EMISSIVITY & EXTINCTION COEFFICIENTS: α_ν & j_ν:

1) b-b TRANSITIONS ($l \rightleftarrows u$)

MONOCHROMATIC VOLUME EXTINCTION COEFFIENT, α_ν^l (cm^{-1})

$$\alpha_\nu^l(\nu-\nu_0) = \frac{h\nu}{4\pi}\left\{n_l B_{lu} \phi_\nu(\nu-\nu_0) - n_u B_{ul} \chi_\nu(\nu-\nu_0)\right\}$$

2nd TERM: "CORRECTION" FOR STIM. EMISSION

- STIM. EM. IS EXACTLY -ve EXTINCTION

$$\therefore \alpha_\nu^l = \frac{h\nu}{4\pi} n_l B_{lu} \phi_\nu \left\{ 1 - \frac{n_u B_{ul} \chi_\nu}{n_l B_{lu} \phi_\nu} \right\}$$

- $\dfrac{B_{ul}}{B_{lu}} = \dfrac{g_l}{g_u}$

- CRD: $\chi_\nu = \phi_\nu$

$$\therefore \alpha_\nu^l = \frac{h\nu}{4\pi} n_l B_{lu} \phi_\nu \left\{ 1 - \frac{n_u g_l}{n_l g_u} \right\}$$

LTE BOLTZMANN DIST:

$$\frac{n_u g_l}{n_l g_u} = e^{-h\nu_0/kT_{KIN}}$$

$$\therefore \alpha_\nu^{l} = \frac{h\nu}{4\pi} n_l B_{lu} \phi_\nu \left(1 - e^{-h\nu_0/kT_{KIN}}\right)$$

$\therefore$ LTE CORRECTION FOR STIM. EM.

$$= \left(1 - e^{-h\nu_0/kT_{KIN}}\right)$$

TOTAL LINE EXTINCTION COEFFICIENT, $\alpha^{l}_{\nu_0}$:

