ATMOSPHERIC APPROXIMATIONS:

SURFACE APPROXIMATIONS ($\tau_\nu = 0$):

- SIGNIFICANT: $I_\nu(0, \mu)$, $\mathcal{F}_\nu(0)$
  OBSERVABLE

1) 2^{nd} EDDINGTON APPROX.:

- 3D HOMOGENEITY
  $\therefore$ ISOTHERMAL ($T_{KIN}(\tau_\nu) = C$)

  (i.e. "LAMBERT RADIATOR")

$$\therefore S_\nu(\tau_\nu) = B_\nu(T_{KIN} = C) = a_0$$

Formal Sol'n at surface ($\tau_r = 0$):

$$I_r^-(0,\mu) = 0, \quad -1 < \mu < 0$$

$$I_r^+(0,\mu) = \frac{1}{\mu}\int_{t_r=0}^{\infty} S_r(t_r)\, e^{-t_r/\mu}\, dt_r$$

$$= a_0 \int_0^{\infty} \frac{e^{-t_r/\mu}}{\mu}\, dt_r = -a_0 \left(e^{-t_r/\mu} \right|_{t_r=0}^{\infty}$$

$$= a_0, \quad 0 < \mu < 1$$

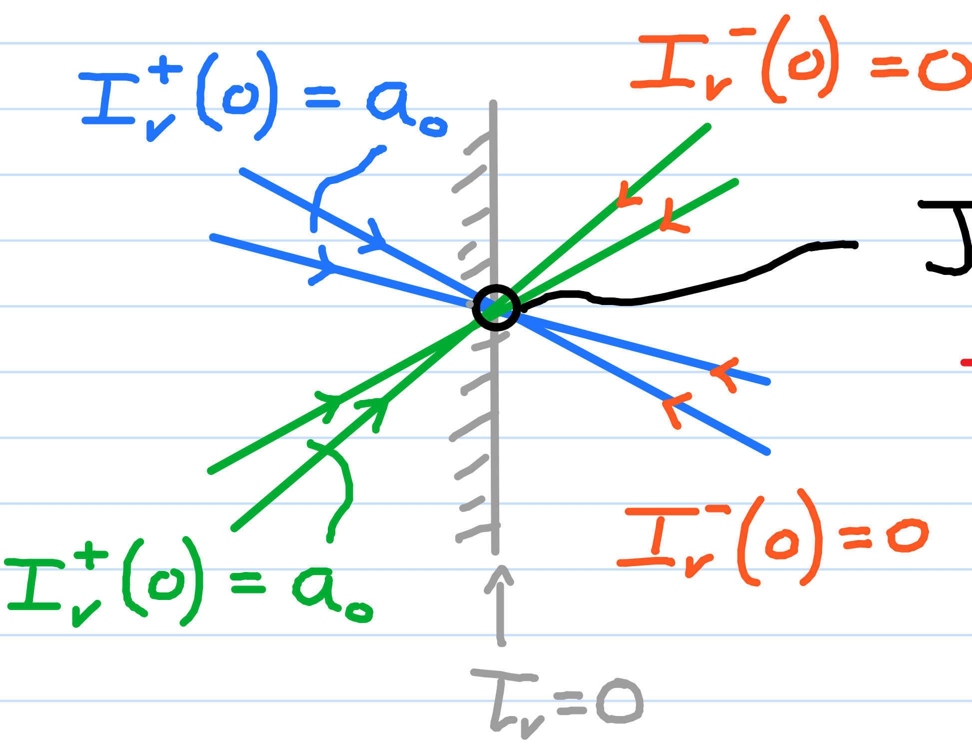
$$\therefore I_r^+(0,\mu) = a_0 = \underline{S_r}, \quad 0 < \mu < 1$$

THEN:

$$J_\nu(0) = \frac{1}{2}\int_{-1}^{1} I_\nu(0,\mu)\,d\mu$$

$$= \frac{1}{2}\int_{-1}^{0} \underbrace{I_\nu^-(0,\mu)}_{0}\,d\mu + \frac{1}{2}\int_{0}^{1} I_\nu^+(0,\mu)\,d\mu$$

$$= \frac{a_0}{2}\int_0^1 d\mu = \underline{\frac{a_0}{2}} = \underline{\frac{I_\nu^+(0)}{2}}$$



$$\underline{J_\nu(0) = \frac{I_\nu(0)}{2}}$$

SIMILARLY:

$$\mathcal{F}_\nu^+(0) = \pi I_\nu^+(0) = 2\pi J_\nu(0)$$

2nd EDDINGTON APPROXIMATION:

ANY REAL STAR:

$$\mathcal{F}_\nu(0) \approx 2\pi J_\nu(0)$$

2) EDDINGTON - BARBIER (E-B) APPROXIMATION

ASSUME:

$$S_\nu(\tau_\nu) = a_0 + a_1 \tau_\nu \; ; \; a_0 \; \& \; a_1 > 0$$

- IMPOSSIBLE FOR $B_\nu(\tau_\nu)$ AT ALL ν VALUES

FORMAL Sol'n AT $\tau_\nu = 0$:

FOR $0 < \mu < 1$:

$$I_\nu^+(0,\mu) = \frac{1}{\mu}\int_0^\infty (a_0 + a_1 t_\nu)\, e^{-t_\nu/\mu}\, d\nu$$

$$= a_0 \int_0^\infty \frac{e^{-t_\nu}}{\mu}\, dt_\nu + a_1 \int_0^\infty \frac{t_\nu e^{-t_\nu/\mu}}{\mu}\, dt_\nu$$

$$I_\nu^+(0,\mu) = a_0 + a_1 \mu$$

$$\boxed{\therefore\ I_\nu^+(0,\mu) = S_\nu(\tau_\nu = \mu)}$$

EDDINGTON-BARBIER (E-B) RELATION

AND:

$$J_\nu(0) = a_0/2 + a_1/4$$
$$= \tfrac{1}{2}\, S_\nu(\tau_\nu = \tfrac{1}{2})$$

$$\mathcal{F}_\nu^+(0) = \pi a_0 + \frac{2\pi}{3} a_1$$
$$= \pi S_\nu(\tau_\nu = \tfrac{2}{3})$$

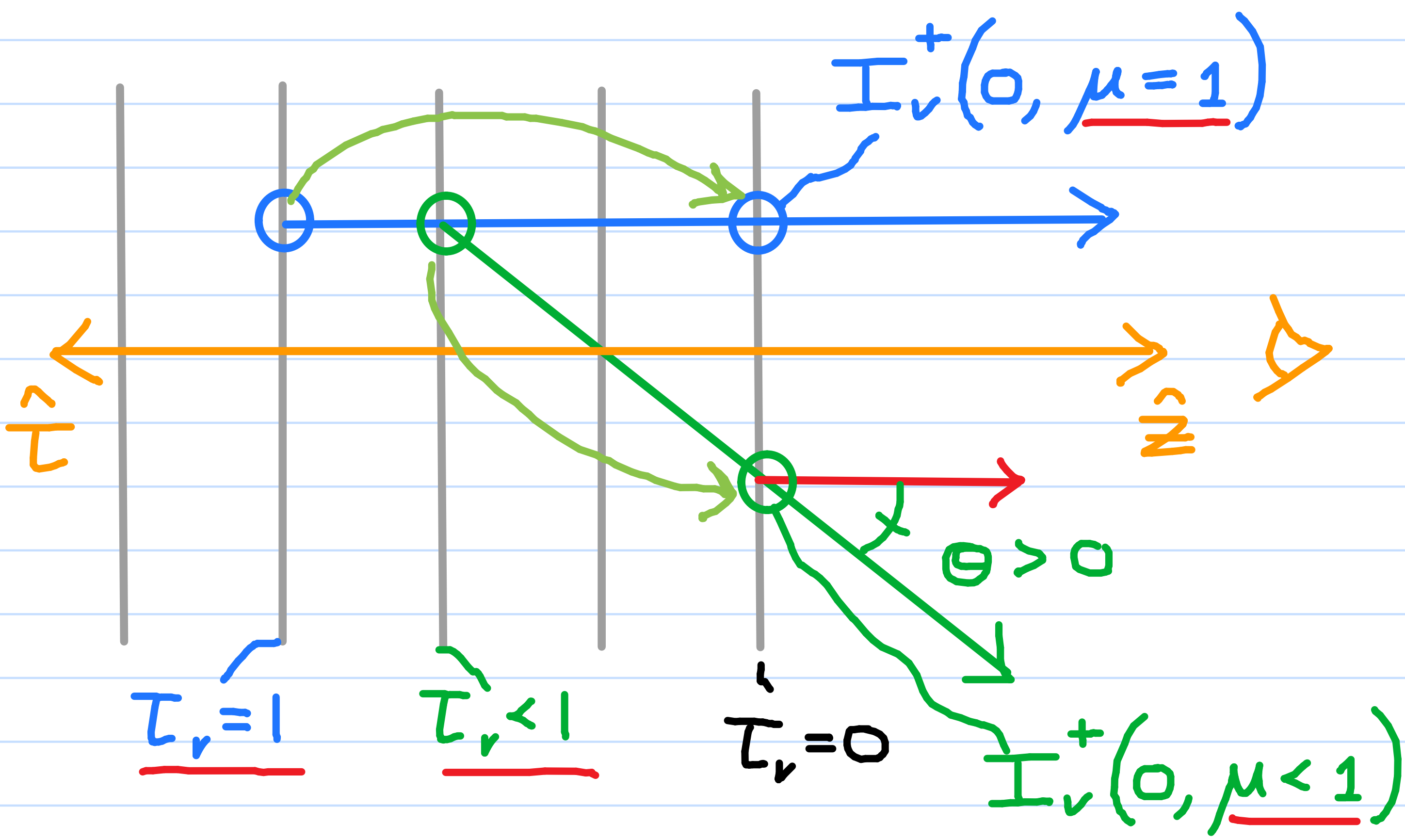
EDDINGTON - BARBIER (E-B) APPROXIMATION:

ANY REAL STAR:

$$\therefore\ I_\nu^+(0, \mu) \approx S_\nu(\tau_\nu = \mu)$$

EDDINGTON-BARBIER (E-B) APPROXIMATION

$$\therefore \; I_\nu^+(0,\mu) \approx S_\nu(\tau_\nu = \mu):$$



FOR $\mu = 1$ (RADIAL BEAM):

