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The constraint of neutron star parameters through pulse profile modelling with NICER*

ABSTRACT

The equation of state of neutron star matter is difficult to constrain, but the behaviour of pulsars and their rotating magnetic poles can reveal critical physical properties. The discovery of a new rotation-powered millisecond pulsar PSR J0343-3511 provides an opportunity to refine our understanding of the neutron star equation of state through approaches in X-ray astronomy.

The collection of observational data using the NICER observatory, adapted to rigorous and intricate computational techniques, will extract a pulse profile that extrapolates to the radius and mass of the respective pulsar. We propose a series of NICER exposures to collect this data, with discussion on technical feasibility, preliminary calculations, and the possible approaches we can take to pulse profile modelling.

Keywords: Neutron stars (1108) — Compact objects (288) — High Energy astrophysics (739)

1. SCIENTIFIC JUSTIFICATION

1.1. *Scientific rationale*

Neutron stars are thought to exhibit some of the most extreme densities in the universe, with densities ρ exceeding the *nuclear saturation density* ρ_{sat}, where most matter deteriorates into neutron-rich nucleons, and even regions exceeding the *neutron drip energy* ρ_{drip}, where neutrons are stripped from nuclei. (R. Kumar et al. 2025). This results in an extremely inhomogeneous equation of state (EoS) that is difficult to generalize.

A refined understanding of neutron star structure has far-reaching implications not only to the composition and structure of neutron stars, but also stellar evolution and cosmology. To the particular interest of nuclear physicists, an advanced understanding of neutron star EoS also contributes to our understanding of pure neutron matter. (A. Verma et al. 2025)

In lower densities ($\rho \lesssim \rho_{sat}$), theoretical nuclear physics has found promising results in modelling neutron star matter. However, theoretical models of dense matter suffer at higher densities due to the introduction of many-body interactions and the introduction of exotic matter that inflate uncertainties for an overall EoS. (L. Lindblom 1992)

Fruitful astrophysical research of neutron stars relies on a derived relation between the mass-radius ratio ($M - R$) and pressure-density ratio ($p - \rho$), as follows:

We can begin with the standard Oppenheimer-Volkoff (OV) equations that define conventional stellar structure:

$$\frac{dm}{dr} = 4\pi r^2 \rho \ ,$$

which can be transformed in terms of pressure:

$$\frac{dp}{dr} = -(\rho + p)\frac{m + 4\pi r^3 p}{r(r - 2m)}$$

Given a barotropic density (which is to say, positively increasing as a function of pressure) such that

$$\rho = \rho(p) \ ,$$

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the OV equations can then be solved for the case of a simple neutron star. This allows us to create a map between the $M-R$ and $P-\rho$ ratios as they relate to each other. For a given neutron star, such a map can then be inverted to determine an EoS (L. Lindblom 1992; F. Özel & D. Psaltis 2009).

For a pulsar (a rapidly spinning neutron star), R and M can be individually constrained from modelling the behaviour of the X-ray pulses coming from its rotating magnetic poles. (E. Bootsma et al. 2025) Models of this data are referred to as *pulse profile modelling* (PPM).

Particularly good candidates for observational study are the *"rotation-powered" millisecond pulsars* (MSPs). MSPs have rotational periods less than 10 milliseconds - and such small periods have been seen to correlate to a low *period derivative* ($\dot{P}$). A small $\dot{P}$ means stability in the analysis of an MSP's pulse profiles, and less ambiguity in the behaviour of the MSP's magnetic poles. (E. Bootsma et al. 2025; M. C. Miller et al. 2019; S. Bogdanov et al. 2019; D. R. Lorimer & M. Kramer 2012)

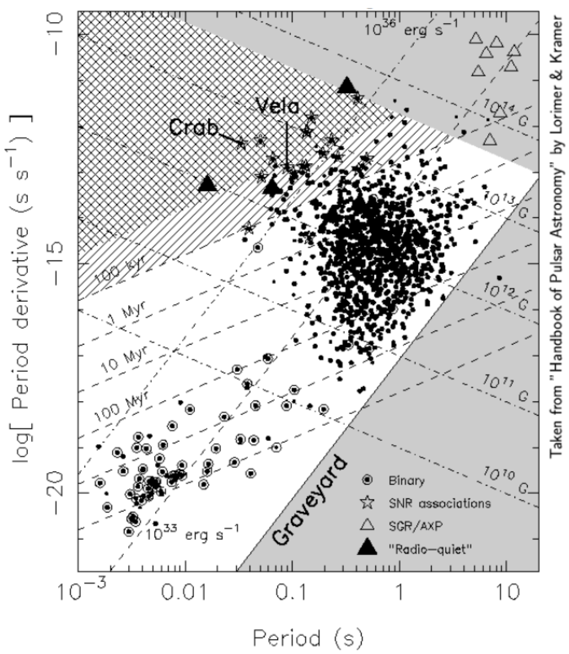

Figure 1. $P-\dot{P}$ diagram displaying the relation between period P and the period derivative $\dot{P}$ (D. R. Lorimer & M. Kramer 2012). Millisecond pulsars ($P<0.01$) are shown on the bottom-left; they have a significantly lower $\dot{P}$ than other conventional pulsars.