- ASSUMING CRD: $\chi_\nu = \phi_\nu$

$$\alpha^{l}_{\nu_0} =$$

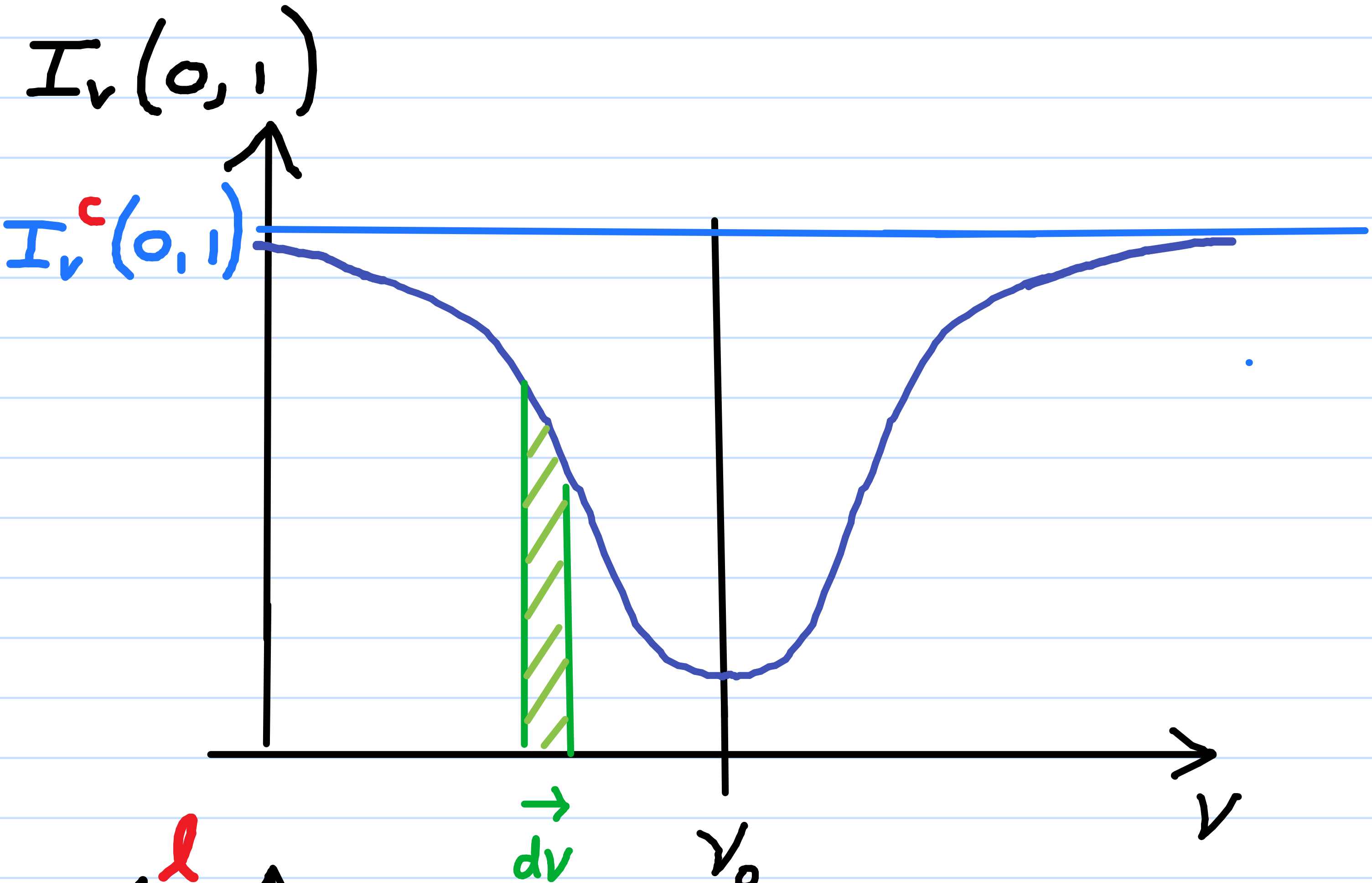
$$\frac{1}{4\pi}\left(n_l B_{lu} - n_u B_{ul}\right) \int_{\nu=0}^{\infty} h\nu\, \phi_\nu(\nu-\nu_0)\, d\nu$$

