ISSN 1063-7737, Astronomy Letters, 2014, Vol. 40, No. 4, pp. 193–200. © Pleiades Publishing, Inc., 2014.
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Search for Evolutionary Changes in the Periods of Cepheids: U Sgr

L. N. Berdnikov[1*] and D. Majaess[2]

[1]Sternberg Astronomical Institute, Universitetskii pr. 13, Moscow, 119992 Russia
[2]St. Mary's University, 923 Robie Str., Halifax, Nova Scotia B3H 3C3, Canada

Received October 2, 2013

Abstract—For the classical Cepheid U Sgr, we have constructed an $O-C$ diagram spanning a time interval of 144 years. The $O-C$ diagram has the shape of a parabola, which has made it possible to determine for the first time the quadratic light elements and to calculate the rate of evolutionary increase in the period, $dP/dt = 0.39$ (± 0.10) s yr^{-1}, in agreement with the results of theoretical calculations for the third crossing of the instability strip. The available data reduced by the Eddington–Plakidis method reveal small random period fluctuations that do not distort the evolutionary trend in the $O-C$ residuals.

DOI: 10.1134/S1063773714030025

Keywords: *Cepheids, period variability, stellar evolution.*

INTRODUCTION

The detection of parabolas on the $O-C$ diagrams allows the rates of observed evolutionary changes in periods to be calculated. Comparing them with the theoretical rates calculated for different crossings of the instability strip allows the crossing number to be identified. In prospect, this will make it possible to construct the period–luminosity relation separately for each crossing, which, in turn, will lead to a more accurate determination of the distances to Cepheids.

In 1994, we began a long-term project to study the variability of Cepheid periods. Our experience has shown that when the time interval spanned by the $O-C$ diagram reaches a century, more than 90% of the Cepheids studied (in the entire range of periods encountered in the Galaxy) exhibit evolutionary changes in their periods (Turner et al. 2006). Therefore, when the variability of Cepheid periods is studied, the longest possible time interval should be covered by observations.

In this paper, we investigate the behavior of the pulsations for the Cepheid U Sgr, with the period of its brightness variations being $6^{\rm d}.75$. The importance of such an investigation also stems from the fact that U Sgr is a member of the open star cluster M25 (Doig 1925), i.e., it belongs to the small group of Cepheids based on which the period–luminosity relation is calibrated.

*E-mail: berdnik@sai.msu.ru

THE TECHNIQUE AND OBSERVATIONAL MATERIAL

To study the Cepheid period variability, we use the universally accepted technique of analyzing the $O-C$ diagrams; the most accurate method for determining the $O-C$ residuals is the method of Hertzsprung (1919), whose computer implementation was described by Berdnikov (1992a). To confirm the reality of the detected period changes, it should be shown that the random fluctuations in the pulsation period, if present, are not dominant on the $O-C$ diagram; we search for these random fluctuations using the technique described by Eddington and Plakidis (1929).

The variability of U Sgr was discovered in 1866 by Schmidt (1868), who also found the period of its brightness variations, $6^{\rm d}.74784$. Nielsen (1954) and Szabados (1989) studied the behavior of the period based on the 1866–1954 and 1898–1986 observations, respectively, but no evolutionary changes in the period were detected.

For a new study of the period of U Sgr, we added the early (beginning in 1867) visual observations made by Winnecke (Zinner and Wachmann 1931) and Schoenfeld (1900) and also used observations published after 1986.

The oldest visual observation was made in 1866 and the latest CCD frame was taken in 2010. Consequently, our data span a time interval of 144 years.