$$I_\nu^+(0,1) \approx S_\nu(1)$$

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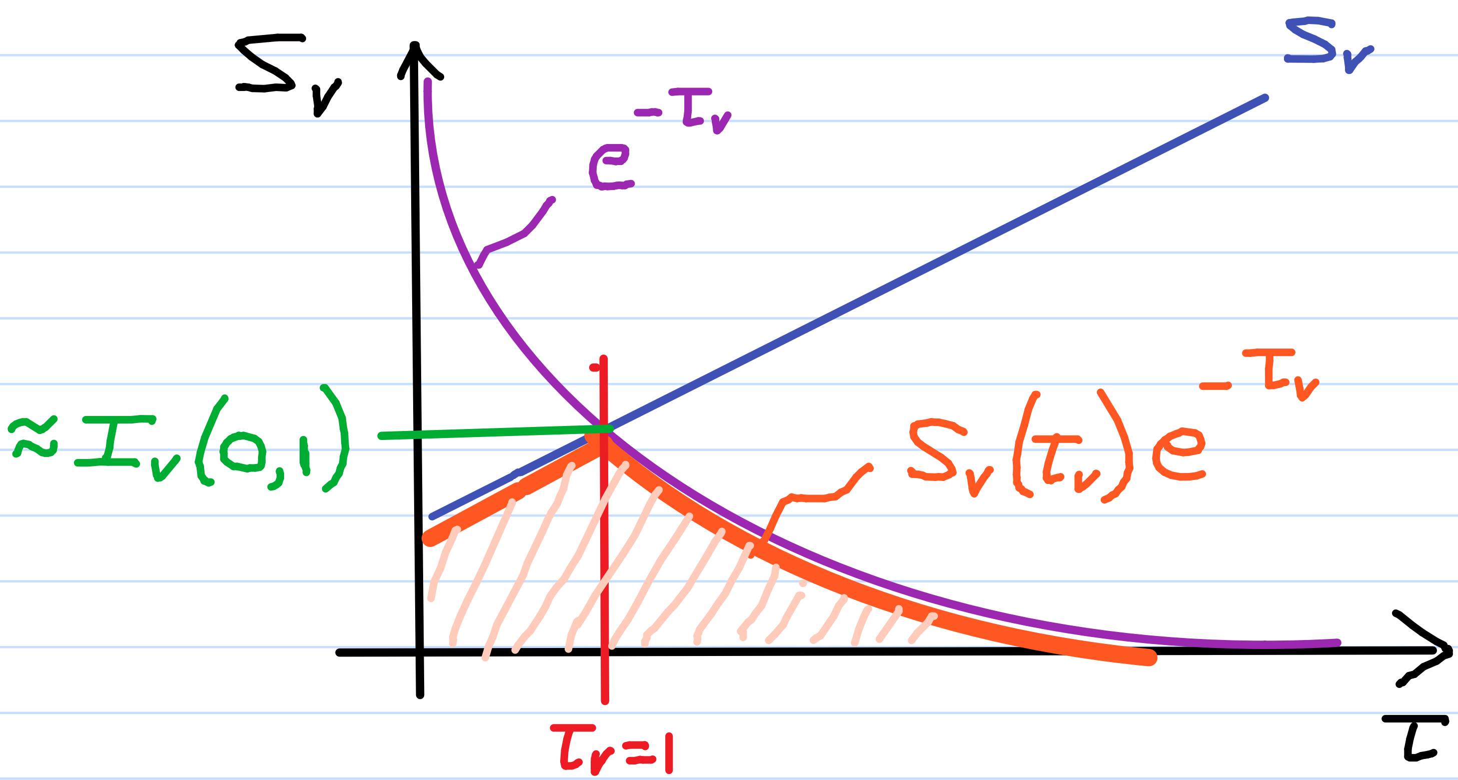
FORMAL Soln:

$$I_\nu^+(0,1) = \int_0^\infty S_\nu(t_\nu)\, e^{-t_\nu}\, dt_\nu \approx S_\nu(\tau_\nu = 1)$$



FOR $0 < \mu < 1$:

$$I_\nu^+(0,\mu) \approx S_\nu(\mu)$$

FORMAL Soln.

$$I_\nu^+(0,\mu) = \int_0^\infty S_\nu(t_\nu)\, e^{-t_\nu/\mu} \frac{dt_\nu}{\mu} \approx S_\nu(\tau_\nu/\mu = 1)$$

$$= S_\nu(\tau_\nu = \mu)$$

LTE E-B RELATION:

LTE: $S_\nu(\tau_\nu) = B_\nu(T_{KIN}(\tau_\nu)) =$ "$B_\nu(\tau_\nu)$"

$\therefore$ E-B RELATION:

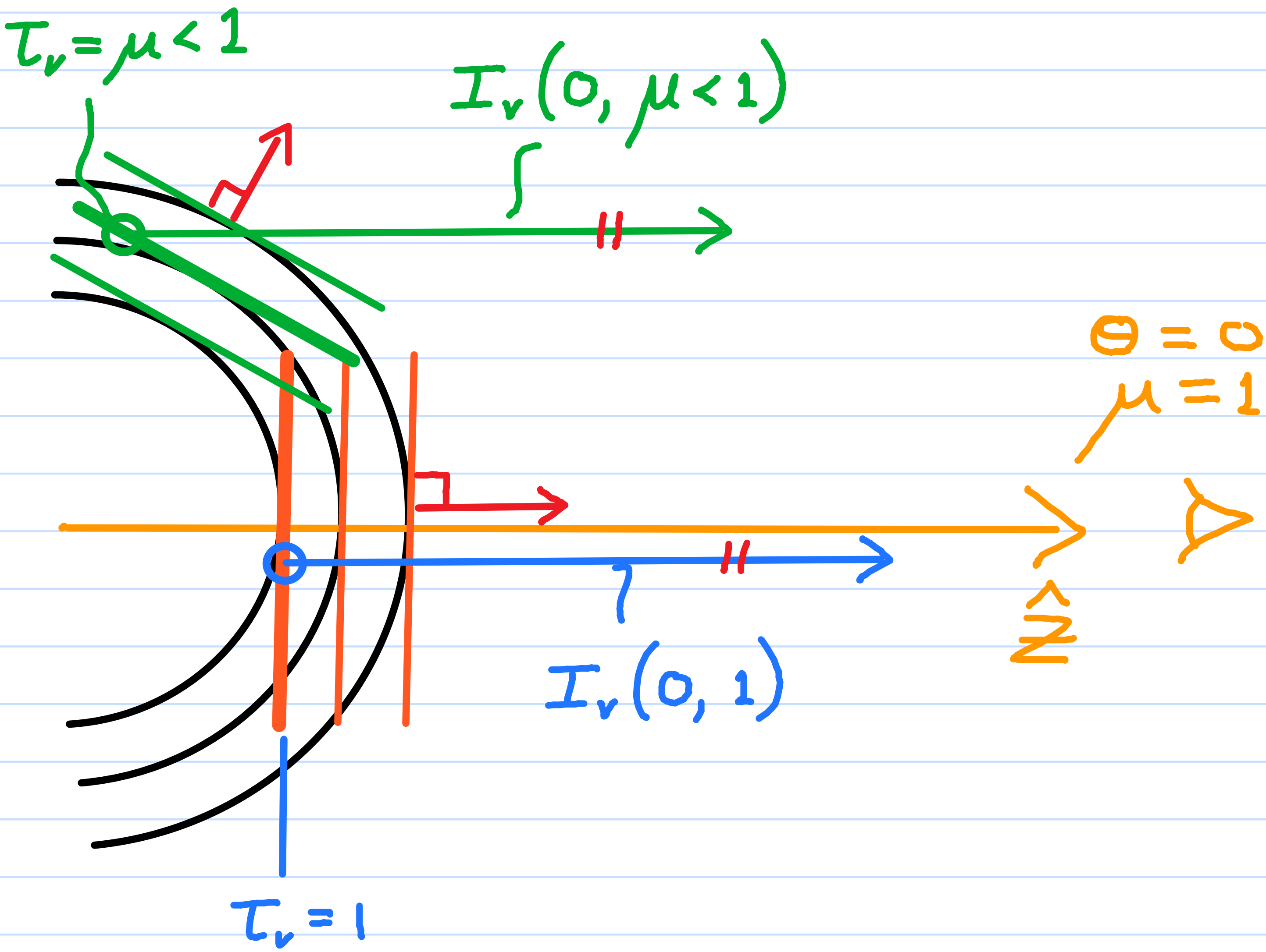
$$I_\nu^+(0,\mu) = B_\nu(T_{KIN}(\tau_\nu = \mu)) = B_\nu(\mu)$$

CAN MEASURE $T_{KIN}(\tau_\nu)$

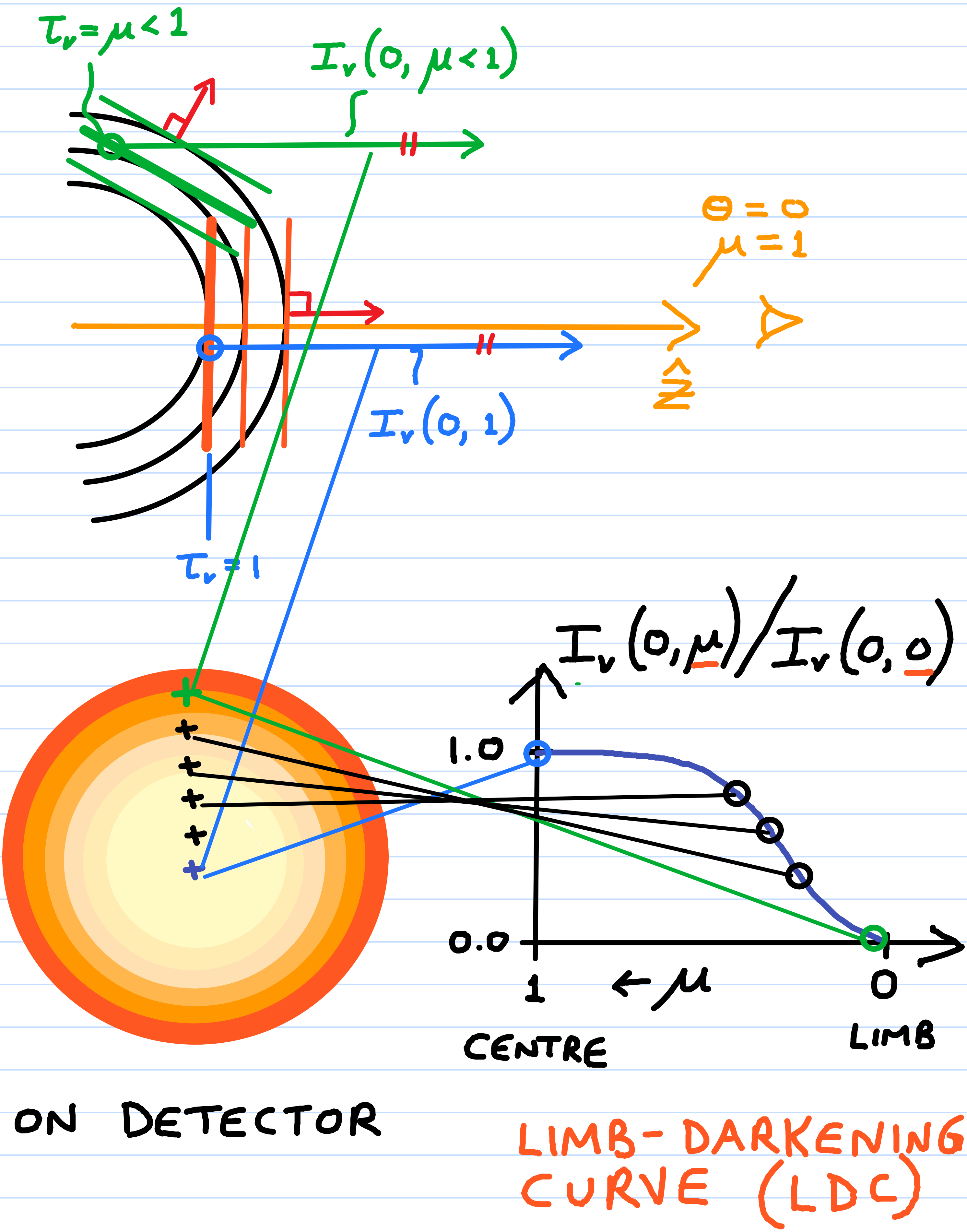
APPLICATIONS OF THE LTE E-B RELATION:

1) SOLAR LIMB DARKENING:

SUN IS SPATIALLY RESOLVED:



1) SOLAR LIMB DARKENING:



LTE E-B RELATION:

$$I_\nu^+(0,\underline{\mu}) \approx B_\nu(\tau_\nu=\underline{\mu})$$

IN SOLAR ATMOSPHERE:

$$\frac{dT_{KIN}}{d\tau_\nu} > 0$$

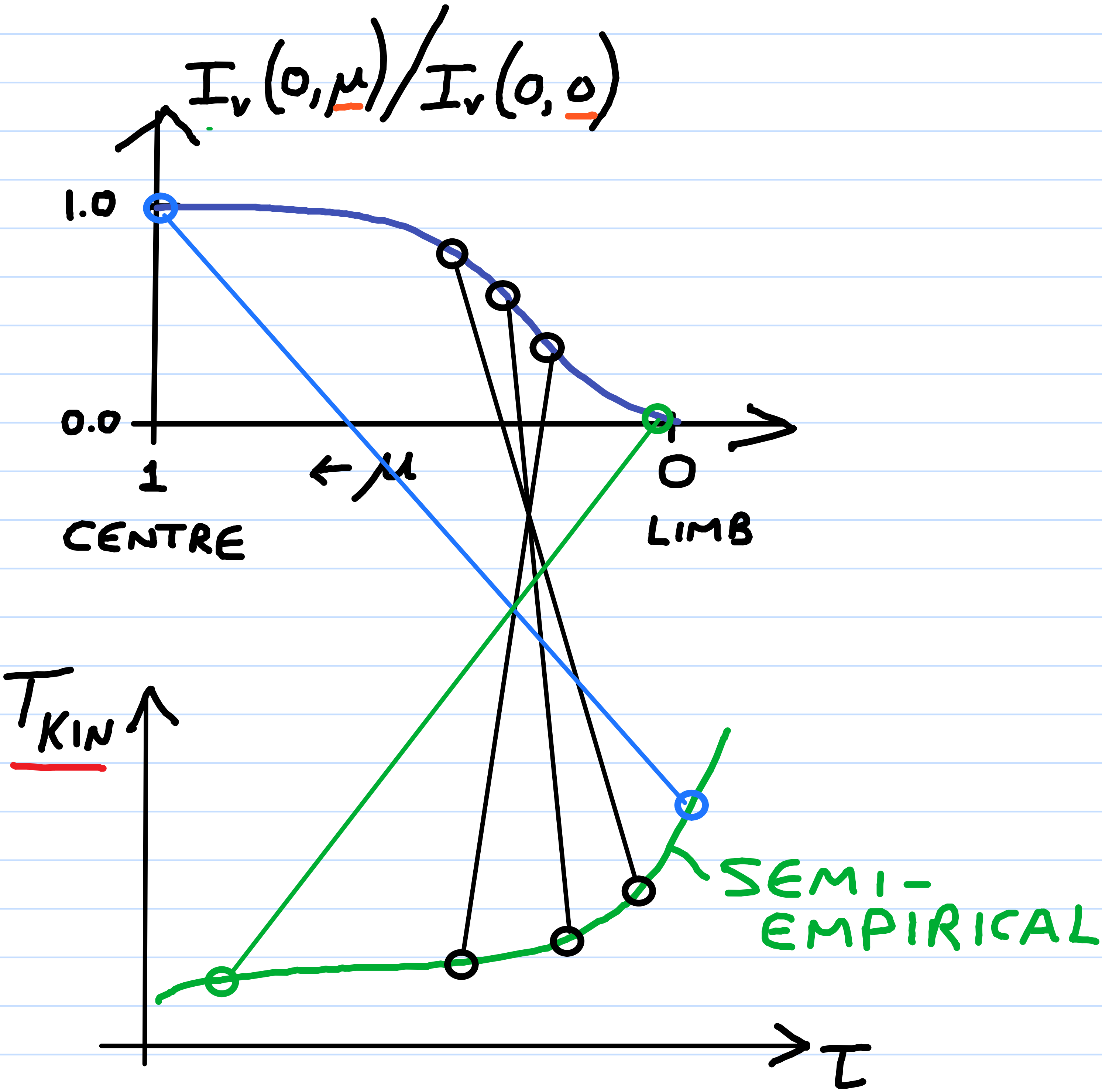
$$\therefore T_{KIN}(\tau_\nu=1) > T_{KIN}(\tau_\nu=\mu<1)$$