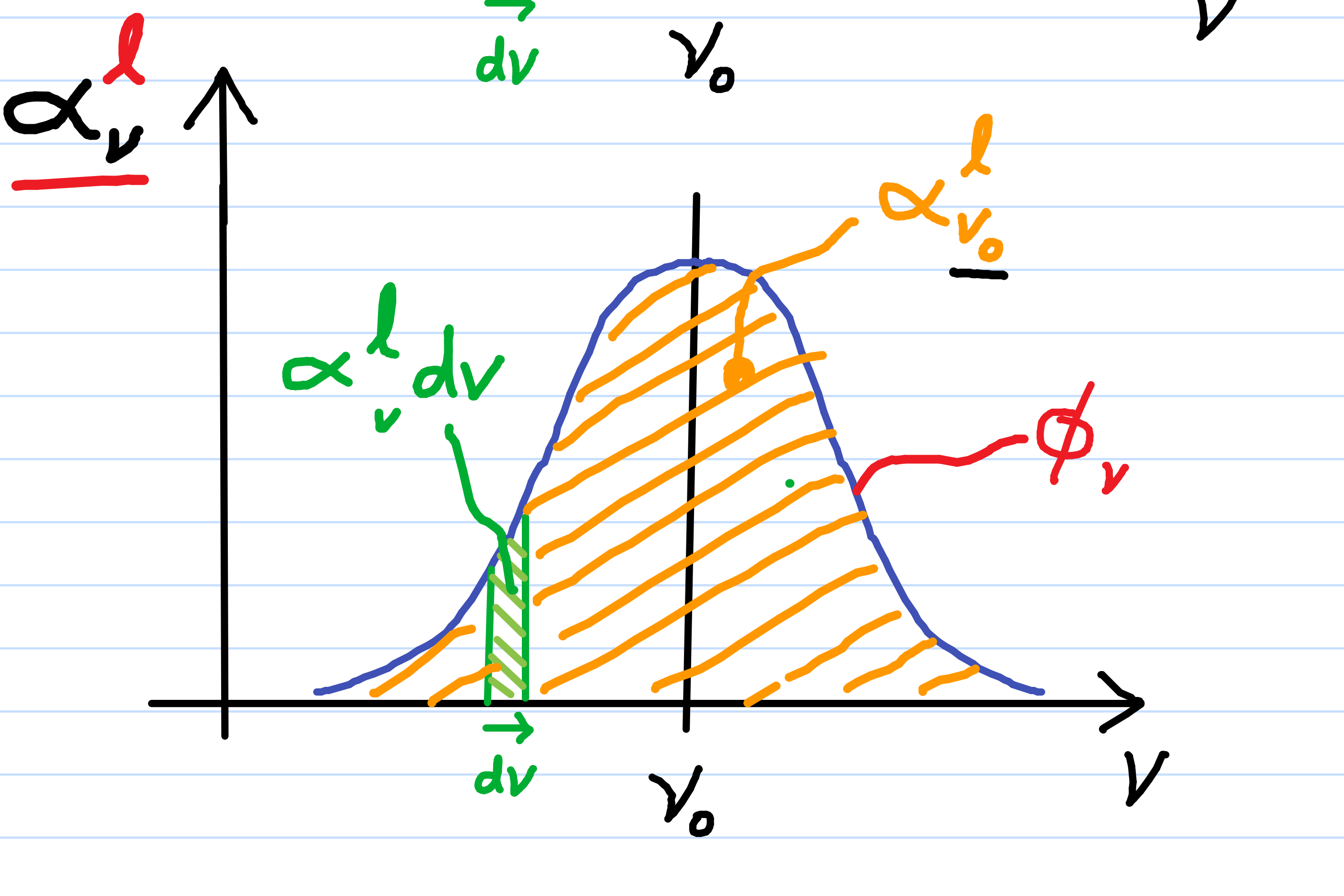
IF LINE NARROW & SYMMETRIC:

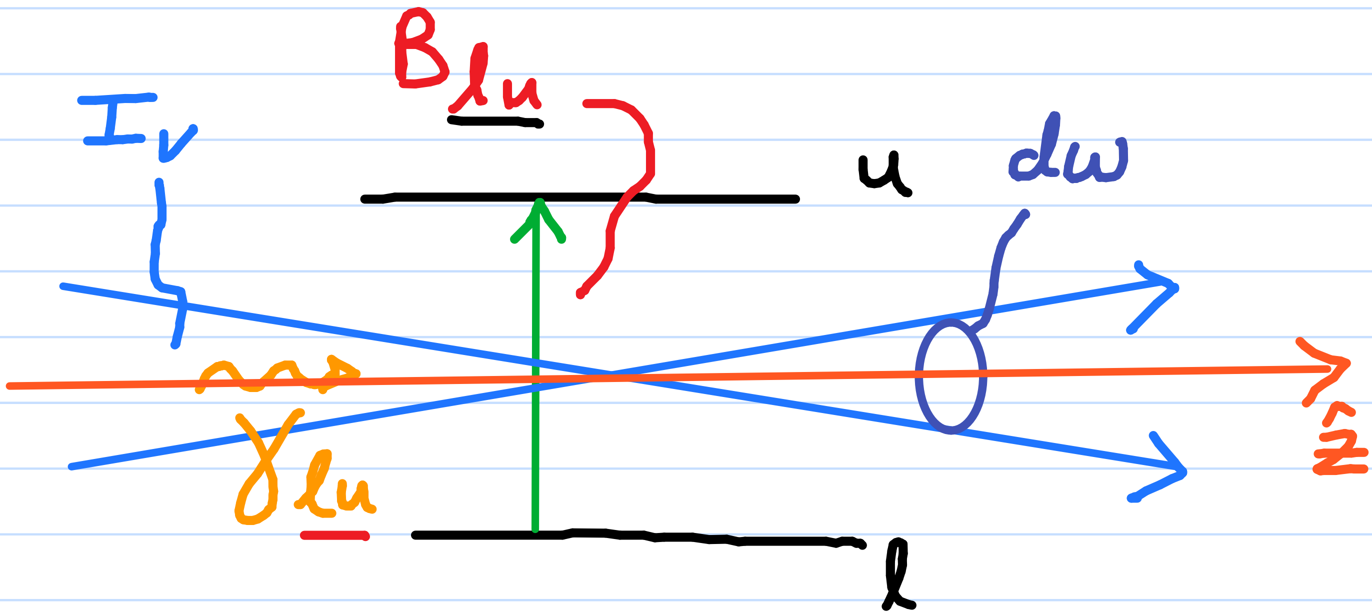
$$\alpha^{l}_{\nu_0} \approx \frac{h\nu_0}{4\pi}\left(n_l B_{lu} - n_u B_{ul}\right)$$