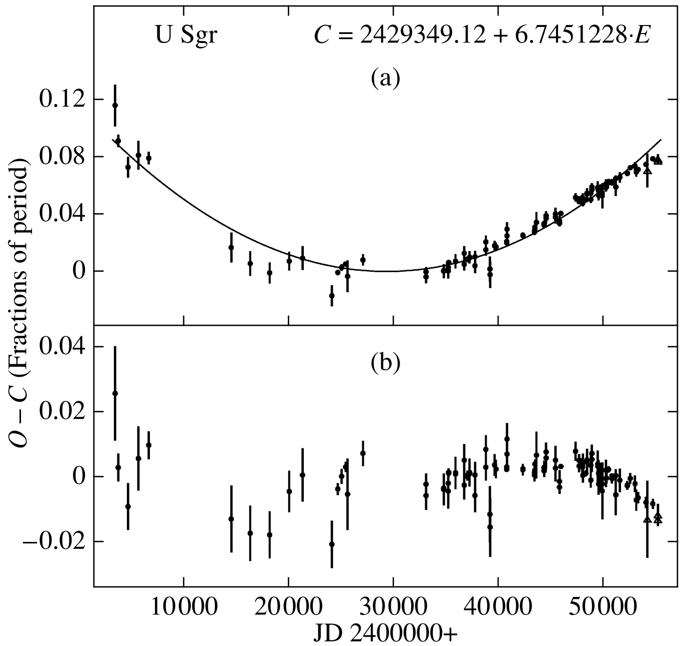

Fig. 1. $O-C$ diagrams for the Cepheid U Sgr relative to the linear (a) and quadratic (b) light elements (1). The curve in Fig. 1a is a parabola corresponding to the elements (1).

RESULTS AND DISCUSSION

The results of our reduction of the seasonal light curves for U Sgr are presented in Table 1. Columns 1 and 2 give the times of maximum light and their errors; column 3 gives the type of observations used (see Table 1); columns 4 and 5 contain the epoch number E and the $O-C$ residual; columns 6 and 7 contain the number of observations N and the data source. The data from Table 1 are shown on the $O-C$ diagram (Fig. 1) by the circles with vertical bars indicating the limits of the errors in the $O-C$ residuals. The $O-C$ residuals in Fig. 1 are expressed in fractions of the period.

Figure 1 shows that the $O-C$ diagram has the shape of a parabola. Based on the times of maximum light from Table 1, we derived the quadratic light elements for the Cepheid U Sgr:

$$\text{Max JD hel} = 2429349.122(\pm 0.007) \quad (1)$$
$$+ 6.^{\mathrm{d}}7451228(\pm 0.0000025)E + 0.413 \times 10^{-7}$$
$$\times (\pm 0.106 \times 10^{-8})E^2;$$

their linear part was used to calculate the $O-C$ residuals in the fifth column of Table 1. The elements (1) were used to draw the parabola in Figs. 1a and 1b shows the deviations from this parabola.

Based on our photoelectric observations, we found the maxima in the B band to occur earlier than those in the V band by $0.^{\mathrm{d}}001$. We applied this correction when constructing Fig. 1 and determining the elements (1), which, thus, refer to the V band.

The $O-C$ residuals from the elements (1) for each rth maximum, $a(r)$, were analyzed for the presence of random fluctuations in the pulsation period using the method published by Eddington and Plakidis (1929). For this purpose, we calculated the absolute values of all delays $u(x) = |a(r+x) - a(r)|$ for maxima separated by x cycles. According to Eddington and Plakidis (1929), the means $\langle u(x)\rangle$ of all accumulated delays should be related to the random period fluctuation ε by the formula

$$\langle u(x)\rangle^2 = 2\alpha^2 + x\varepsilon^2, \quad (2)$$

where α characterizes the random errors in the measured times of maximum light.

Strictly speaking, the Eddington—Plakidis method can be applied to reduce the $O-C$ residuals of individual successive brightness maxima; as a last resort, the brightness maxima determined from the mean curve spanning a time interval of several periods can be used. Since photographic and visual observations do not satisfy these conditions, we used the $O-C$ residuals derived only from V-band photoelectric observations.

The results of our calculations presented in Fig. 2 reveal a linear trend in $\langle u(x)\rangle^2$ for a cycle difference

