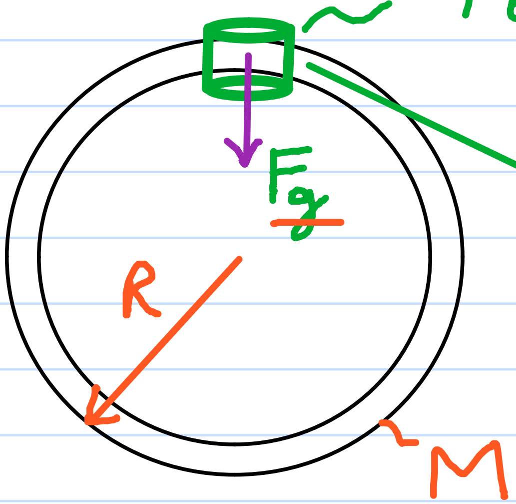
HYDROSTATIC EQUILIBRIUM (HSE):

→ TOTAL FLUID PRESSURE, $P(z)$

$$a_z(z) = 0 \rightarrow F_{NET\,z}(z) = 0$$

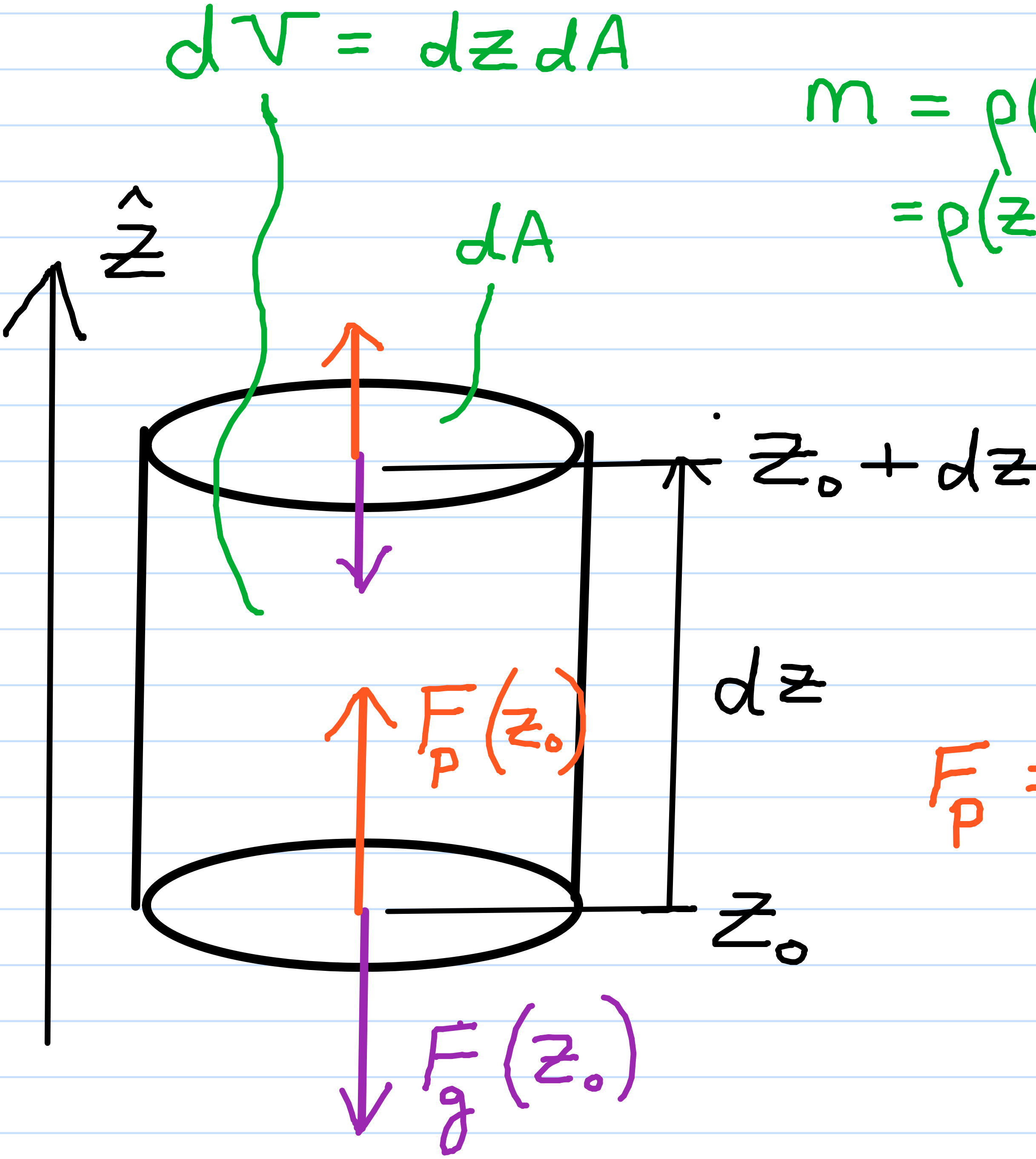
CYLINDRICAL TEST MASS, m



$$g(z) = g = \frac{GM}{R^2}$$

- INPUT PARAMETER

$$\therefore F_g = mg$$



$$m = \rho(z_0)\,dV$$
$$= \rho(z_0)\,dz\,dA$$

$$F_p = P \cdot dA$$

$$dF_g \equiv F_g(z_0) - F_g(z_0 + dz) \; \underline{> 0}$$

$$= gm = g\rho(z_0) \cdot (-dz)\,dA$$

$$\therefore \; dF_g(z) = -g\rho(z)\,dA\,dz$$

GRAVITATIONAL PRESSURE, P_{grav}

$$dP_{grav}(z) = \frac{dF_g(z)}{dA} = -g\rho(z)\,dz$$

HSE: $dP(z) = dP_{grav}(z)$

$$\therefore \boxed{\frac{dP(z)}{dz} = -g\rho(z)} \quad \underline{< 0}$$

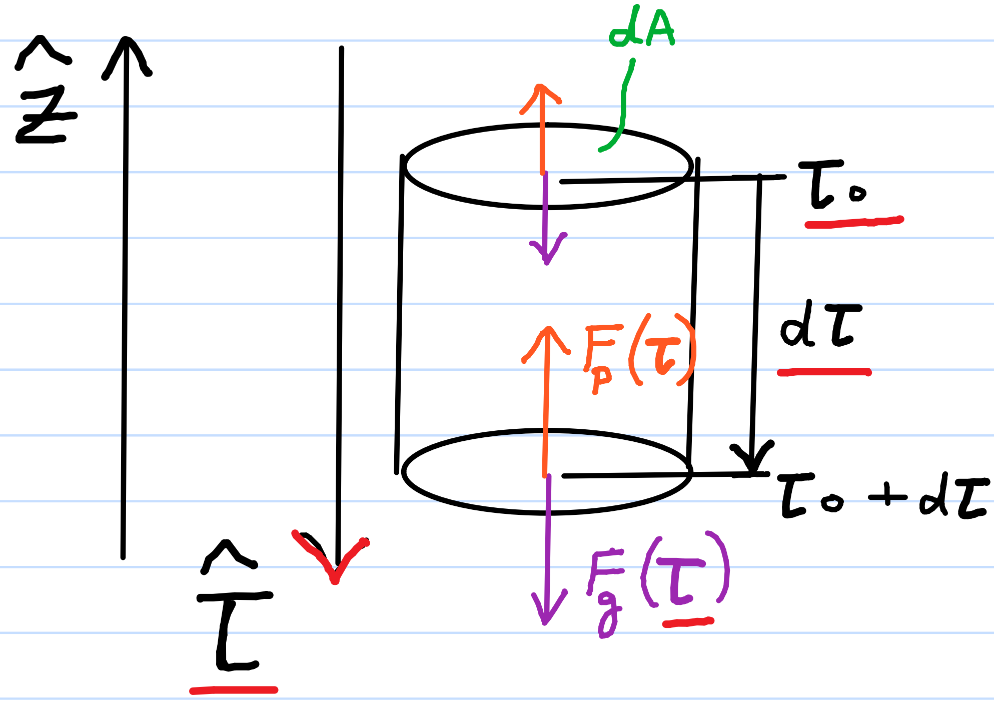
HSE ON GRAY OPTICAL DEPTH SCALE IN -ve $\hat{z}$ DIRECTION,