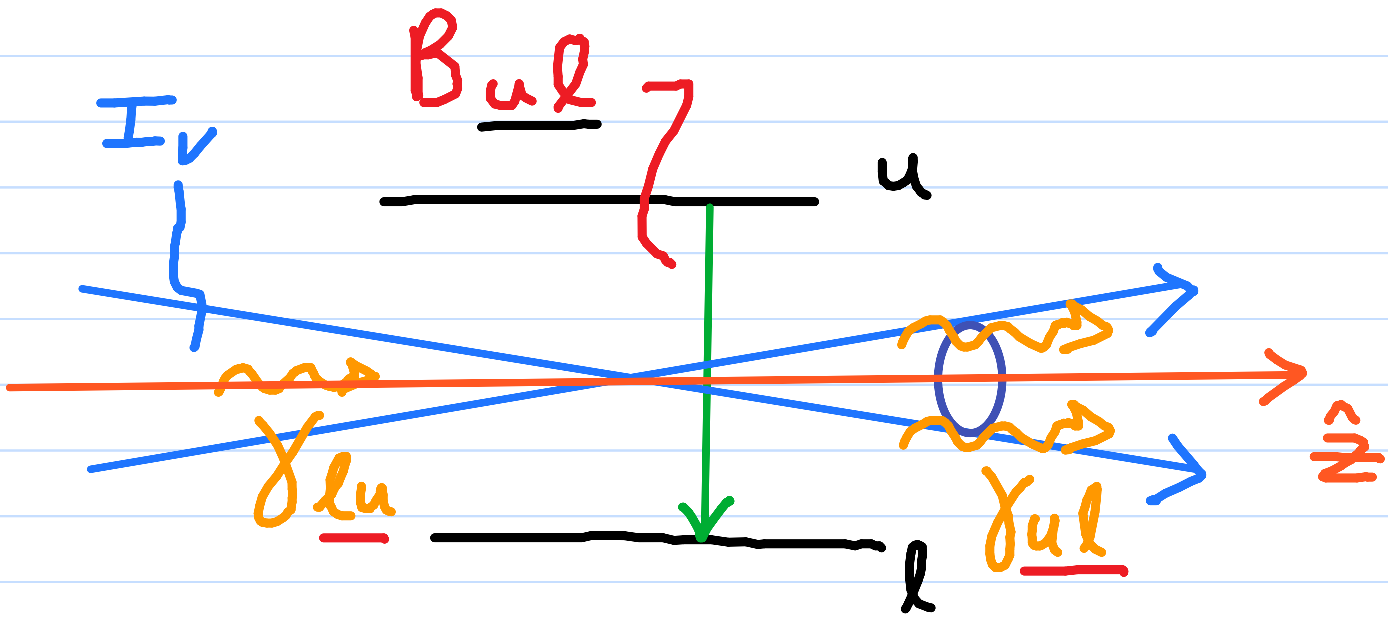
- TOTAL TRANSITION "STRENGTH"