Table 1. Times of maximum light for U Sgr

Max JD hel	Error, days	Band	E	$O-C$, days	N	Data source
2403408.164	0.098	VIS	−3846	0.785	10	Zinner and Wachmann (1931)
2403698.037	0.029	VIS	−3803	0.617	1619	Nielsen (1954)
2404642.230	0.048	VIS	−3663	0.493	131	Schoenfeld (1900)
2405633.820	0.067	VIS	−3516	0.549	74	Schoenfeld (1900)
2406618.594	0.029	VIS	−3370	0.535	1644	Nielsen (1954)
2414503.213	0.069	PG	−2201	0.106	32	Pickering (1904)
2416317.576	0.057	PG	−1932	0.031	33	Shapley (1930)
2418179.186	0.048	PG	−1656	−0.013	39	Shapley (1930)
2420054.385	0.042	PG	−1378	0.042	63	Shapley (1930)
2421322.482	0.055	PG	−1190	0.056	29	Shapley (1930)
2424128.276	0.049	PG	−774	−0.121	39	Shapley (1930)
2424694.976	0.012	PG	−690	−0.011	120	Voute and Bruggencate (1927)
2425059.236	0.015	PG	−636	0.012	56	Voute and Bruggencate (1927)
2425436.980	0.010	PG	−580	0.028	110	Bruggencate (1931)
2425625.792	0.073	VIS	−552	−0.023	154	Zakharov (1954)
2427096.305	0.026	VIS	−334	0.054	200	Florya and Kukarkina (1953)
2433133.104	0.029	B	561	−0.032	16	Eggen (1951)
2433133.134	0.022	V	561	−0.002	16	Eggen (1951)
2434812.669	0.025	B	810	−0.002	9	Walraven et al. (1958)
2434812.673	0.033	V	810	0.001	9	Walraven et al. (1958)
2435251.107	0.036	V	875	0.002	5	Irwin (1961)
2435251.117	0.025	B	875	0.012	5	Irwin (1961)
2435291.611	0.007	B	881	0.036	22	Irwin (1961)
2435291.616	0.008	V	881	0.040	22	Irwin (1961)
2435945.892	0.031	B	978	0.039	13	Johnson (1960)
2435945.901	0.033	V	978	0.049	13	Johnson (1960)
2436782.275	0.029	B	1102	0.027	19	Sandage (1960)
2436782.333	0.033	V	1102	0.085	19	Sandage (1960)
2437092.572	0.008	B	1148	0.049	39	Wampler et al. (1961)
2437092.582	0.009	V	1148	0.059	39	Wampler et al. (1961)
2437294.935	0.021	B	1178	0.058	25	Mitchell et al. (1964)
2437294.944	0.028	V	1178	0.067	25	Mitchell et al. (1964)
2437807.533	0.034	V	1254	0.027	6	Williams (1966)
2437807.569	0.026	B	1254	0.063	6	Williams (1966)
2438846.358	0.027	V	1408	0.102	29	Wisniewski and Johnson (1968)
2438846.388	0.029	B	1408	0.133	29	Wisniewski and Johnson (1968)
2439237.477	0.058	B	1466	0.005	12	Breger (1967)
2439250.948	0.061	V	1468	−0.015	14	Breger (1967)
2439676.021	0.022	B	1531	0.115	6	Schmidt (1971)
2439817.669	0.019	V	1552	0.116	18	Schmidt (1971)
2440836.196	0.004	B	1703	0.129	41	Pel (1976)
2440836.208	0.004	V	1703	0.141	41	Pel (1976)
2440869.960	0.035	V	1708	0.168	35	Feltz and McNamara (1980)
2440869.985	0.033	B	1708	0.193	35	Feltz and McNamara (1980)
2442387.608	0.011	B	1933	0.163	19	Dean et al. (1977)
2442387.617	0.010	V	1933	0.172	19	Dean et al. (1977)
2443493.830	0.010	B	2097	0.185	10	Dean (1981)
2443493.831	0.013	V	2097	0.186	10	Dean (1981)
2443534.313	0.011	V	2103	0.197	36	Moffett and Barnes (1984)
2443534.319	0.009	B	2103	0.203	36	Moffett and Barnes (1984)
2443682.740	0.048	V	2125	0.232	16	Harris (1980)
2444411.200	0.003	B	2233	0.219	45	Gieren (1981)

Table 1. (Contd.)

