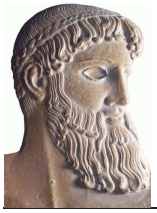

ZEUS-3D 3-D Gallery #1: Sedov Blast Wave

Introduction

The Sedov blast wave (Sedov, L. I., 1959, *Similarity and Dimensional Methods in Mechanics*, New York: Academic Press) is essentially a strong shock wave launched in spherical polar coordinates. This problem, which ZEUS-3D can do in 1-D spherical polar and 3-D Cartesian coordinates, is a strong test on isotropy and energy conservation. This is a hydrodynamical blast; for an MHD blast, see the 2-D MHD blast page.

This page was created, in part, to answer criticisms of *ZEUS* raised by Tasker *et al.* (2008, MNRAS, 390, 1265) which, as shown herein, were largely misdirected or incorrect.

Simulation

A 200^3 uniform Cartesian grid is initialised in the octant $0 < x, y, z < 5$ with a gas ($\gamma = 5/3$) of density ($\rho = 1$) and pressure ($p = 10^{-5}$). Reflecting boundary conditions are imposed on the $x = 0$, $y = 0$, and $z = 0$ planes (`dzeus36` maintains both reflecting boundary conditions and octal symmetry to machine accuracy), outflow conditions elsewhere (though the simulation is stopped before the blast wave reaches the outflow boundaries which are therefore never engaged). At $t = 0$, 1.25×10^4 units of energy are released in the eighth-sphere of radius 0.0875 centred at the origin (resolved with 3.5 zones along each axis, 2 zones along the diagonal). This creates a pressure jump of 2.4×10^{12} at $t = 0$, and thus poses an extreme test, even for double precision.

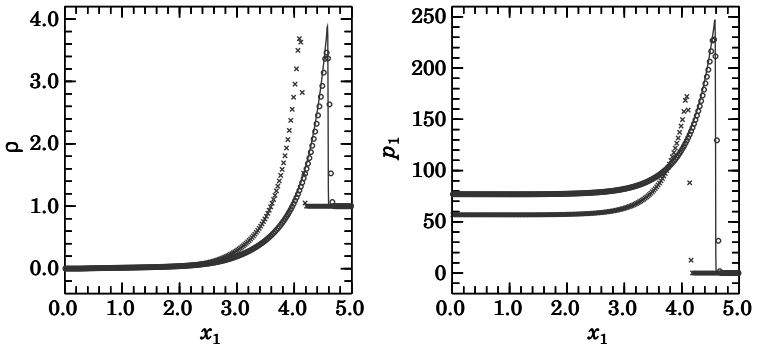

Figure 1: Density (left) and pressure (right) profiles at $t = 0.1$ using the internal ($\times$; `itote=0`) and total ($\circ$; `itote=1`) energy equation.

Figure 1 shows radial (x_1) profiles of density, ρ, and pressure, p_1, at $t = 0.1$ when the internal and total energy equation is used. (See §3.1 of the document What is *ZEUS-3D*? for a discussion on internal *vs.* total energy equations.) Lines represent the analytical solution at $t = 0.1$, where $r_{\rm sph} = 4.59$, $\rho_{\rm max} = 4.00$, and $p_{1,\rm max} = 252$. (Sedov, 1959). Evidently, the solution using the total energy equation agrees with the analytical one, but the solution using the internal energy equation does not.

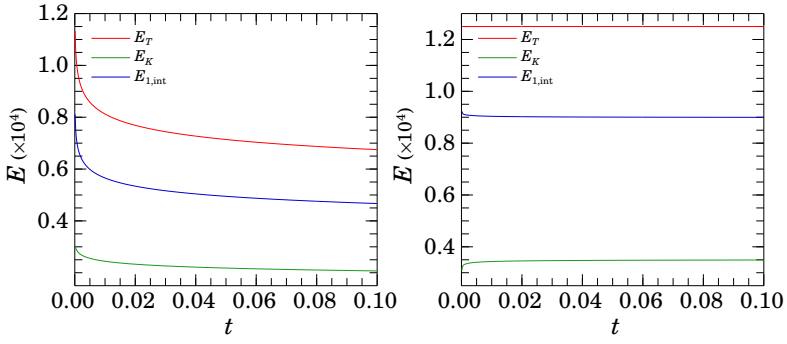

Figure 2: Grid-integrated total, internal, and kinetic energies as a function of time using the internal (left) and total (right) energy equations.

