SPECTRAL LINE BROADENING (DAMPING)

→ LINE PROFILE, $\phi_\nu(\nu-\nu_0)$:

RECALL: b-b TRANSITIONS:

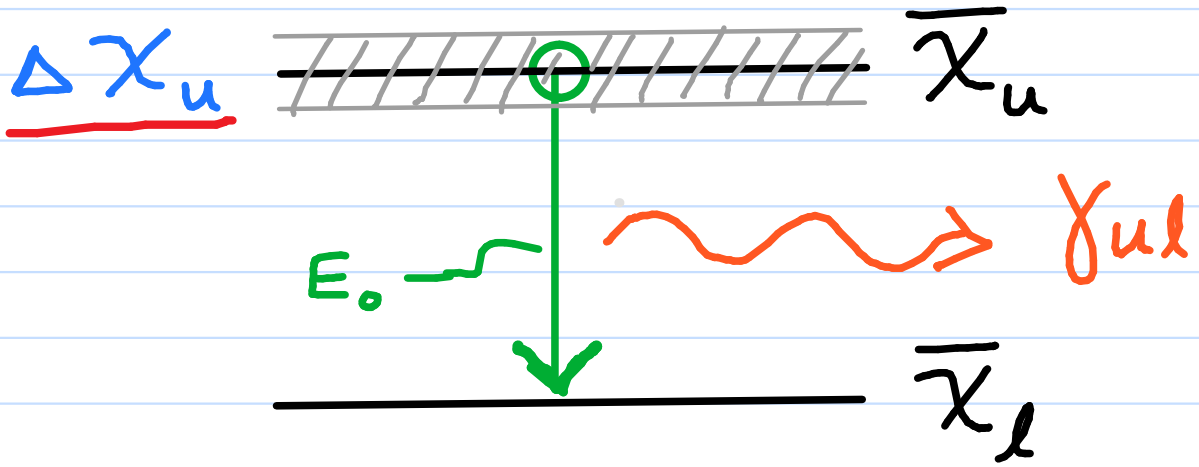
IN CRD: $\psi_\nu = \chi_\nu = \phi_\nu$

$$\alpha_\nu^{l}(\nu) = \alpha_{\nu_0}^{l}\,\phi_\nu(\nu-\nu_0) \neq \alpha_{\nu_0}\,\delta(\nu-\nu_0)$$

SPONTANEOUS DE-EXCITATION, $u \to l$

ASSUME $\Delta\chi_l = 0$:

2-LEVEL ATOM:

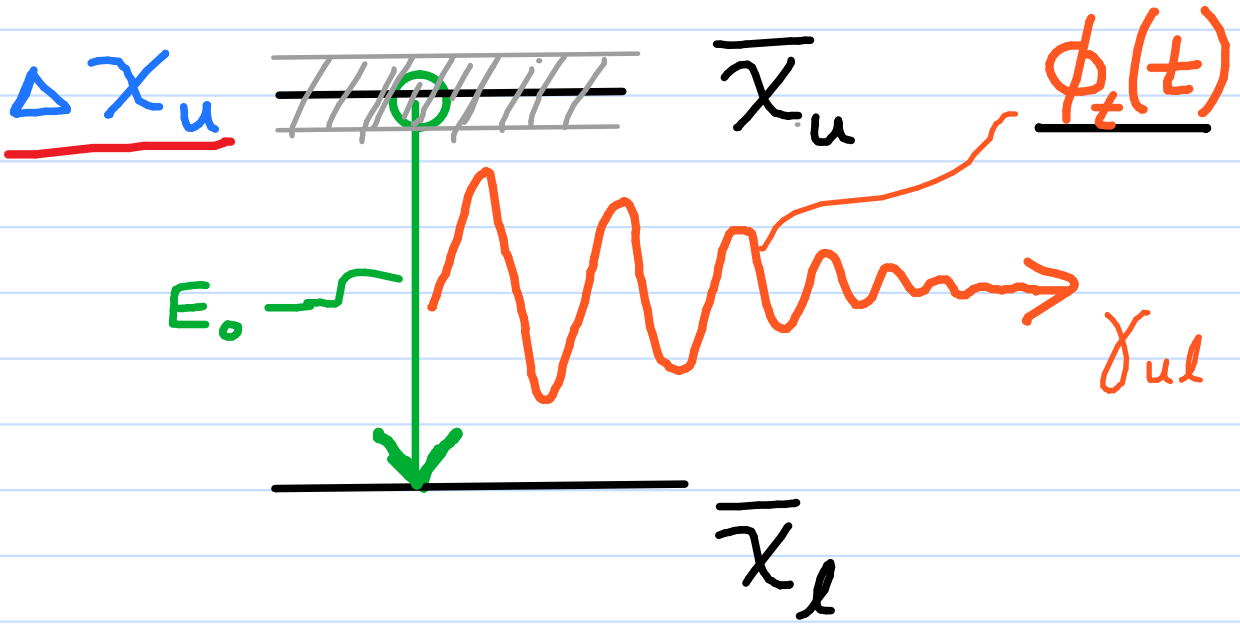


PHOTON $E_0 = h\nu_0 = \bar{\chi}_u - \bar{\chi}_l$

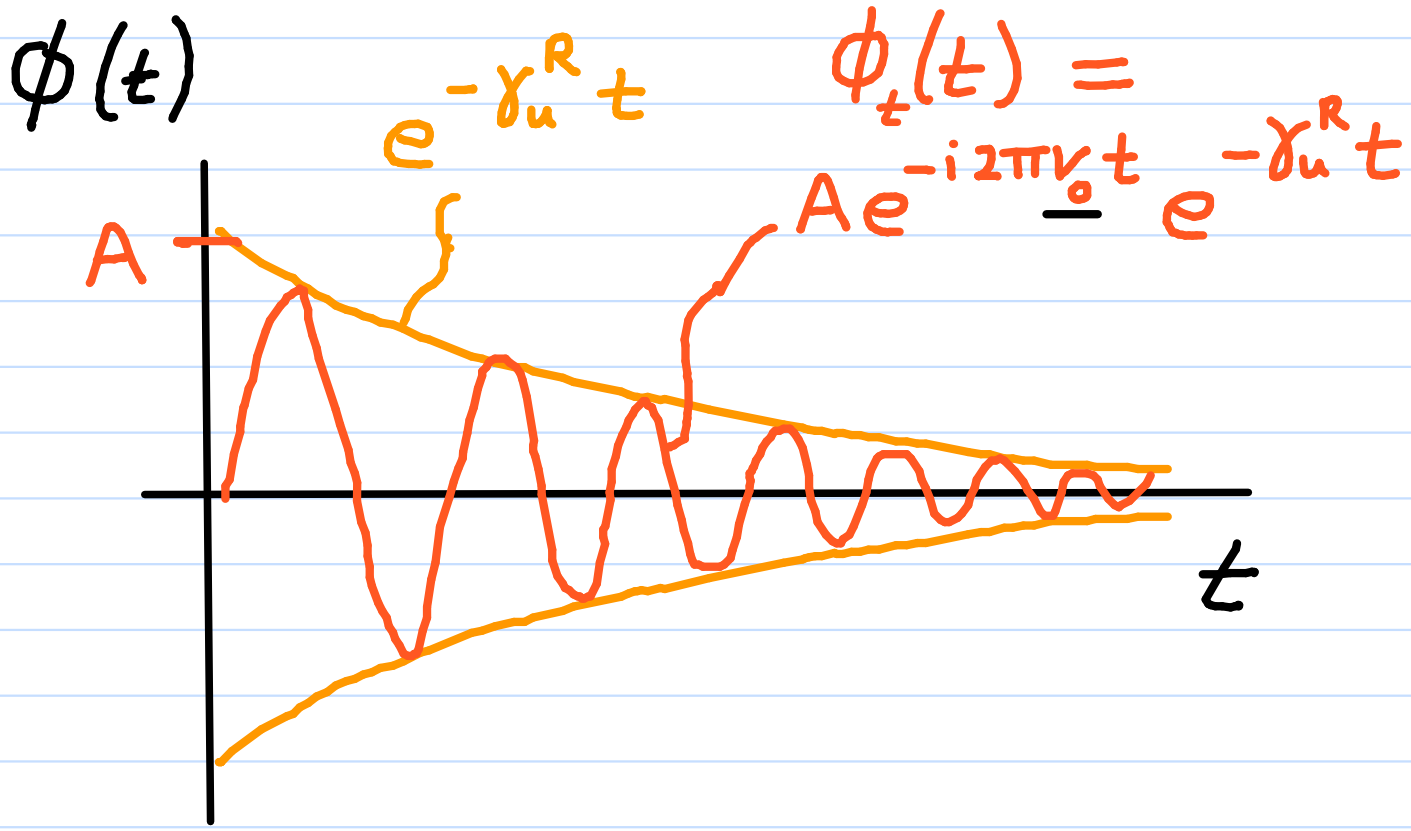
RADIATION (NATURAL) BROADENING:

MODEL DE-EXCITING ATOM AS DAMPED, DRIVEN HARMONIC OSCILLATOR

- ν_0 = NATURAL (RESONANT) FREQ.
- ν = DRIVING FREQ.
- γ_u^R = DAMPING PARAMETER OF E-LEVEL u
- $\phi(t)$ = UNDER-DAMPED Sol'n FOR EMITTED CLASSICAL EM WAVE



EMITTED EM WAVE:



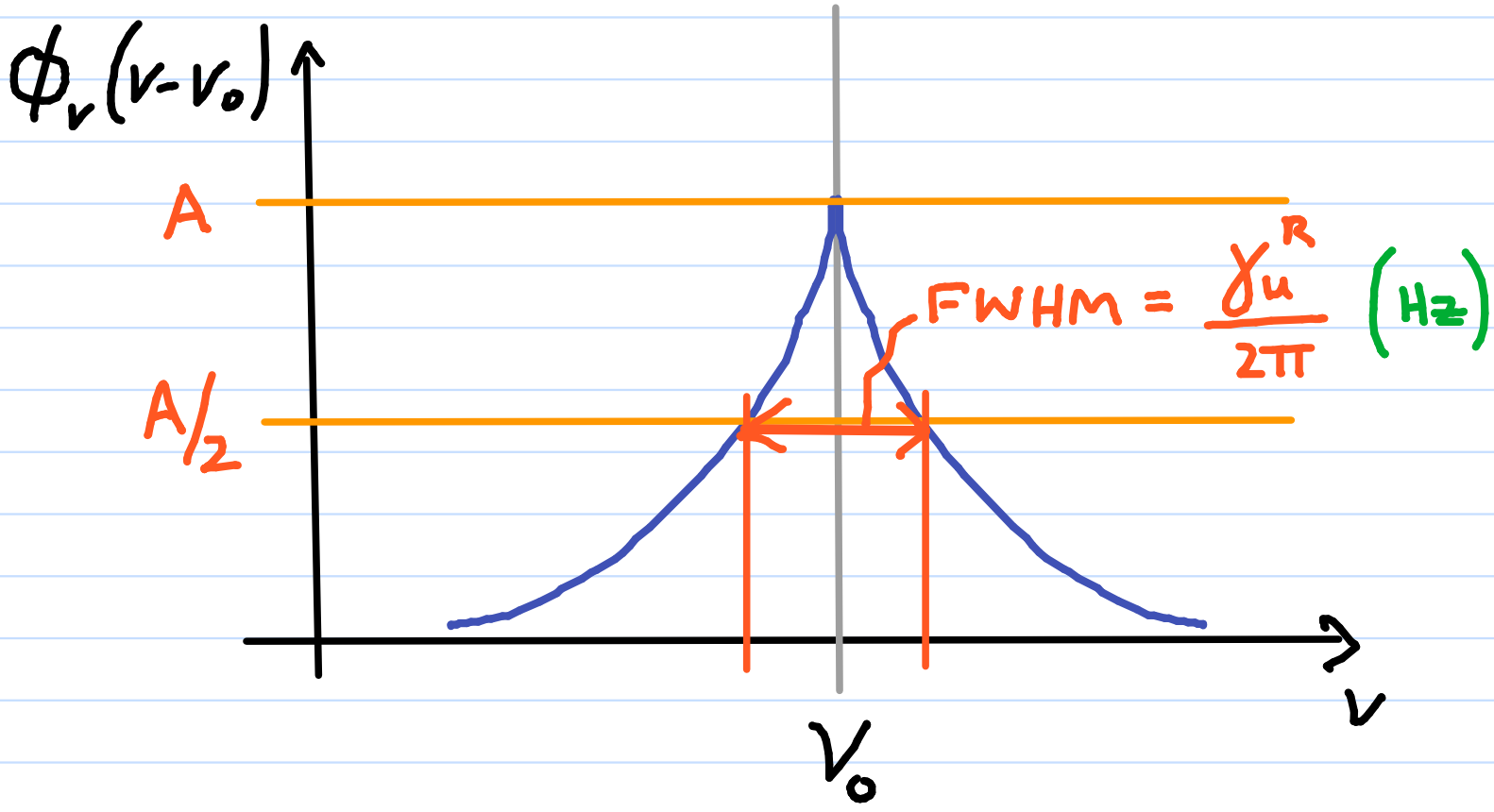
$$\phi_\nu(\nu-\nu_o) = F_T\left(\phi_t(t)\right)$$

$$= A^* F_T\left(e^{-\gamma_u^R t}\right) * F_T\left(e^{-i2\pi\nu_o t}\right)$$

$$= A^* F_T\left(e^{-\gamma_u^R t}\right) * \delta(\nu-\nu_o)$$

$$\therefore \quad \phi_\nu(\nu-\nu_0) = \frac{\gamma_u^R/4\pi^2}{(\nu-\nu_0)^2 + (\gamma_u^R/4\pi)^2}$$

- LORENTZIAN DAMPING PROFILE



NORMALIZED: $\int_{\nu=0}^{\infty} \phi_\nu \, d\nu = 1$

QUANTUM MECH. INTERPRETATION (γ_u^R):

HEISENBERG $\Delta E - \Delta t$ UNCERTAINTY PRINCIPLE FOR E-LEVEL u:

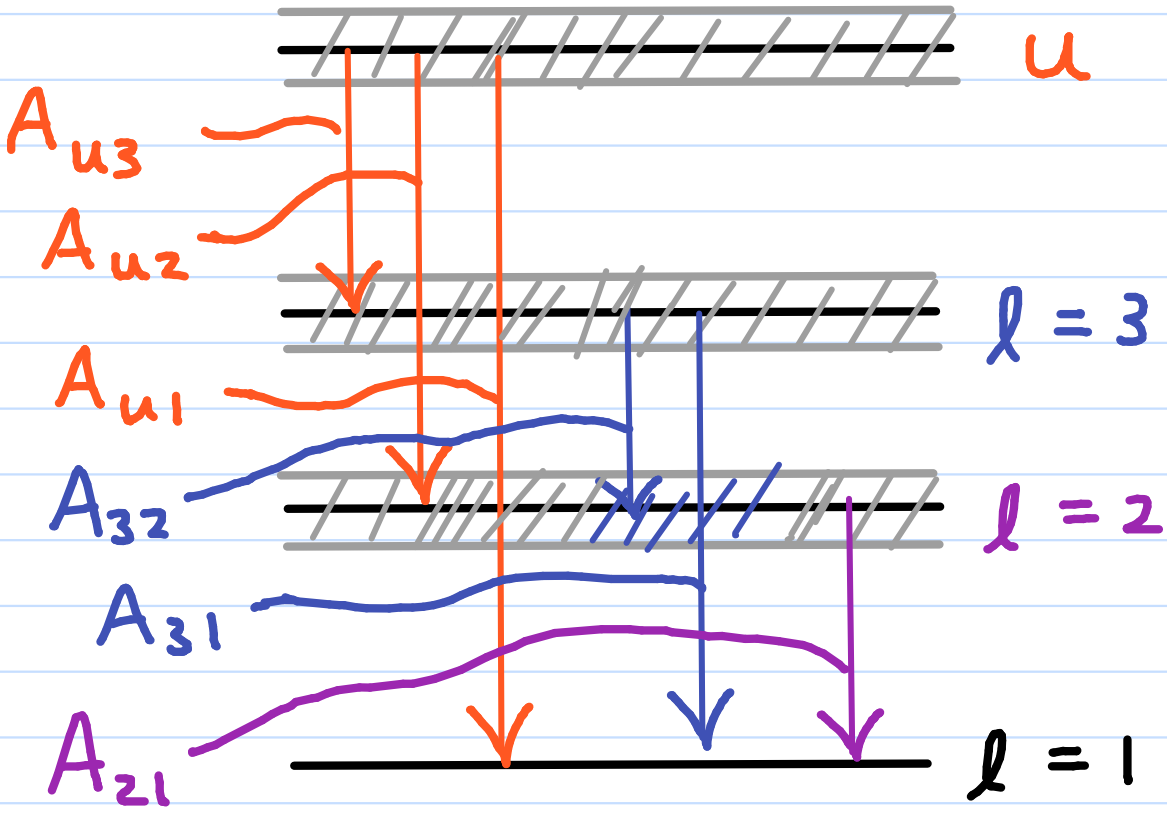
$$\Delta\chi_u \, \overline{\Delta t} = h/2\pi$$

→ $\overline{\Delta t}$ = MEAN LIFE-TIME IN LEVEL u

$$= A_{ul}^{-1} \quad (s)$$

$$\gamma_u^R \equiv \frac{2\pi}{h} \Delta\chi_u = A_{ul} \quad (Hz)$$

MULTI-LEVEL ATOM:



$$\gamma_u^R = \sum_{l<u} A_{ul}$$

FOR $u \leftrightarrow l$: $\gamma^R = \gamma_u^R + \gamma_l^R$

$l = 1$ (GROUND):

$\overline{\Delta t} = \infty \therefore \Delta x_1 = 0 \therefore \gamma_1^R = 0$

COLLISIONAL (PRESSURE) BROADENING:

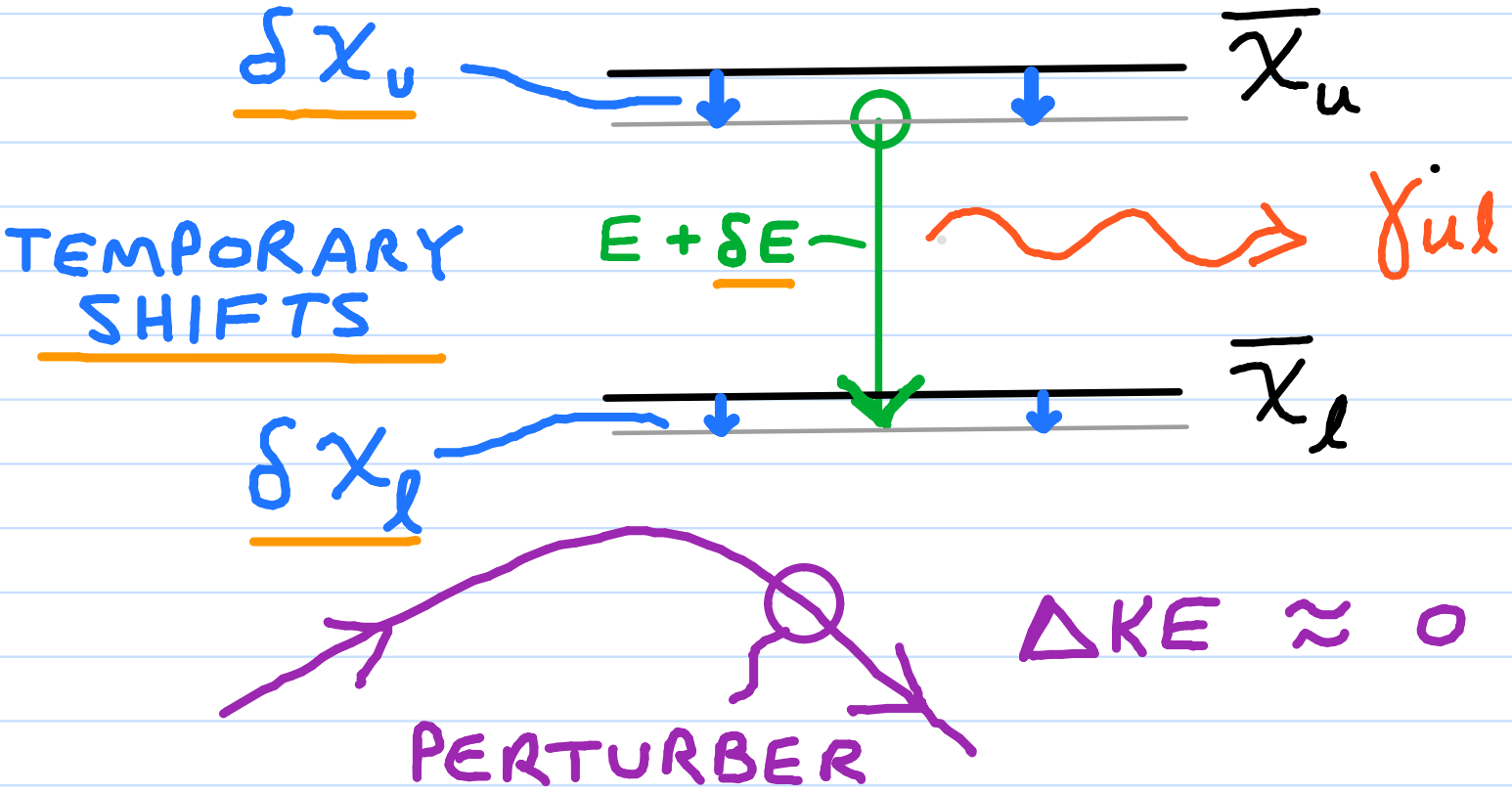
- ELASTIC EM INTERACTION WITH A PERTURBER
  - → $\Delta KE \approx 0$