Max JD hel	Error, days	Band	E	$O-C$, days	N	Data source
2444411.201	0.004	V	2233	0.219	45	Gieren (1981)
2444424.685	0.012	B	2235	0.213	6	Caldwell et al. (2001)
2444424.698	0.014	V	2235	0.226	6	Caldwell et al. (2001)
2444613.587	0.023	V	2263	0.251	21	Eggen (1985)
2444613.593	0.019	B	2263	0.257	21	Eggen (1985)
2445503.947	0.027	V	2395	0.255	11	Berdnikov (1986)
2445503.957	0.029	B	2395	0.265	11	Berdnikov (1986)
2445881.639	0.013	B	2451	0.221	13	Berdnikov (1986)
2445881.658	0.023	V	2451	0.239	14	Berdnikov (1986)
2446016.595	0.005	V	2471	0.274	48	Bersier et al. (1994)
2447419.650	0.018	B	2679	0.343	25	Berdnikov (1992b)
2447419.657	0.019	V	2679	0.350	25	Berdnikov (1992b)
2447756.886	0.012	B	2729	0.324	41	Berdnikov (1992c)
2447756.905	0.012	V	2729	0.342	40	Berdnikov (1992c)
2448080.667	0.014	V	2777	0.339	7	Shobbrook (1992)
2448080.671	0.008	B	2777	0.343	7	Shobbrook (1992)
2448114.371	0.015	B	2782	0.317	17	Berdnikov (1992d)
2448114.401	0.021	V	2782	0.346	18	Berdnikov (1992d)
2448411.177	0.006	V	2826	0.338	58	ESA (1997)
2448512.375	0.020	B	2841	0.359	21	Berdnikov (1992d)
2448512.385	0.023	V	2841	0.369	21	Berdnikov (1992d)
2448883.332	0.015	B	2896	0.333	21	Berdnikov (1993)
2448883.369	0.024	V	2896	0.371	21	Berdnikov (1993)
2448984.555	0.025	B	2911	0.380	25	Arellano Ferro et al. (1998)
2448984.574	0.018	V	2911	0.399	22	Arellano Ferro et al. (1998)
2449544.411	0.023	B	2994	0.391	15	Berdnikov and Turner (1995a)
2449544.413	0.032	V	2994	0.393	15	Berdnikov and Turner (1995a)
2449625.314	0.022	B	3006	0.352	11	Berdnikov and Vozyakova (1995)
2449625.342	0.027	V	3006	0.381	11	Berdnikov and Vozyakova (1995)
2449814.193	0.020	B	3034	0.368	12	Berdnikov and Turner (1995b)
2449814.206	0.028	V	3034	0.381	12	Berdnikov and Turner (1995b)
2449969.320	0.059	V	3057	0.357	10	Berdnikov et al. (1997)
2449969.328	0.038	B	3057	0.365	10	Berdnikov et al. (1997)
2450320.105	0.017	V	3109	0.396	19	Berdnikov et al. (1998)
2450320.116	0.021	B	3109	0.407	19	Berdnikov et al. (1998)
2450374.069	0.011	V	3117	0.399	28	Berdnikov and Turner (1998a)
2450576.449	0.006	V	3147	0.425	25	Berdnikov and Turner (1998b)
2450906.953	0.005	B	3196	0.418	26	Berdnikov and Turner (2000)
2450906.953	0.009	V	3196	0.418	26	Berdnikov and Turner (2000)
2451271.164	0.042	B	3250	0.393	27	Berdnikov and Turner (2001)
2451271.211	0.013	V	3250	0.439	27	Berdnikov and Turner (2001)
2451655.688	0.024	V	3307	0.445	23	Berdnikov and Caldwell (2001)
2452370.685	0.008	B	3413	0.458	35	Berdnikov and Turner (2004)
2452370.689	0.008	V	3413	0.463	35	Berdnikov and Turner (2004)
2452654.012	0.011	V	3455	0.490	99	Pojmanski (2002)
2453105.943	0.017	V	3522	0.498	15	Berdnikov and Turner (2006)
2453227.327	0.022	V	3540	0.470	11	Pojmanski (2002)
2453402.713	0.011	V	3566	0.482	103	Pojmanski (2002)
2454158.190	0.011	V	3678	0.506	93	Pojmanski (2002)
2454293.063	0.079	V	3698	0.477	21	Henden (2013)
2454805.748	0.010	V	3774	0.532	121	Pojmanski (2002)
2455284.635	0.009	B	3845	0.515	20	Henden (2013)
2455284.651	0.023	V	3845	0.531	20	Henden (2013)