Figure 2 shows the total (red), internal (blue) and kinetic (green) energies integrated over the grid as a function of time when the internal (far left) and total (near left) energy equation is solved. While the total energy equation conserves total energy to machine accuracy, the internal energy equation loses a third of the total energy by $t = 0.1$. The accompanying loss of internal and kinetic energies then explains the disagreements in the density and pressure profiles in Fig. 1, and why the blast front reaches only $x_1 = 4.35$ by $t = 0.1$, 5% less than the analytical value.

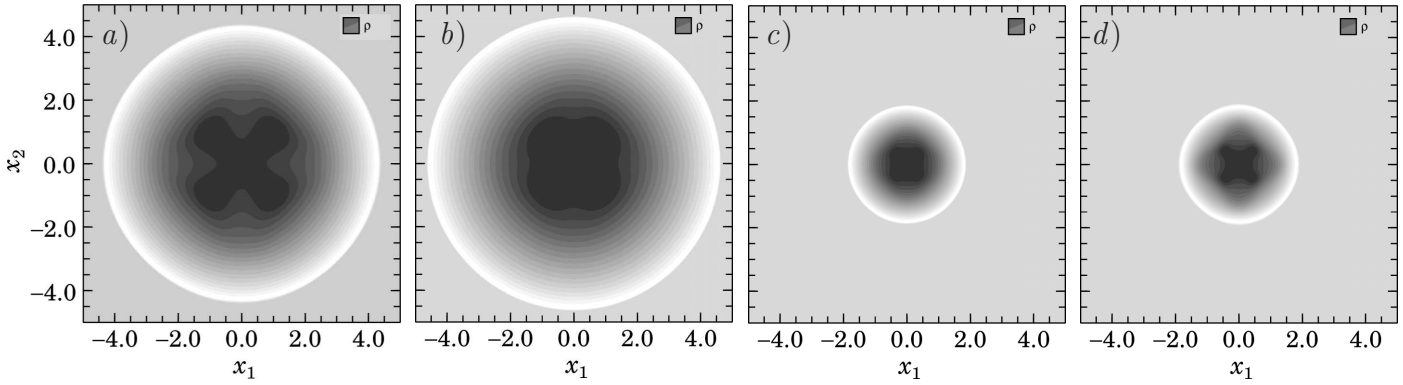

Figure 3: Density slices of the blast front at $t = 0.1$ [panels a) and b)] and $t = 0.01$ [panels c) and d)] using `itote=0` [panel a)] and `itote=1` [panels b), c) and d)]. Parameter `qcon` is 1 for panels a) and d), 0 for panels b) and c). The internal energy equation slows the advance of the shock wave, but does not introduce the non-isotropic "cross-shaped" central contours; the latter is caused by the quadratic term of the artificial viscosity.

Figure 3 shows a density slice through the blast wave using a) the internal energy equation (`itote=0`) with quadratic artificial viscosity parameter `qcon=1` at $t = 0.1$; b) the total energy equation (`itote=1`) with `qcon=0` at $t = 0.1$, c) `itote=1` with `qcon=0` at $t = 0.01$, and d) `itote=1` with `qcon=1` at $t = 0.01$. This series shows definitively that the aspherical contours especially near the centre are caused by a non-zero `qcon` and, in particular, have nothing to do with energy conservation as claimed by Tasker *et al.* (2008) since energy is conserved to machine accuracy in panels c) and d) regardless of `qcon`. The much smaller asymmetries in panels b) and c) result from the initial anisotropy caused by resolving the initial over-pressured region with 3.5 zones along the grid axes, but only 2 zones diagonal to them.