IMPORTANT PERTURBERS:

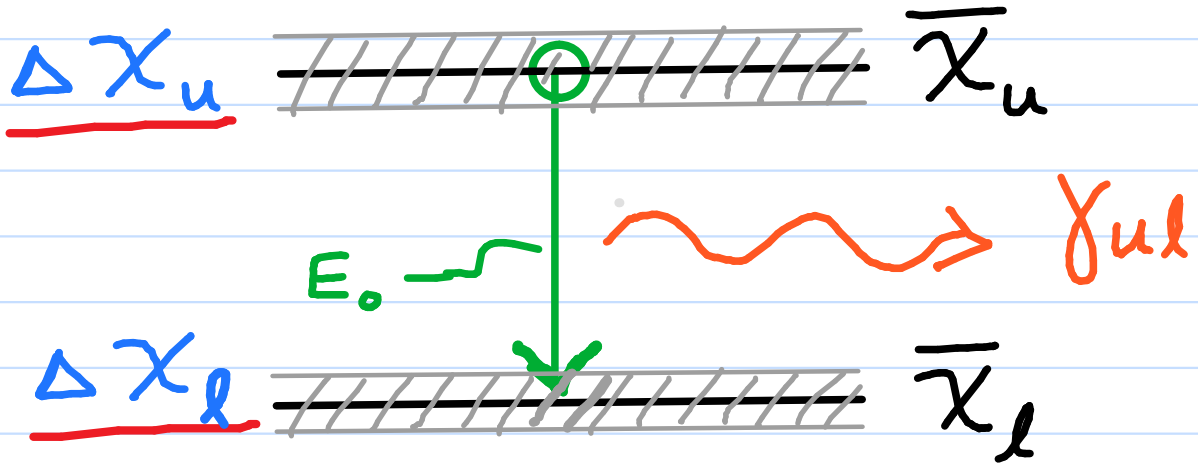
OBA STARS: e^-, HII

FGKM STARS: HI

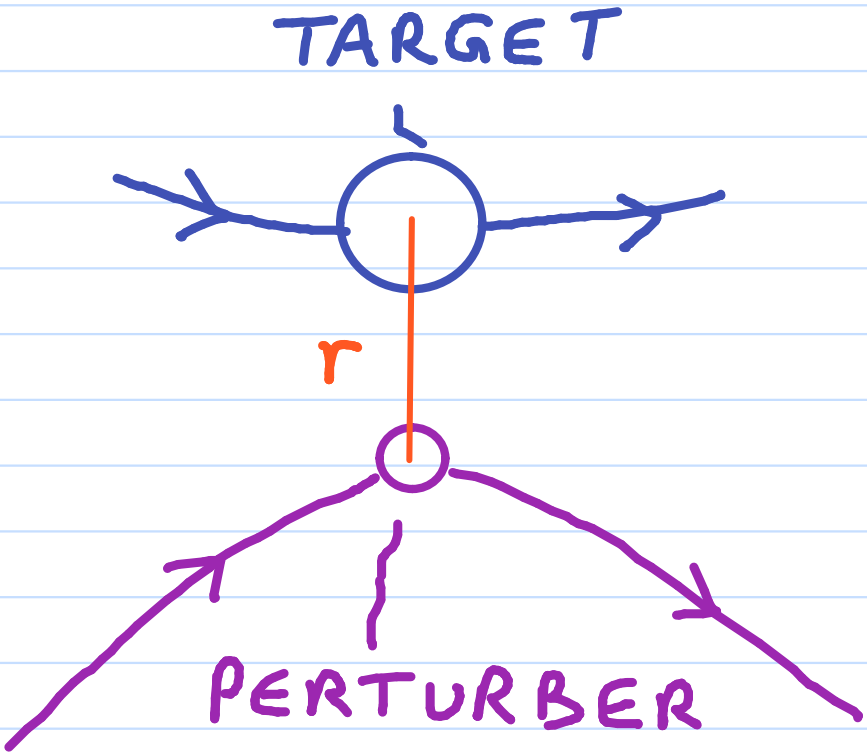
2-LEVEL ATOM, SINGLE COLLISION:



2-LEVEL ATOM: MANY COLLISION, ENSEMBLE REPRENTATIVE



COLLISION: EM INTERACTION



r = MINIMUM SEPARATION

$\phi(r)$ = EM POTENTIAL

n = POWER-LAW INDEX (MONOPOLE, DIPOLE, QUADRUPOLE...)

$$\phi(r) \propto r^{-n}$$

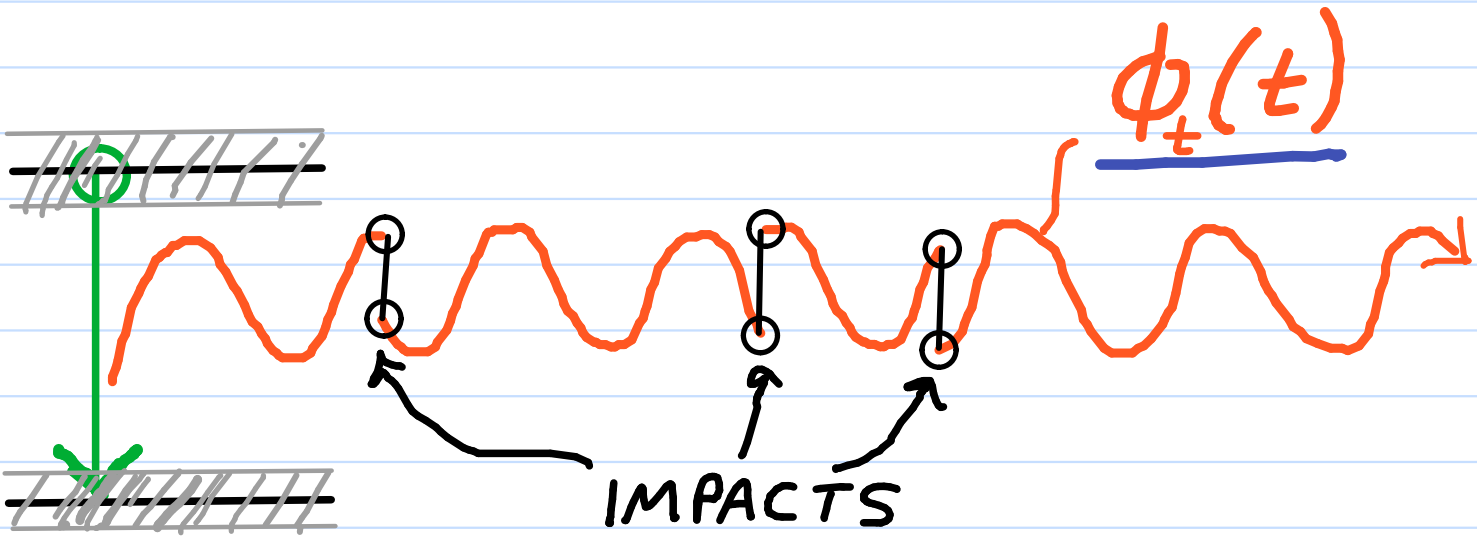
C_n = INTERACTION CONSTANT

$$\text{SHIFT } \Delta V = C_n / r^n$$

δt = MEAN IMPACT DURATION

Δt = MEAN INTERVAL BETWEEN COLLISIONS

IMPACT APPROXIMATION: $\delta t \ll \Delta t$



$$\phi_v(v-v_0) = F_T(\phi_t(t))$$

$$= \frac{\gamma_n/4\pi^2}{(v-v_0)^2 + (\gamma_n/4\pi)^2}$$

- LORENTZIAN AGAIN!

$$\text{FWHM} = \gamma_n/4\pi$$

COLLISIONAL BROADENING TYPES WELL-DESCRIBED BY IMPACT APPROX.:

- LARGER PERTURBER v
  - LOWER PERT. m
- LARGER n,

1) LINEAR STARK BROADENING:

PERTURBER: e^-

TARGET: HYDROGENIC SPECIES (INC. HI BALMER LINES)