LINE EXTINCTION, α_ν^l :









RAD. TRANS. Eq., LINE EXTINCTION ONLY:

$$dI_\nu(z) = -\alpha^l_\nu(z)\, I_\nu(z)\, dz$$

MONOCHROMATIC LINE EXTINCTION

PER PARTICLE, σ_ν^ℓ :

$$\sigma_\nu^\ell(\nu-\nu_0) = \frac{h\nu}{4\pi} B_{lu} \phi_\nu(\nu-\nu_0)$$

TOTAL LINE EXTINCTION

CROSS-SECTION, $\sigma_{\nu_0}^\ell$ (cm²)

$$\sigma_{\nu_0} = \frac{B_{lu}}{4\pi} \int_0^\infty h\nu \, \phi_\nu(\nu-\nu_0) \, d\nu$$

$$\approx \frac{h\nu_0}{4\pi} B_{lu}$$

OSCILLATOR STRENGTH, f_{lu}

(UNITLESS)

$$\frac{\pi e^2}{m_e c} f_{lu} \equiv \frac{h\nu_0}{4\pi} B_{lu} \approx \sigma^{l}_{\nu_0}$$

$$\text{"gf-VALUE"} \equiv g_l f_{lu}$$

RECALL: $A_{ul} = B_{lu} \frac{g_l}{g_u} \frac{2h\nu_0^3}{c^2}$

$$\therefore A_{ul} = \frac{(g_l f_{lu})}{g_u} (h\nu_0)^2$$

MONOCHROMATIC VOLUME EMISSION COEFFICIENT, j_ν^l

(erg/s/cm^3/STER/Hz)

