply a dwarf, luminosity class V. Best match is near B0 - B0.5, H-line wings as strong as in class V.
i.e. Sp.T. = B0-B0.5 V