$\therefore$ REGIME: B, A STARS

$\eta = 2$

$\therefore \Delta\nu = C_2/r^2$

DAMP. PARAM., γ_2

2) RESONANCE (SELF) BROADENING:

PERTURBER: H I

TARGET: H I

∴ REGIME: GK STARS

$n = 3$

$\therefore \Delta\nu = C_3/r^3$

DAMP. PARAM., γ_3

3) QUADRATIC STARK BROADENING:

PERTURBER: e^-, HII

TARGET: NON-HYDROGENIC SPECIES

$\therefore$ REGIME: B, A STARS

$\eta = 4$

$\therefore \Delta V = C_4 / r^4$

DAMP. PARAM., γ_4

4) VAN DER WAALS (VdW) BROADENING:

PERTURBER: H I

TARGET: NON-HYDROGENIC SPECIES

$\therefore$ REGIME: GK STARS

$\eta = 6$

$\therefore \Delta V = C_6 / r^6$

DAMP. PARAM., γ_6

Eg. VdW DAMPING DOMINATES BROAD LORENTZIAN WINGS OF MANY FRAUNHOFER LINES IN SOLAR SPECTRUM

- CaII HK, Mg II hk, Na I D LINES

TOTAL LORENTZIAN DAMPING :

$$\phi_\nu = \phi_\nu^{R} * \phi_\nu^{n=2} * \phi_\nu^{n=3} * \phi_\nu^{n=4} * \phi_\nu^{n=6}$$

$$= \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + \left(\gamma/4\pi\right)^2}$$

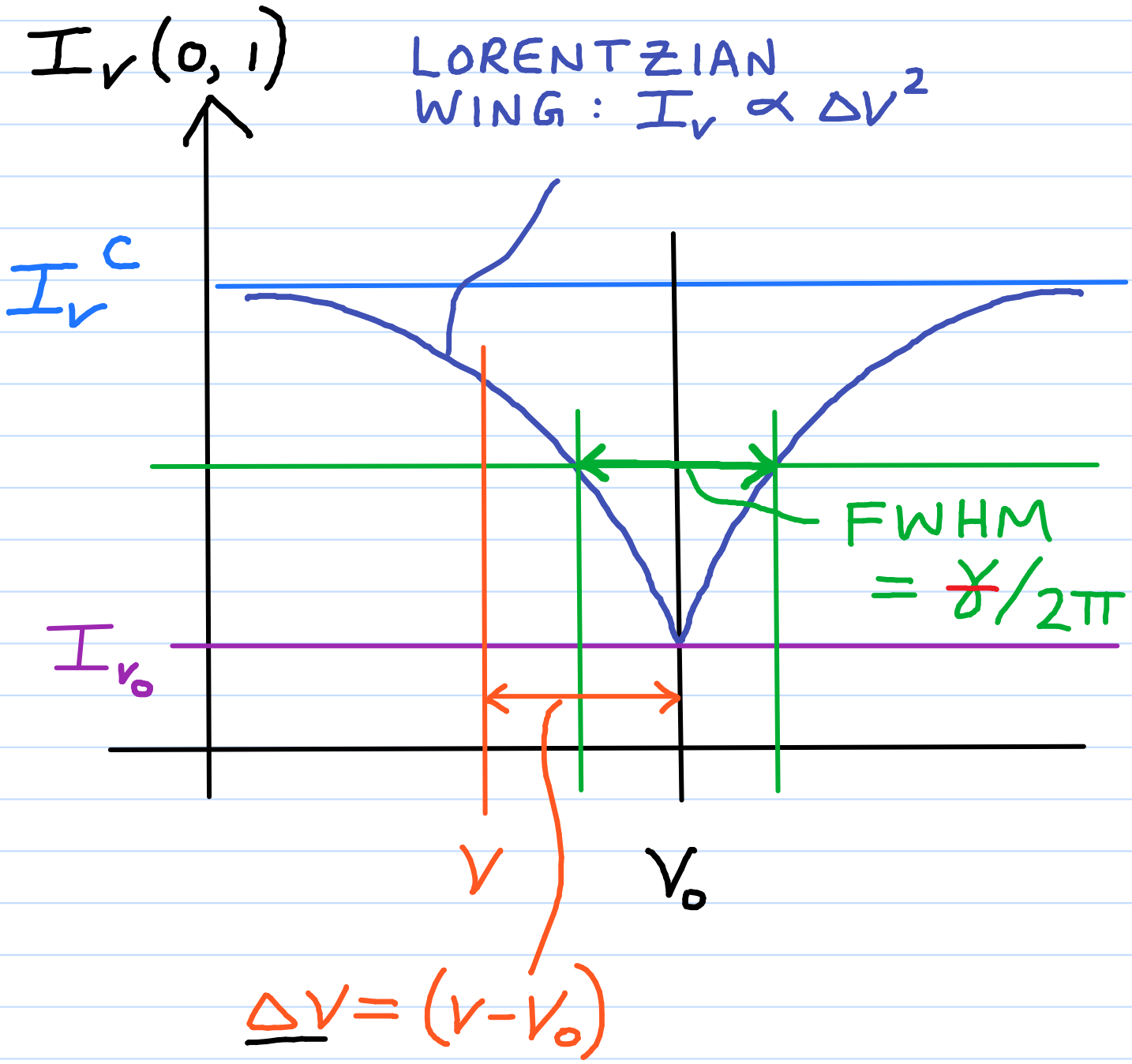
- STILL LORENTZIAN !