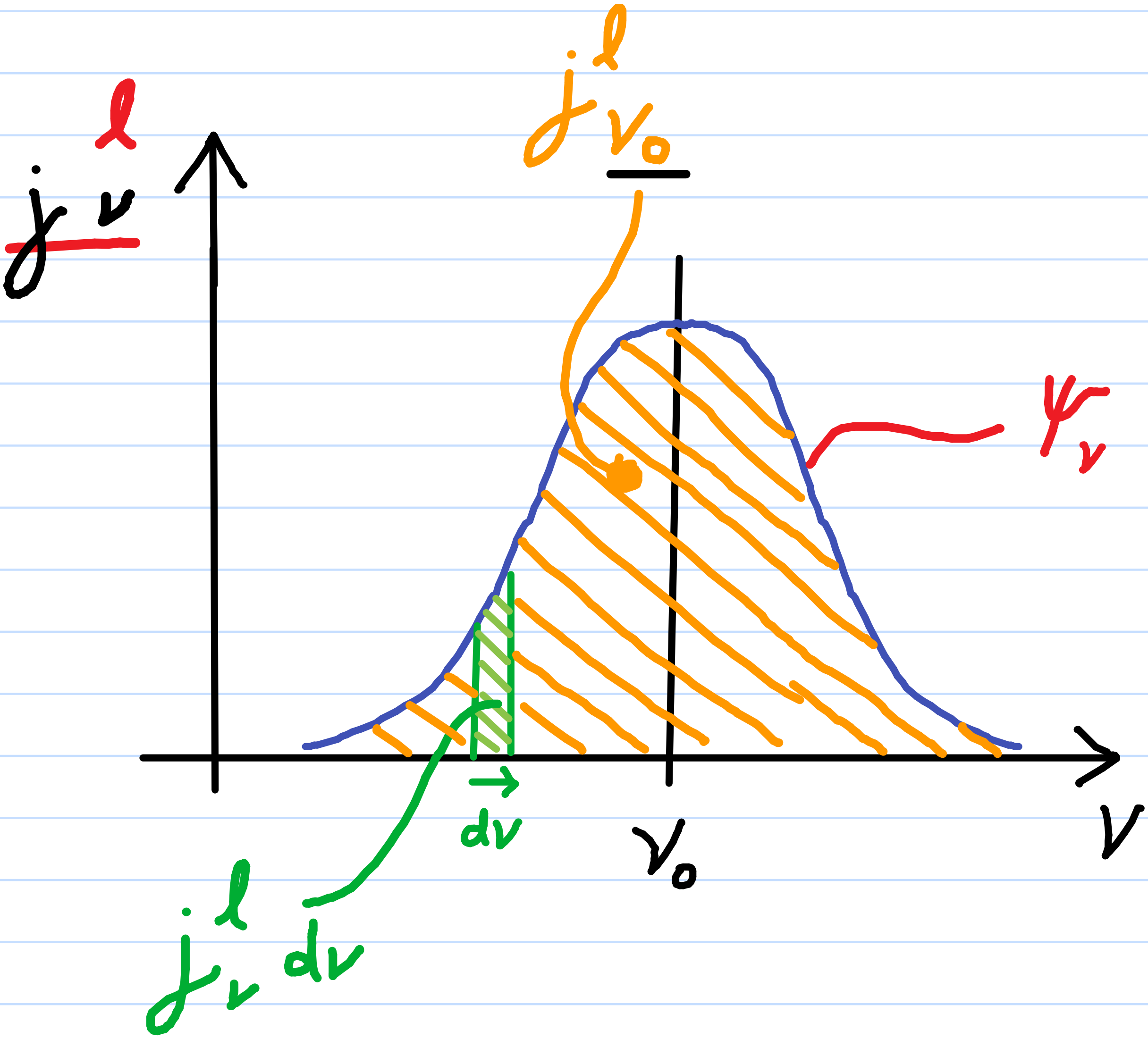
$$j_\nu^l(\nu-\nu_0) = \frac{h\nu}{4\pi}\left(n_u A_{ul}\,\psi_\nu(\nu-\nu_0)\right)$$

TOTAL LINE EMISSIVITY, $j_{\nu_0}^l$:

$$j_{\nu_0}^l = \frac{n_u}{4\pi} A_{ul} \int_0^\infty h\nu\,\psi_\nu(\nu-\nu_0)\,d\nu$$

$$\approx \frac{h\nu_0}{4\pi} n_u A_{ul}$$

LINE EMISSIVITY:



MONOCHROMATIC LINE SOURCE fn, S_ν^ℓ

$$S_\nu^\ell(\nu-\nu_0) \equiv \frac{j_\nu^\ell(\nu-\nu_0)}{\alpha_\nu^\ell(\nu-\nu_0)}$$

$$= \frac{\left(h\nu/4\pi\right) n_u A_{ul}\, \psi_\nu}{\left(h\nu/4\pi\right)\left\{n_\ell B_{\ell u}\,\phi_\nu - n_u B_{ul}\,\chi_\nu\right\}}$$

- ASSUME CRD: $\psi_\nu = \chi_\nu = \phi_\nu$

$$\therefore S_{\nu_0}^\ell = \frac{n_u A_{ul}}{n_\ell B_{\ell u} - n_u B_{ul}}$$

$$= \frac{A_{ul}/B_{ul}}{\dfrac{n_\ell B_{\ell u}}{n_u B_{ul}} - 1}$$

RECALL:

$$\frac{A_{ul}}{B_{ul}} = \frac{2h\nu_0^3}{c^2} \; ; \quad \frac{B_{lu}}{B_{ul}} = \frac{g_u}{g_l}$$

$$\therefore \; S_{\nu_0}^{l} = \frac{2h\nu_0^3}{c^2} \frac{1}{\frac{g_u n_l}{g_l n_u} - 1}$$