For a *"rotation-powered"* MSP, the observed X-ray emission derives from return currents that heat up the pulsar's surface at the magnetic poles (E. Bootsma et al. 2025). It is for this reason why rotation-powered MSPs are also known as *"recycled"* MSPs. This type of rotation-powered emission is preferable to other emission types because it tends to produce rapid rotation rates that are exceptionally stable and thought to be well understood compared to other forms of pulsar emission. (S. Bogdanov et al. 2019)

1.2. *Immediate objectives*

We propose observation of a recently discovered millisecond pulsar, PSR J0343-3511[2] in order to collect the observational data with which to create and analyze PPM. Using this data, we then seek to obtain estimates for the mass and radius of the pulsar. Approximately 500 MSPs have been discovered in total as of April 2025[3], which makes the discovery of new MSPs an important opportunity to collect new PPM data and refine our models.

The newly discovered PSR J0343-3511 is an isolated $205.44 Hz$ MSP. Its early observations have also been catagorized as a rotation-powered MSP, which indicates it satisfies our intended criteria for study. Under these considerations, we

[2] Please note that this pulsar, alongside any data attributed to it, has been fabricated for the sake of this mock proposal.

[3] From the ATNF Pulsar Catalogue: https://www.atnf.csiro.au/research/pulsar/psrcat

seek to observe PSR J0343-3511, model its X-ray pulse profile, and analyze the data to construct an estimated mass and radius.

2. TECHNICAL JUSTIFICATION

2.1. *Observing plan*

The observational data of MSPs sought by this proposal would ideally be collected using the *Neutron star Interior Composition ExploreR* (NICER) observatory.

NICER is an external observatory payload attached to the International Space Station that particularly specializes in high-energy observation of neutron stars. Furthermore, NICER's primary science instrument is the *X-Ray Timing Instrument* (XTI), which enables the collection of high quality phase and pulse profiles that are critical to our methods for analysis (S. Bogdanov et al. 2019; K. Gendreau et al. 2016).

Since launch, NICER has been used to collect many pulse profiles from rotation-powered MSPs. It has been documented that, in observation of rotation-powered MSPs, NICER maintains low uncertainties where other varieties of MSPs (such as *accretion-powered* and *burst-oscillation*) suffer from bias (D. Choudhury et al. 2024; S. Bogdanov et al. 2019). These considerations make NICER the clear choice for studying this specific variety of neutron stars.

Since its inauguration, NICER has been already used to catalogue and model pulse profiles for many MSPs. As such, our observations will follow previous conventions for the observation of rotation-powered neutron stars.

While a definitive pulse profile model cannot be designed before further study of the MSP, we can consider commonly adopted techniques. For analyzing a rotation-powered MSP, our pulse profile modelling will take the form of an intricate program that performs *ray-tracing* computations. From past literature, the following techniques have been applied to pulse profile models (L. Brandes & W. Weise 2025; D. Choudhury et al. 2024; V. Loktev et al. 2020; D. Leahy et al. 2007; N. Kardashev & L. Marochnik 1990):

1. Relativistic ray-tracing of the surface emission using the *oblate Schwarzschild approximation* (accounting for the oblate shape of the pulsar)
2. Derivation of the orbital inclination
3. Measuring pulsar distance through the *Shklovskii effect* (accounting for proper motion of pulsar across the sky)
4. Machine learning methods

Because these exposures will be the first NICER exposures of PSR J0343-3511, there are not previously documented observations to reference. However, we expect to assemble pulse profiles as well as histograms for pulse phase and photon energy, resembling previous NICER observations of MSPs in S. Bogdanov et al. (2019) in Figure 2 and 3.

2.2. *Technical feasibility*

After collecting the observational data from the proposed NICER exposures, we will conduct data processing and filtering using the *HEASoft* aand *NICERDAS* programs, as is conventional for work done with the NICER observatory.[4]

Windows for observation using NICER are limited by the 92-minute orbit of the ISS, which will allow for largely separated windows of exposure. From referencing similar observation plans of MSPs completed with NICER, we can estimate around 1-2 years to collect the necessary data.(S. Bogdanov et al. 2019; M. C. Miller et al. 2019).

Typical exposures for NICER observations of MSPs range from several hundred to 2000 seconds (M. C. Miller et al. 2019), which we similarly anticipate for observations of PSR J0343-3511. Using spectral simulation from the *WebSpec* program, we modelled a NICER spectrum for an assumed 1000 second exposure in Figure 4.