$$\therefore B_\nu(\tau_\nu=1) > B_\nu(\tau_\nu=\mu<1)$$

$$\therefore \underline{I_\nu^+(0,\tau_\nu=1) > I_\nu^+(0,\mu<1)}$$

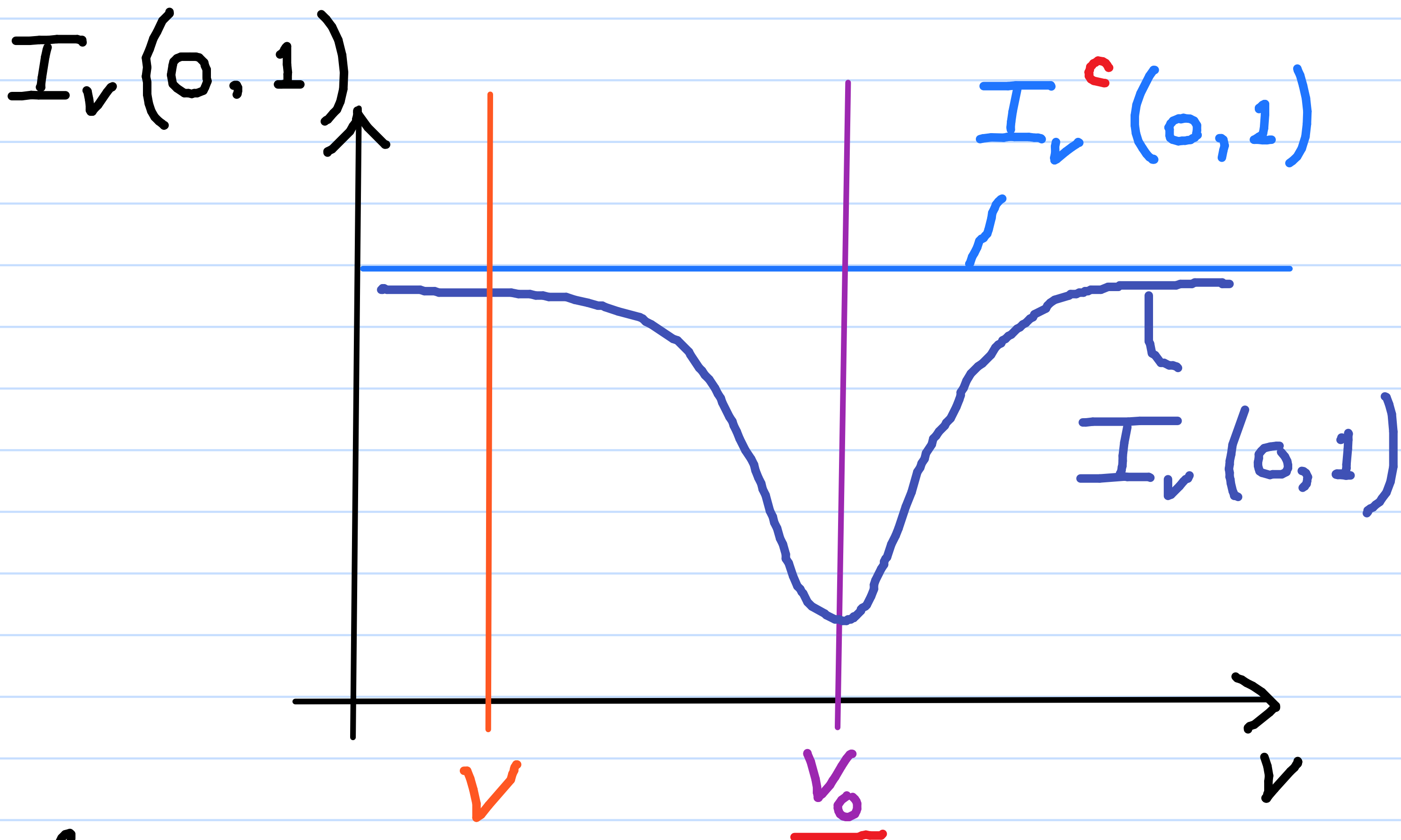
USEFUL: USE LDC TO MEASURE $T_{KIN}(\tau_\nu)$ STRUCTURE

