# PHYS3300 - Assignment 2

**Due in my mailbox on Fri, Oct 13, 2023 (Time allowed=two weeks)**

**Show all working to receive full credit, especially when performing integrals via substitution. If it is acceptable to quote a standard result then it will be mentioned in the question.**

**(1) (2 marks)** For the double pendulum problem we discussed that there are two holonomic constraints. What are they in Cartesian coordinates?

**(2)** Consider a massless spring hanging from a ceiling in the presence of gravity, that has an unstretched length of $y_0$. A mass $m$ is placed on the end of the spring, and we assume motion occurs only in the vertical direction.
(a) **(1 mark)** What is the extension of the spring in terms of the stretched length $\hat{y}$ and the unstretched length $y_0$?
(b) **(1 mark)** Using the extension of the spring as the degree of freedom for the system, what is the kinetic energy of the system?
(c) **(1 mark)** What is the potential energy of the system?
(d) **(1 mark)** Derive the E-L equation for the system.

**(3)** Suppose we have a pendulum of length $l$ that has its pivot attached to a moving point on the y-axis (the mass $m$ on the end of the pendulum is free to swing in the usual manner). This point on the y-axis accelerates uniformly along the axis with acceleration $a$.
(a) **(1 mark)** Assuming that at $t = 0$ the velocity of the pivot of the pendulum is $v = 0$, what is the position of the pivot of the pendulum at time $t$?
(b) **(3 marks)** Using $\theta$ as the swing angle for the pendulum, show that the kinetic energy and potential energy (taking $y = 0$ to be $V = 0$) are given by,

$$T = \frac{1}{2}m(l^2\dot{\theta}^2 + a^2t^2 + 2alt\dot{\theta}\sin\theta)$$

$$V = mg(\frac{1}{2}at^2 - l\cos\theta).$$

(c) **(3 marks)** From the Lagrangian, use the E-L equation to show that the equation of motion is

$$\ddot{\theta} + \frac{a+g}{l}\sin\theta = 0.$$

(d) **(3 marks)** Assume that the oscillations of the pendulum are small. Using a trial solution of angular frequency $\omega$,

$$\theta = Ae^{i\omega t}$$

derive the period of the pendulum. A normal pendulum has a period

$$T = 2\pi\sqrt{\frac{l}{g}}$$

what is the physical implication of the result you have just derived?

**(4)** To the spring and mass system in Q2 we now add a second spring and mass, this time attached to the mass at the end of the first spring. Assume that the second spring and mass have the same spring constant and mass. Also assume motion occurs only in the vertical direction.
(a) **(4 marks)** Construct the Lagrangian for the system. (HINT: there will now be two degrees of freedom.)
(b) **(2 marks)** Derive the E-L equations for the system.

(c) **(4 marks)** Assume that solutions of the form $Ae^{i\omega t}$ exist. Find the allowed values of $\omega$. (HINT: look at the coupled pendulum example in the notes.)

**(5)** Now consider a pendulum where the mass is no longer constrained to be in a plane.
(a) **(3 marks)** Using spherical polar coordinates with $\theta$ as the polar angle and $\phi$ as the longitudinal angle what is the velocity of the mass?
(b) **(1 mark)** What is the potential energy?
(c) **(4 marks)** Derive the equations of motion for the system.

**(6)** Suppose a particle of mass $m$ is constrained to move on a circle of radius R. In addition, the circle itself rotates in the same plane about a point on its circumference. Suppose that the rotation of the circle has constant angular speed $\omega$, and that there is no gravity acting on the system.
(a) **(3 marks)** Let $\theta$ describes the relative position of the mass on the hoop. It's best to measure it relative to the line drawn through the centre of the circle and the point on the circumference about which the circle rotates. Show that the components of the velocity of the particle are given by

$$\dot{x} = -\omega R \sin \omega t - (\dot{\theta} + \omega) R \sin(\theta + \omega t),$$

$$\dot{y} = \omega R \cos \omega t + (\dot{\theta} + \omega) R \cos(\theta + \omega t).$$

(b) **(4 marks)** Construct the Lagrangian and derive the E-L equation for this system (there is only one true degree of freedom here).
(c) **(1 mark)** This looks similar to a system we've seen before-which one?