Based on a resultant count rate of $3638,381cts/s$ over the fitted energy range in Figure 4, and a calculated column density of $8.12 \times 10^{20} cm^{-2}$[5], we also created a preliminary simulation of NICER count rates for soft and hard bands from the *WebPIMMS* program, in Table 1.

This data provides us with preliminary expectations on count rates for a NICER exposures for this MSP. Particularly relevant is the separation of data for count rates for the soft and hard band - it has been established that rotation-powered MSPs, which are very old neutron stars exhibiting recycled thermal emission, emit harder X-rays than young pulsars (T. Zhao et al. 2025).

[4] Details on HEASoft and NICERDAS programs can be found at https://heasarc.gsfc.nasa.gov/docs/software/lheasoft

[5] Please recall this value is fabricated but in the range of values for similar existing MSPs.

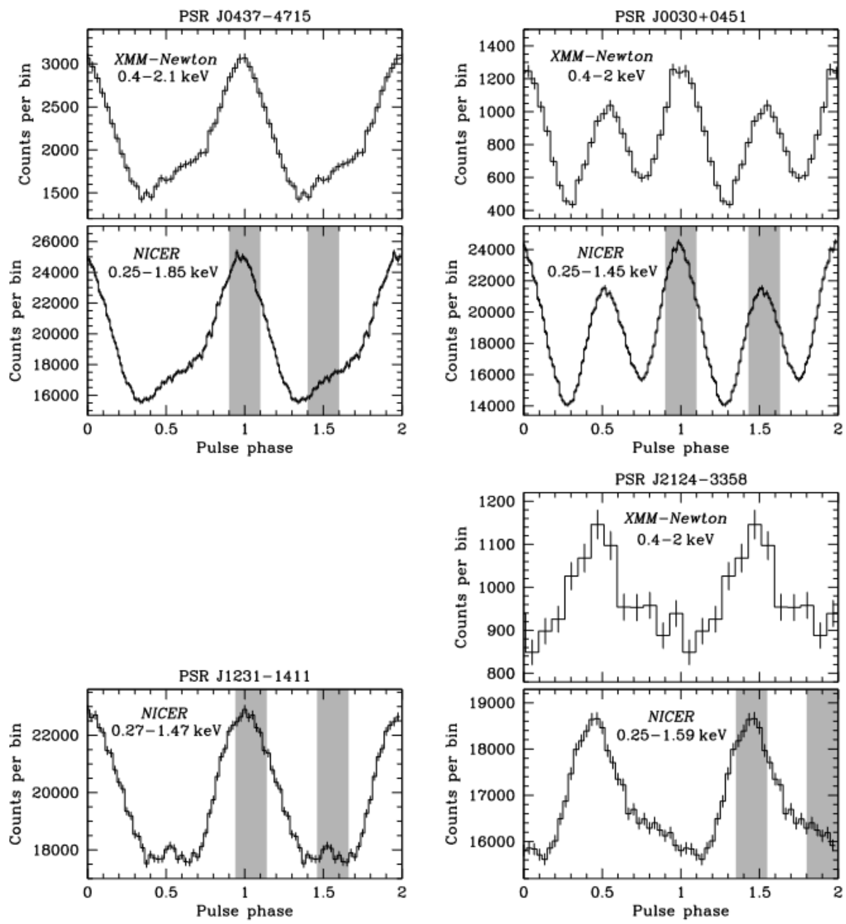

Figure 2. Previously recorded pulse profiles for NICER and XMM-Newton for the PSRs J0437-4715, J0030+0451, J1231-1411, and J2124-3358, represented for two phase cycles (S. Bogdanov et al. 2019)

E range	Source (cps)	Background (cps)	5-sigma detection (s)	(+10%)
0.4-2 keV	2331.667	0.64	0.011	0.011
2-8 keV	1209.805	0.55	0.021	0.021

Table 1. Basic predicted NICER data for source count rates, background count rates, time for 5-sigma detection, and 10% additional systematic uncertainty, that should be observed in soft and hard bands (bottom and top respectively) using WebPIMMS. Assuming Power Law model, photon index 1.0, and calculated count rate 3638.381ct/s.

3. CONCLUSIONS

With comprehensive observational data and further analysis, our primary challenge will be the design of appropriate pulse profile models for PSR J0343-3511. Adapting the resulting pulse profile models will allow us to ultimately infer measurements for radius and mass.

Ultimately, the study of the neutron star EoS is limited by candidates for observational study; this makes the collection of new data from new candidates all the more valuable. With successful observations from NICER, our pulse models and their inferences will seek to contribute to our understanding of the constraints of neutron star radius and mass, and ultimately the equation of state of neutron star matter.