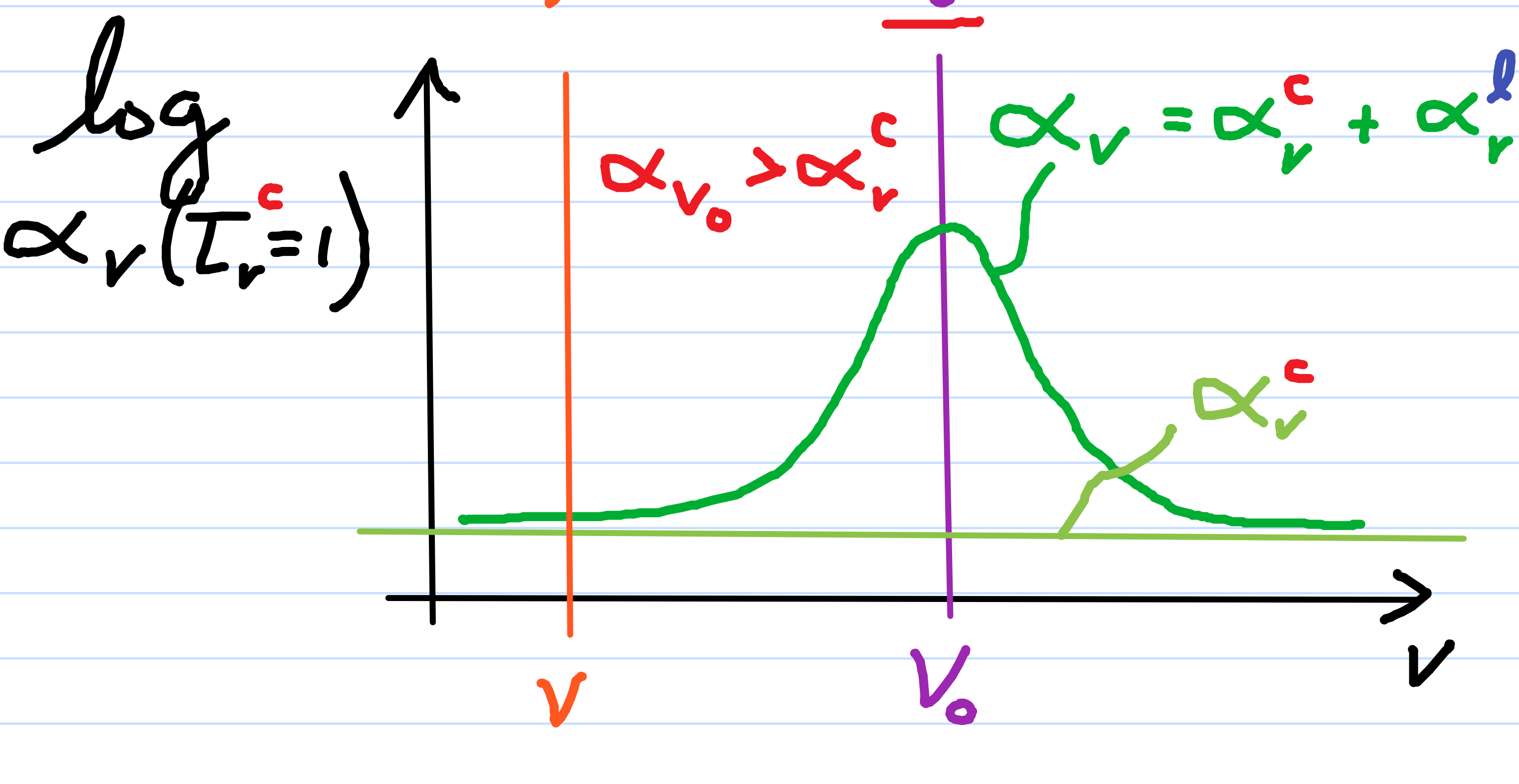
$$I_\nu^+(0,\mu) \approx B_\nu(T_{KIN}(\tau_\nu = \mu))$$



2) SPECTRAL ABSORPTION LINES:

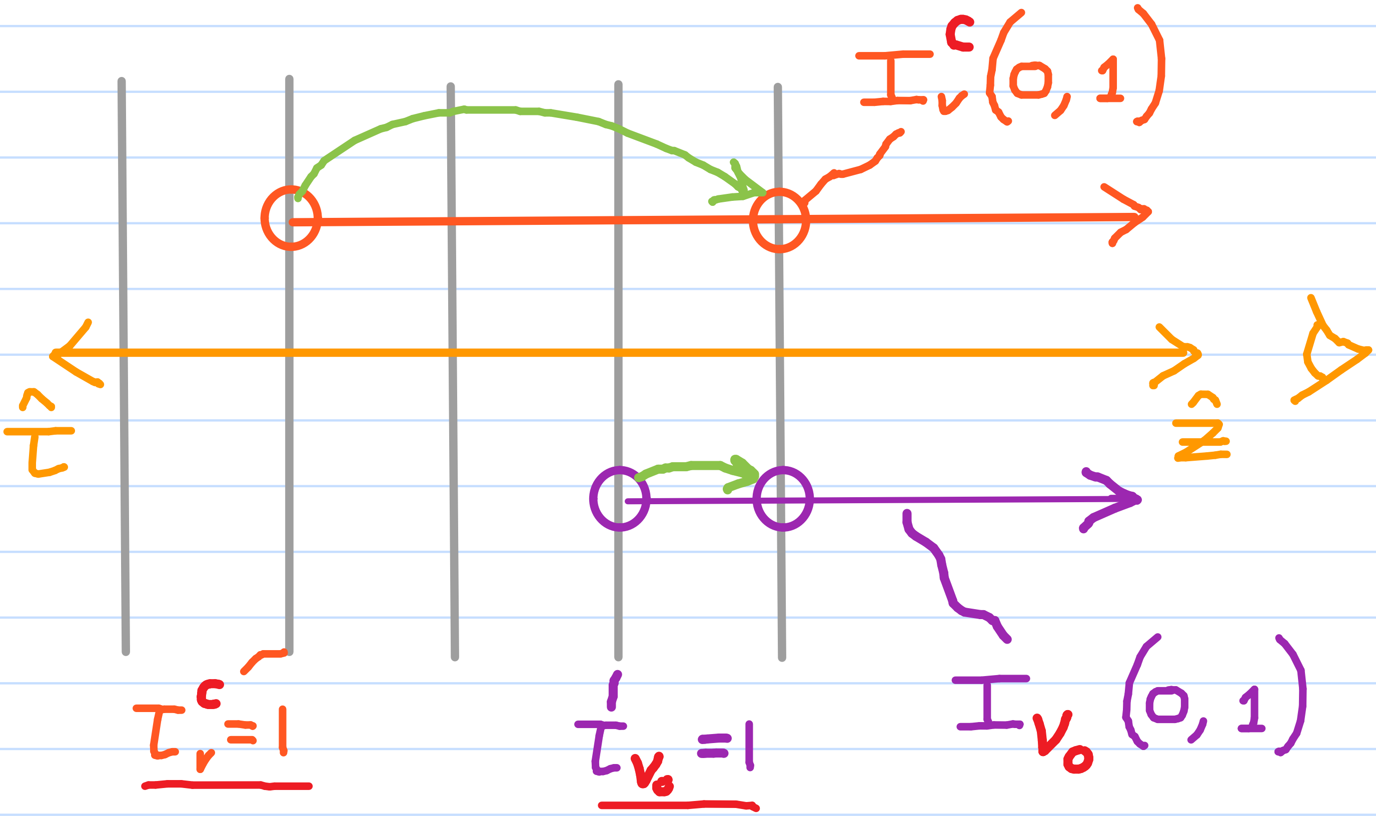
$\mu = 1$ (RADIAL):