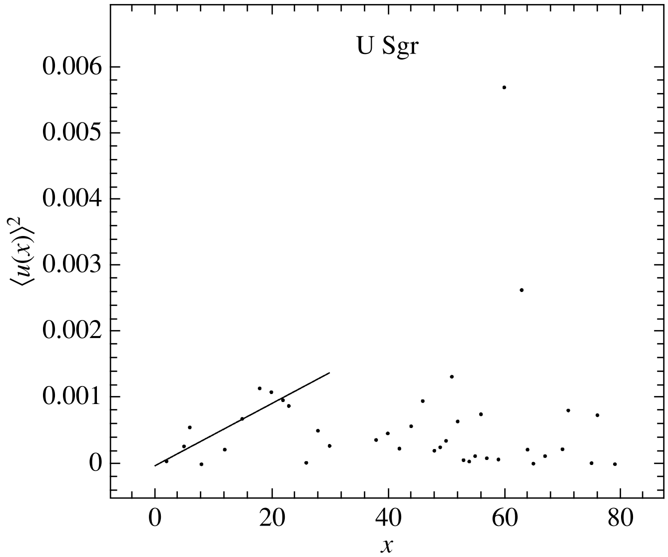

Fig. 2. Square of the mean accumulated delay $\langle u(x)\rangle$ versus cycle difference x for the Cepheid U Sgr. The line is the fit to Eq. (2) for $x < 24$, which gives the random period fluctuation $\varepsilon = 0^{\rm d}\!.0069 \pm 0^{\rm d}\!.0032$.

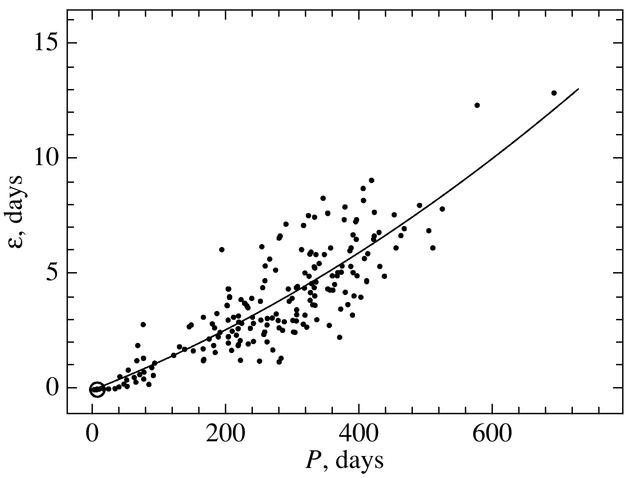

Fig. 3. Random period fluctuation ε versus period P for pulsation variables. The curve is the parabola fitting the data points. The large circle is the Cepheid U Sgr.

$x < 24$, where the formal fitting of Eq. (2) gives a solution in the form

$$\langle u(x)\rangle^2 = -0.41 \times 10^{-4}(\pm 0.16 \times 10^{-3}) + 0.48 \times 10^{-4}(\pm 0.11 \times 10^{-4})x,$$

whence $\alpha = 0^{\rm d}\!.005 \pm 0^{\rm d}\!.009$, which is close to the mean error in the times of maximum light for photoelectric and CCD observations (the second column of Table 1). The random period fluctuation $\varepsilon = 0^{\rm d}\!.0069 \pm 0^{\rm d}\!.0032$ to which the large circle in Fig. 3 corresponds satisfies the general period dependence of ε for all pulsating variables (Turner et al. 2009; Berdnikov and Stevens 2009).

Thus, our data provide evidence for the existence of small random period fluctuations whose amplitude is considerably lower than the amplitude of the evolutionary changes, i.e. the parabola on the $O-C$ diagram (Fig. 1a) reflects the real evolutionary increase in period.

The quadratic term of the elements (1) makes it possible to calculate the rate of evolutionary increase in the period, $dP/dt = 0.39$ (± 0.10) s yr^{-1}, in agreement with the theoretical calculations for the third crossing of the instability strip (Berdnikov et al. 2000; Turner et al. 2006).

