THERMODYNAMIC STATE & ATOMIC ENERGY:

PRESSURE EQUATION OF STATE

$$P_{gas} = P(T, \rho, \mu \ldots) \qquad \text{(EOS)}$$

STELLAR ATMOSPHERE: ρ LOW

$\therefore$ IDEAL GAS LAW GOOD

$$P(\tau) = N(\tau)\, k\, T_{kin}(\tau) \quad (\text{dyne}/\text{cm}^2)$$

$$= \frac{\rho(\tau)\, k\, T_{kin}(\tau)}{(\mu(\tau)\, m_H)}$$

- N (cm^{-3})
- μ = MEAN "MOLECULAR" "WEIGHT" OF MIXTURE (a.m.u.) – RUTTEN
- m_H = 1 a.m.u. (g)

N_Z = NO. DENSITY OF ELEMENT Z

"SPECIES" k : IONIZATION STATE OF ELEMENT Z

- e.g. Ca II (Ca^{+1})

$$\therefore N_Z = \sum_{k}^{\text{STAGES}} N_k$$

$$N = \sum_{Z}^{\text{ELEMENTS}} N_Z + N_e$$

PARTIAL PRESSURE OF SPECIES, k:

$$P_k(\tau) = N_k(\tau)\, k T_{KIN}(\tau)$$

N_e = NO. DENSITY OF FREE ELECTRONS (e^-)

$$P_e(\tau) = N_e(\tau)\, k T_{KIN}(\tau)$$

$$P(\tau) = \sum_{Z}^{\text{ELEMENTS}} \sum_{k}^{\text{STAGES}} N_{Z,k}\, k\, T_{\text{KIN}}(\tau)$$

$$+ N_e(\tau)\, k\, T(\tau)$$

ATOMIC MASS NUMBER OF ELEMENT Z, $\underline{a}_Z$

$$\rho = \sum_{Z}^{\text{ELEMENTS}} (a_Z m_H) N_Z$$

$$\underline{\mu(\tau)} = \frac{1}{m_H} \frac{\rho(\tau)}{N(\tau)}$$

$$= \frac{1}{m_H} \frac{\sum_Z m_H a_Z N_Z(\tau)}{\sum_Z N_Z(\tau) + \underline{\underline{N_e(\tau)}}}$$

ELECTRON (e^-) DONORS

- ELEMENTS, Z, OF:
  - 1^{ST} IONIZATION E, $\chi_I < kT$
  - RELATIVELY LARGE A_Z

- F, G, K STARS ($kT < 13.6\ \text{eV}$):

  Na, Mg, Ca, Al, Si, Fe, (H^-)

- O, B, A STARS ($kT \gtrsim 13.6\ \text{eV}$):

  H

DETERMINING N_e:

- ASSUME
  - $T_{KIN}(\tau)$, $N(\tau)$ KNOWN
  - NEUTRAL (I), SINGLE (II), DOUBLE (III) +ve ION. STAGES ONLY
  - LTE: $T_{IONIZATION} = T_{KIN}$

FOR IONIZATION STAGE k OF ELEMENT Z:

$(k = \mathrm{I}, \mathrm{II}, \text{ OR } \mathrm{III})$

- IONIZATION FRACTION, f_k:

$$f_k = \frac{N_k}{N_Z} = \frac{N_k}{N_{\mathrm{I}} + N_{\mathrm{II}} + N_{\mathrm{III}}}$$

SAHA DISTRIBUTION:

FOR $k \rightleftarrows (k+1) + e^-$:

$$\frac{N_{k+1}}{N_k} = \frac{1}{N_e} C\, T_{KIN}^{3/2}\, e^{-\chi_{I,k}/kT_{KIN}}$$

$$= f(T_{KIN}, N_e)$$

Eg. $k = \mathrm{II}$:

$$f_{\mathrm{II}} = \frac{N_{\mathrm{II}}}{N_{\mathrm{I}} + N_{\mathrm{II}} + N_{\mathrm{III}}}$$

$$= \frac{N_{\mathrm{II}}/N_{\mathrm{I}}}{1 + N_{\mathrm{II}}/N_{\mathrm{I}} + \left(N_{\mathrm{III}}/N_{\mathrm{II}}\right)\left(N_{\mathrm{II}}/N_{\mathrm{I}}\right)}$$

$$= f(N_e)$$

ITERATIVE PROCEDURE:

- INITIAL GUESS $N_e^{(0)}(\tau)$

- FOR EACH e^--DONOR, Z:

$$f_{II}(\tau) = \frac{N_{II}(\tau)}{N_Z(\tau)} = f(\underline{N_e}(\tau))$$

$$f_{III}(\tau) = \frac{N_{III}(\tau)}{N_Z(\tau)} = f(\underline{N_e}(\tau))$$

- $\underline{\underline{N_e^{(1)}}}(\tau) = \sum_Z N_Z(\tau) f_{\underline{II},Z}(\tau)$

$$+ \underline{2} \sum_Z N_Z(\tau) f_{\underline{III},Z}(\tau)$$

UNTIL:

$$\frac{N_e^{(n)}(\tau) - N_e^{(n-1)}(\tau)}{N_e^{(n)}(\tau)} < \varepsilon \ll 1$$

- ALL τ

$$\frac{N}{N_e} = \frac{\sum_z N_z + N_e}{N_e} = \frac{\sum_z N_z}{N_e} + 1$$

$$= \frac{\sum_z N_z}{\sum_z N_z f_{II,z} + 2\sum_z N_z f_{III,z}}$$

RECALL: $N_z = A_z N_H$

$$\frac{N}{N_e} = \frac{\sum_z \underline{A_z}}{\sum_z \underline{A_z} f_{II,z} + 2\sum_z \underline{A_z} f_{III,z}}$$

$\{A_z\}$ VALUES ARE <u>INPUT</u> PARAMETERS

Eg.

GAS COMPOSED OF H
+ ONE "METAL", M

- $A_M \ll 1$
- STAGES I & II ONLY

STUDY N_e/N IN LIMITS OF
HIGH- & LOW-T_{KIN}