$$d\tau(z) = -\bar{K}(z)\rho(z)\,dz :$$

$$\frac{dP(\tau)}{d\tau} = -\frac{(-g\rho(z))}{K(\tau)\rho(z)}$$

$$\frac{dP(\tau)}{d\tau} = \frac{g}{K(\tau)} > 0$$

HSE



HSE FORMAL SOLUTION:

$$P(\tau) = \left(\frac{3g}{2} \int_{t=0}^{\tau} \frac{P^{1/2}(t)}{K(t)}\, dt \right)^{2/3}$$

ITERATIVE PROCEDURE:

1) INITIAL GUESS $P^{(1)}(\tau)$

2) IMPROVED ESTIMATE
$P^{(n)}(\tau)$ FROM $P^{(n-1)}(\tau)$

3) IMPROVED ESTIMATE
$K^{(n)} = K(P^{(n)})$

ITERATE

4) CONVERGENCE CRITERION:

$$\frac{\Delta P}{P} = \frac{P^{(n)} - P^{(n-1)}}{P^{(n)}} < \varepsilon \ll 1$$

- ALL τ

$$P(\tau) = P_{gas}(\tau) + P_{RAD}(\tau)$$

$$+ P_{MAG}(\tau) + P_{TURB}(\tau) + P_{CENTRIFUGE}(\tau)$$

MOST STARS, MOST τ VALUES:

$$\underline{P_{gas}(\tau)} = P(\tau) - P_{RAD}(\tau)$$