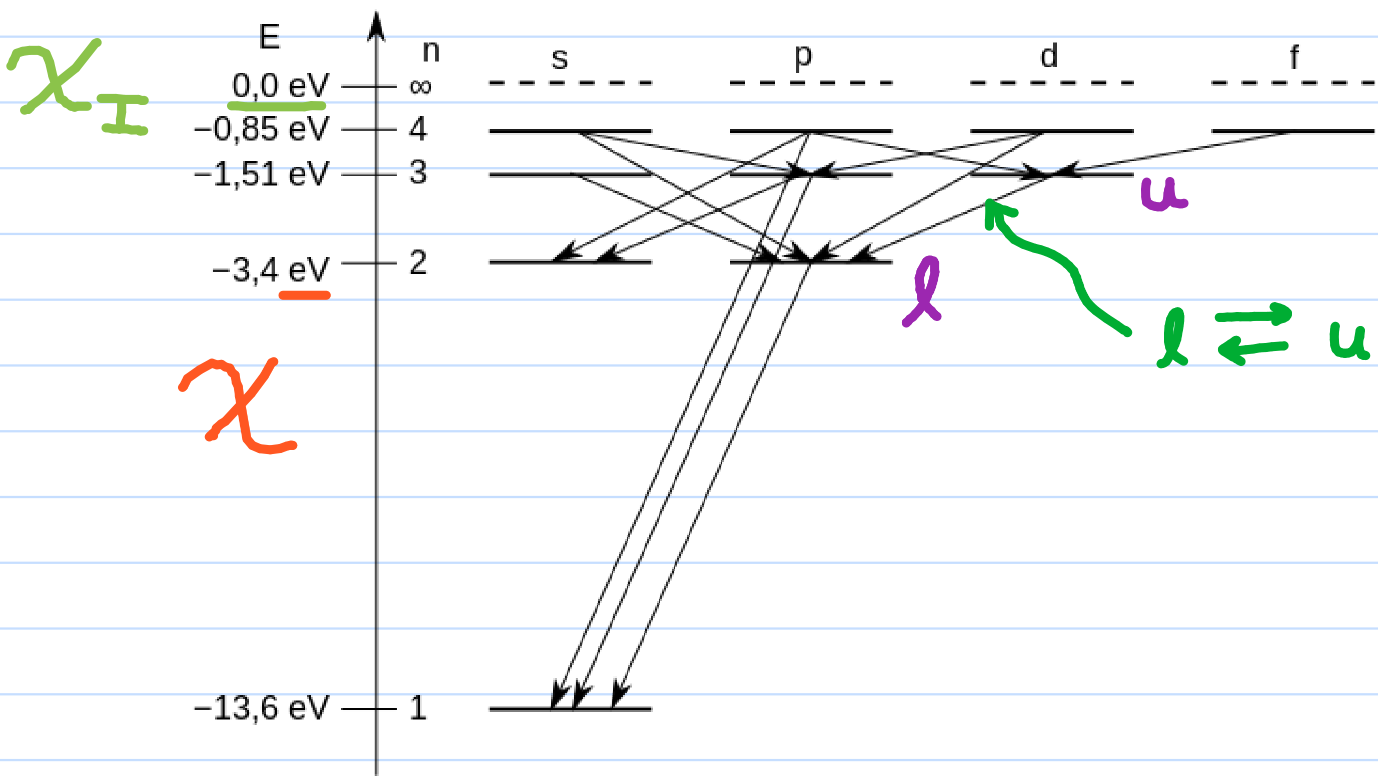
IN LTE: $\frac{g_u n_l}{g_l n_u} = e^{h\nu_0/kT_{KIN}}$

$$\therefore \; S_{\nu_0}^{l} = \frac{2h\nu_0^3}{c^2} \frac{1}{e^{h\nu_0/kT} - 1} = B_{\nu_0}(T_{KIN})$$

Eg. H I:

ATOMIC E-LEVELS &

b-b TRANSITIONS



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