FOR $\mu = 1$ (RADIAL BEAM):

$$I_\nu^+(0,1) \approx S_\nu(1)$$



MONOCHROMATIC τ_ν-SCALES AT

$\nu = \nu_0$ & $\nu \lesssim \nu_0$:

$$d\tau_{\nu_0}(z) = \alpha_{\nu_0}(z)\,dz$$

$$d\tau_\nu^c(z) = \alpha_\nu^c(z)\,dz$$

RECALL: $\alpha_{\nu_0}(z) > \alpha_\nu^c(z)$, ALL z

$\therefore\ z(\tau_{\nu_0}=1) > z(\tau_\nu^c=1)$

$$\frac{dT_{KIN}(z)}{dz} < 0$$

$$\therefore\ \frac{dB_\nu(z)}{dz} < 0$$

LTE E-B RELATION:

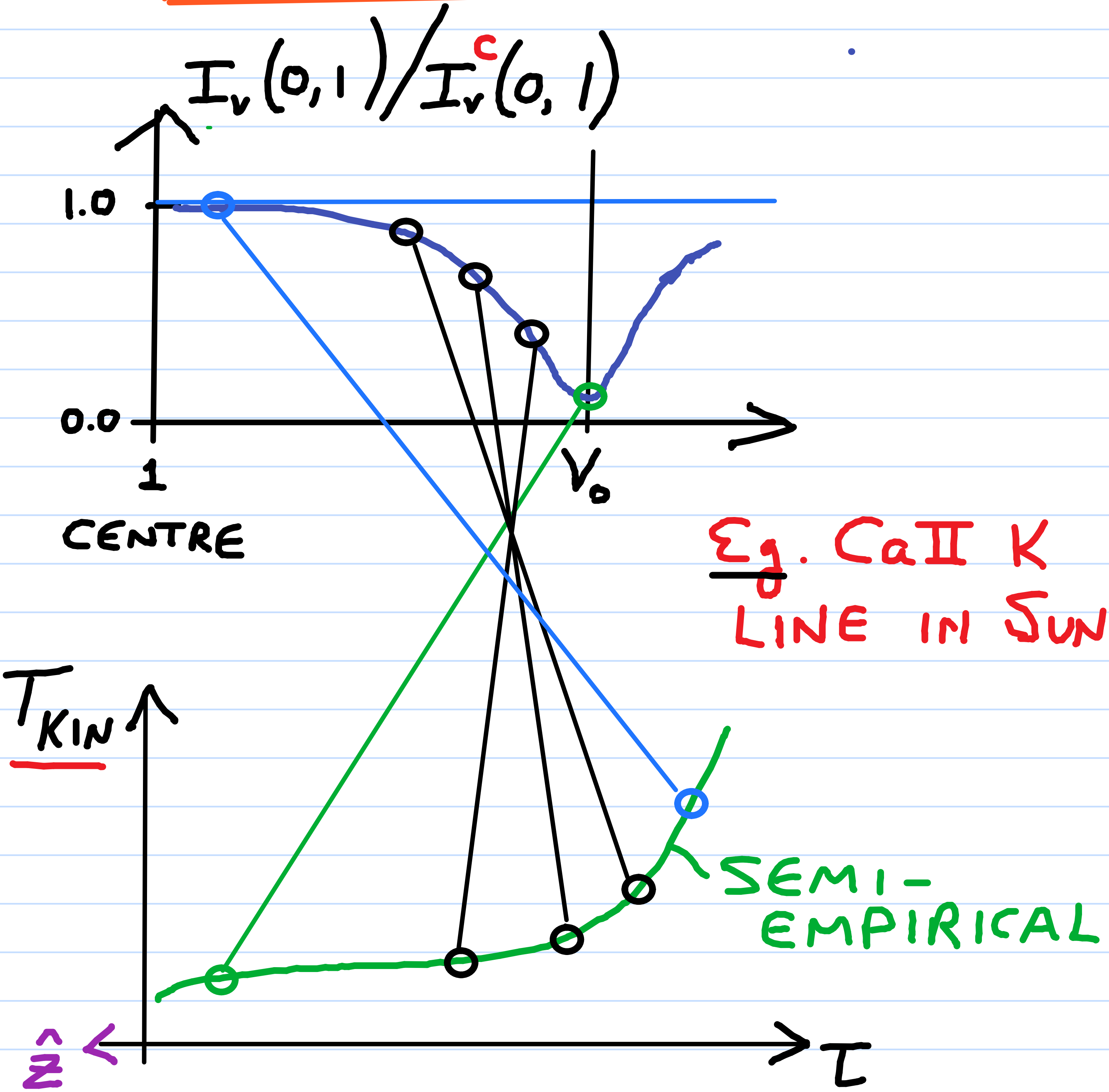
$$I_\nu^+(0,1) \approx B_\nu(z(\tau_\nu=1))$$

$$\therefore\ B_\nu(z(\tau_{\nu_0}=1)) < B_\nu(z(\tau_\nu^c)=1)$$

$$\therefore\ I_{\nu_0}(0,1) < I_\nu^c(0,1)$$

USEFUL: USE BROAD LINE TO MEASURE $T_{KIN}(\tau_\nu)$ STRUCTURE

$$I^+_\nu(0,1) \approx B_\nu(T_{KIN}(z(\tau_\nu=1)))$$



Eg. Ca II K LINE IN SUN