Despite its intended purpose, quadratic viscosity is not actually needed here to stabilise the shock. Because the time step is governed by the enormous temperature at the core, the shock takes many time steps to cross each zone, and grid dissipation is sufficient to stabilise it. Further, the total energy equation solution is virtually independent of `qcon`, and one can safely use `qcon=0`[1]. On the other hand, the internal energy equation solution is quite sensitive to `qcon` (final values of $r_{\rm sph}$, $\rho_{\rm max}$, and $p_{\rm max}$, but not shock stability), with `qcon` between 1 and 2 being optimal. The price paid for a non-zero `qcon`, however, is the loss of isotropy on the inner-most contours.

ZEUS-3D uses the von Neumann & Richtmyer (1950, J. Appl. Phys., 21, 232) form of the artificial viscosity in which the vicous tensor is diagonal and thus behaves anisotropically, particularly for problems such as this. A full tensor treatment of the artificial viscosity (*e.g.*, Richtmyer & Morton, 1967, *Difference Methods for Initial Value Problems*, New York: Wiley), if installed, should improve the isotropy of the Cartesian solution.

Figure 4 provides links to `MPEG` animations of the density, pressure, and velocity divergence integrated along the line of sight through the 200^3 total energy equation simulation of the Sedov blast wave.

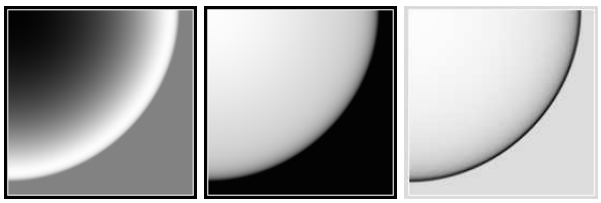

Figure 4: Animations of an octant of the Sedov blast wave for the variables listed.

[1]While the *quadratic* viscous parameter, `qcon`, can be set to 0 for `itote=1`, there is still a need for the linear viscous term controlled by `qlin`, without which violent instabilities start at the origin at around $t = 0.015$ that consume the solution with `NaNs`. For all Sedov simulations, I used `qlin=0.2`; `qlin=0.1` gives virtually identical results.

Conclusions

Tasker *et al.* (2008) used a version of *ZEUS* (either *ZEUSMP* or the module of *ZEUS* in *ENZO*) that did not include the total energy equation. Thus, they obtained essentially the internal energy equation solution presented in Fig. 1 (crosses) and rightfully noted it did not agree with the analytical solution. However, the cause they identified—that the disagreement stemmed from the first-order accuracy of that particular version of the code—is incorrect. In fact, the disagreement is caused by solving the wrong energy equation, the same as my response to one of Sam Falle's (2002) critiques.

With an initial pressure jump of *twelve orders of magnitude* and the absence of perturbations to break the octal symmetry, this must be regarded as a severe test for which, evidently, use of the internal energy equation is inappropriate. This, however, cannot rule out the use of the internal energy equation for all applications. The advantage of the internal energy equation and why it remains an option in `dzeus36` is it guarantees a positive-definite pressure so long as the CFL condition is obeyed. In many applications, pressure jumps are rarely over a few orders of magnitude in which case the internal energy equation can normally conserve total energy to within a percent and provide a perfectly acceptable numerical solution for the problem.

On the other hand, the total energy equation—while guaranteeing total energy conservation—does not guarantee a positive definite pressure. And while the code resets negative pressures to just above zero when they arise, this is not always the desirable correction. Now while the Sedov problem itself generated no negative pressures, the need for positive-definite pressures in other applications (without arbitrarily resetting them to zero) may trump the need for strict conservation of total energy, and the internal energy equation remains in the code as a viable alternative. For an example, the reader is directed to the Kelvin-Helmholtz instability page where, in comparing simulations done with the internal and total energy equations, only the slightest of quantitative differences can be discerned.