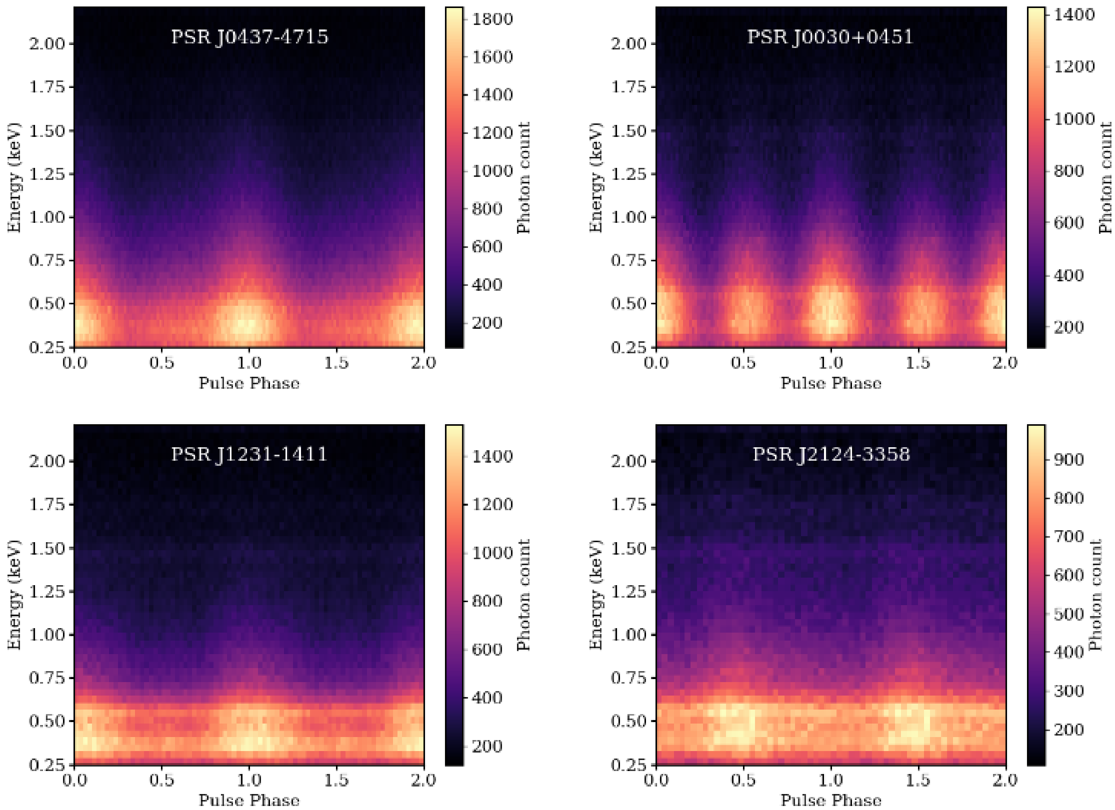

Figure 3. Previously recorded histograms for NICER counts versus pulse phase and energy for the PSRs J0437-4715, J0030+0451, J1231-1411, and J2124-3358, represented for two phase cycles (S. Bogdanov et al. 2019)

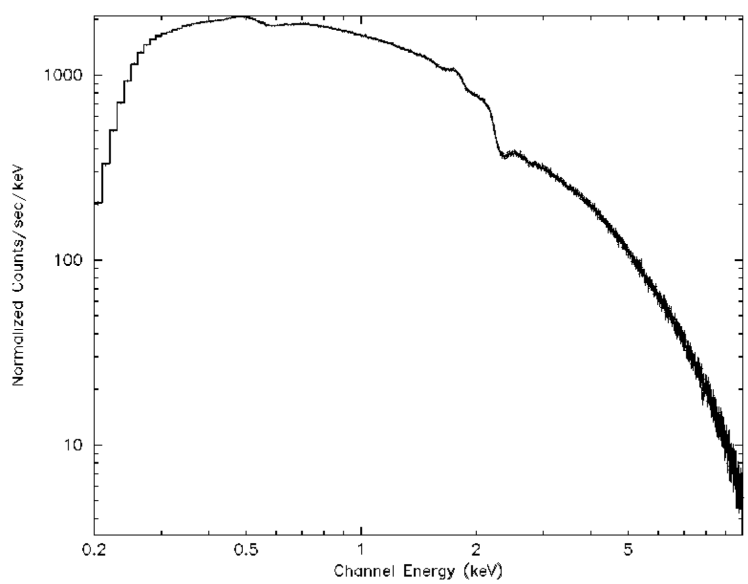

Figure 4. Predicted NICER spectrum simulated for 1000s exposure using WebSpec. Assuming Power Law model, photon index 1.0

Facility: NICER

Software: HEASoft, WebSpec, WebPIMMS

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