γ = TOTAL DAMPING PARAMETER

$$= \gamma^{R} + (\gamma_2 + \gamma_3 + \gamma_4 + \gamma_6)$$

$$= \gamma^{R} + \gamma^{C}$$

EMERGENT LINE PROFILE FROM SINGLE REPRENTATIVE ATOM AT REST :



Eg. Ca II HK LINES, SUN:

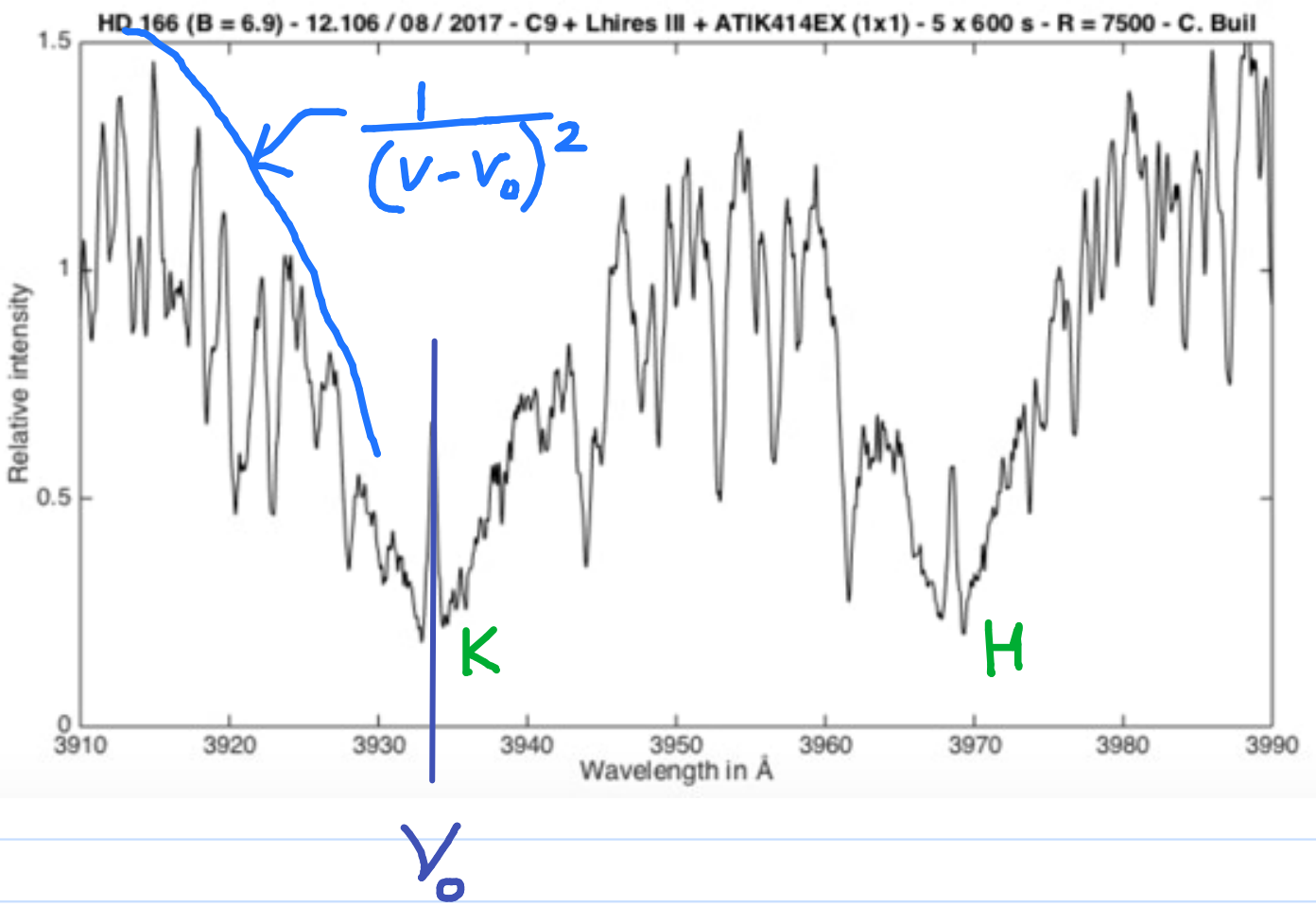


IMAGE CREDIT: ASTROSURF, CHRISTIAN BUIL

SPECIAL EXCEPTION:

LINEAR STARK BROADENING BY H II

PERTURBER: H II (p^+)

TARGET: HYDROGENIC SPECIES (INC. HI BALMER LINES)

$\therefore$ REGIME: B, A STARS

$\delta t \gg \Delta t$

$\therefore$ NEED STATIC APPROXIMATION

$\rightarrow \phi_\nu \approx$ HOLTSMARK PROFILE

- NON-LORENTZIAN