It should be noted that our results are based on specific standard light curves. Therefore, we give them in Table 2 to to be used in future studies and to

Table 2. Standard B and V light curves for U Sgr

Phase	B	V	Phase	B	V	Phase	B	V	Phase	B	V
0.000	7.258	6.337	0.250	7.734	6.623	0.500	8.129	6.880	0.750	8.299	7.045
0.005	7.259	6.337	0.255	7.738	6.626	0.505	8.135	6.884	0.755	8.287	7.039
0.010	7.262	6.339	0.260	7.741	6.628	0.510	8.140	6.888	0.760	8.273	7.031
0.015	7.266	6.342	0.265	7.743	6.629	0.515	8.145	6.892	0.765	8.259	7.022
0.020	7.271	6.346	0.270	7.745	6.631	0.520	8.151	6.896	0.770	8.244	7.013
0.025	7.277	6.351	0.275	7.747	6.632	0.525	8.156	6.901	0.775	8.227	7.002
0.030	7.285	6.356	0.280	7.749	6.632	0.530	8.162	6.905	0.780	8.209	6.991
0.035	7.293	6.362	0.285	7.750	6.633	0.535	8.168	6.910	0.785	8.191	6.979
0.040	7.302	6.369	0.290	7.751	6.633	0.540	8.174	6.915	0.790	8.171	6.966
0.045	7.312	6.375	0.295	7.752	6.634	0.545	8.181	6.921	0.795	8.151	6.953
0.050	7.322	6.382	0.300	7.754	6.635	0.550	8.188	6.926	0.800	8.130	6.938
0.055	7.332	6.389	0.305	7.756	6.635	0.555	8.195	6.932	0.805	8.108	6.924
0.060	7.343	6.396	0.310	7.758	6.636	0.560	8.203	6.938	0.810	8.085	6.908
0.065	7.354	6.403	0.315	7.760	6.637	0.565	8.210	6.944	0.815	8.062	6.893
0.070	7.365	6.410	0.320	7.763	6.639	0.570	8.218	6.950	0.820	8.038	6.877
0.075	7.375	6.417	0.325	7.767	6.641	0.575	8.227	6.956	0.825	8.013	6.860
0.080	7.386	6.423	0.330	7.772	6.643	0.580	8.235	6.963	0.830	7.988	6.844
0.085	7.397	6.430	0.335	7.777	6.646	0.585	8.244	6.969	0.835	7.963	6.827
0.090	7.407	6.436	0.340	7.783	6.650	0.590	8.252	6.976	0.840	7.937	6.809
0.095	7.418	6.441	0.345	7.790	6.654	0.595	8.261	6.982	0.845	7.911	6.792
0.100	7.429	6.447	0.350	7.798	6.658	0.600	8.269	6.988	0.850	7.884	6.774
0.105	7.439	6.453	0.355	7.807	6.664	0.605	8.277	6.994	0.855	7.857	6.756
0.110	7.450	6.458	0.360	7.817	6.670	0.610	8.285	7.000	0.860	7.829	6.738
0.115	7.460	6.464	0.365	7.827	6.676	0.615	8.293	7.005	0.865	7.801	6.719
0.120	7.471	6.469	0.370	7.839	6.683	0.620	8.301	7.010	0.870	7.773	6.701
0.125	7.482	6.475	0.375	7.851	6.691	0.625	8.308	7.015	0.875	7.745	6.682
0.130	7.493	6.480	0.380	7.863	6.699	0.630	8.314	7.020	0.880	7.716	6.663
0.135	7.504	6.486	0.385	7.877	6.707	0.635	8.321	7.025	0.885	7.688	6.643
0.140	7.515	6.492	0.390	7.891	6.716	0.640	8.327	7.029	0.890	7.659	6.624
0.145	7.527	6.498	0.395	7.905	6.726	0.645	8.332	7.034	0.895	7.630	6.604
0.150	7.539	6.505	0.400	7.919	6.735	0.650	8.337	7.038	0.900	7.602	6.585
0.155	7.551	6.511	0.405	7.934	6.745	0.655	8.342	7.041	0.905	7.574	6.565
0.160	7.563	6.518	0.410	7.948	6.754	0.660	8.346	7.045	0.910	7.546	6.546
0.165	7.575	6.525	0.415	7.963	6.764	0.665	8.350	7.048	0.915	7.518	6.527
0.170	7.588	6.532	0.420	7.977	6.773	0.670	8.353	7.052	0.920	7.492	6.508
0.175	7.600	6.539	0.425	7.991	6.783	0.675	8.355	7.055	0.925	7.466	6.489
0.180	7.612	6.546	0.430	8.004	6.792	0.680	8.357	7.057	0.930	7.441	6.471
0.185	7.624	6.554	0.435	8.017	6.801	0.685	8.359	7.060	0.935	7.417	6.454
0.190	7.636	6.561	0.440	8.030	6.809	0.690	8.359	7.062	0.940	7.394	6.438
0.195	7.647	6.568	0.445	8.042	6.817	0.695	8.359	7.064	0.945	7.373	6.422
0.200	7.658	6.575	0.450	8.053	6.825	0.700	8.359	7.066	0.950	7.353	6.407
0.205	7.669	6.581	0.455	8.063	6.832	0.705	8.357	7.067	0.955	7.335	6.394
0.210	7.679	6.588	0.460	8.073	6.839	0.710	8.355	7.067	0.960	7.319	6.382
0.215	7.688	6.594	0.465	8.082	6.845	0.715	8.351	7.067	0.965	7.305	6.371
0.220	7.697	6.599	0.470	8.090	6.851	0.720	8.347	7.066	0.970	7.292	6.362
0.225	7.705	6.604	0.475	8.098	6.857	0.725	8.342	7.065	0.975	7.282	6.354
0.230	7.712	6.609	0.480	8.105	6.862	0.730	8.335	7.063	0.980	7.273	6.348
0.235	7.719	6.613	0.485	8.112	6.867	0.735	8.328	7.060	0.985	7.267	6.343
0.240	7.724	6.617	0.490	8.118	6.871	0.740	8.319	7.056	0.990	7.262	6.339
0.245	7.730	6.621	0.495	8.124	6.875	0.745	8.309	7.051	0.995	7.259	6.337

