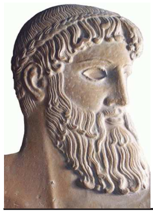
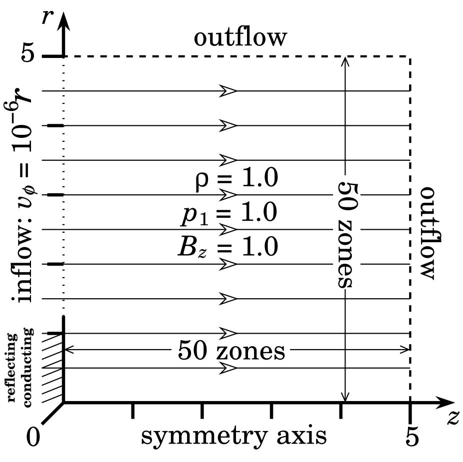
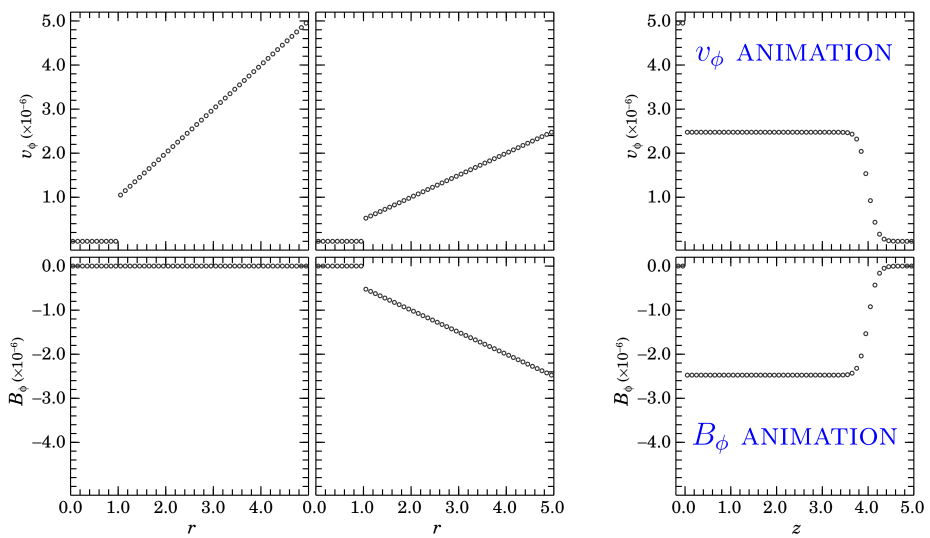


ZEUS-3D 2-D Gallery #3: MHD Boundary Conditions in Cylindrical Coordinates



initial setup



This test problem demonstrates the MHD boundary conditions in `dzeus36`. A 50×50 axisymmetric grid in cylindrical coordinates with domain $0 \le z, r \le 5$ (top left) is initialised with $\rho = 1$, $p_1 = 1$, $v_z = v_r = v_\phi = 0$, $B_z = 1$, and $B_r = B_\phi = 0$ with $\gamma = 5/3$. All variables are evolved to time $t = 4$. The $r = 0$ (bottom) boundary is a symmetry axis (`dzeus36` parameter `nflo=1`) and the $z = 5$ (right) and $r = 5$ (top) boundaries are "outflow" (`nflo=9`). The left boundary ($z = 0$) is *reflecting and conducting* (`nflo=3`) in $0 \le r < 1$, and *inflow* (`nflo=10`) for $r \ge 1$. Thus, the point $(z, r) = (0, 1)$ has "mixed boundary type" which, if not treated carefully, can introduce kinks in the magnetic field. Note that "inflow" doesn't necessarily mean material flows onto the grid. It is the option one picks to maintain the variables at preset values on the boundary, including this problem with zero inflow speed.

Along the inflow boundary, all variables are set to the grid values except for v_ϕ, which is set to:

$$v_{\phi 0} = \begin{cases} 10^{-6} r & r \ge 1; \\ 0 & r < 1, \end{cases} \tag{1}$$

as shown in the top of the panels labelled "at $z = -0.05$". A zero inflow speed is clearly *sub-magnetoslow*, and thus the *emf* needs to "float" on the boundary (*i.e.*, be left as computed by the `CMoC` algorithm) lest the boundary conditions be *over-specified*. This is done by setting `dzeus36` parameter `isetemf=0`.

Specifying v_ϕ by equation (1) twists B_z for $r \ge 1$, and launches an Alfvén wave across the grid as seen in the animations of v_ϕ and B_ϕ. One can show analytically (*e.g.*, by solving the characteristic equations) that the Alfvén wave moves at a speed $B_z/\sqrt{\rho} = 1$, leaving in its wake $v_\phi = \frac{1}{2} v_{\phi 0}$ and $B_z = -\frac{1}{2} v_{\phi 0}$ (panels labelled "at $z = 2.45$"), with all other variables unaffected. The `dzeus36` solution reproduces the analytical solution to within one part in 10^6, with arbitrarily higher accuracy obtained as $t \to \infty$.

Since the Alfvén wave moves along the $r = 5$ boundary, the outflow conditions must be involved in its propagation. *There is no evidence whatever that the outflow conditions adversely affect the Alfvén wave nor the values attained by v_ϕ and B_ϕ behind it*, even immediately inside the boundary (panels labelled "at $r = 4.95$"). Further, the mixed boundary point at $r = 1$ has no detectable effect on the Alfvén wave.

It should be noted that the inflow boundary conditions in all known versions of *ZEUS* prior to version 3.5 implicitly assume a super-magnetofast inflow speed, and cannot do this problem properly because the left boundary is over-specified. For this problem, these earlier versions of *ZEUS* give incorrect grid values for v_ϕ and B_ϕ by a factor of two behind the Alfvén wave.