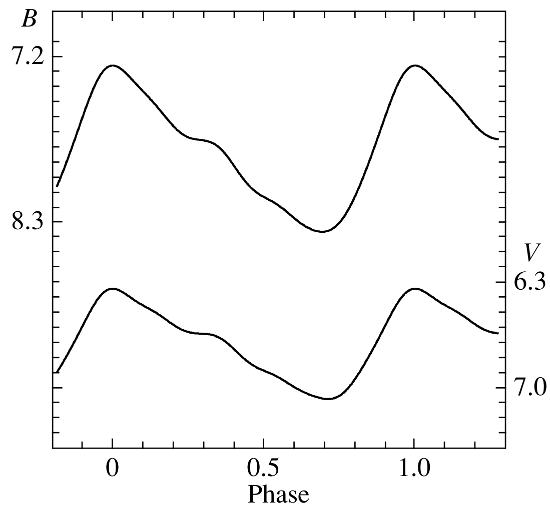

Fig. 4. Standard light curves for the Cepheid U Sgr.

establish a relationship to our data if other standard light curves will be used. Table 2 contains the B and V magnitudes of U Sgr for phases from 0 to 0.995 at 0.005 steps; these standard light curves graphically shown in Fig. 4 were constructed from photoelectric observations.

CONCLUSIONS

Having reduced the published visual, photographic, photoelectric, and CCD observations of the Cepheid U Sgr, we constructed an $O-C$ diagram spanning a time interval of 144 years, which allowed the quadratic light elements (1) to be determined for the first time. This made it possible to calculate the rate of evolutionary increase in the period, $dP/dt = 0.39$ (± 0.10) s yr^{-1}, in agreement with the results of theoretical calculations for the third crossing of the instability strip. The available data reduced by the method of Eddington and Plakidis (1929) are indicative of small random period fluctuations, $\varepsilon/P = 0^{\mathrm{d}}.0010 \pm 0^{\mathrm{d}}.005$, that do not mask the dominant evolutionary increase in period.

ACKNOWLEDGMENTS

This study was financially supported by the Russian Foundation for Basic Research (project no. 13-02-00203). We are grateful to the American Association of Variable Star Observers (AAVSO) for the opportunity to use the CCD observations from their International Database.

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Translated by N. Samus