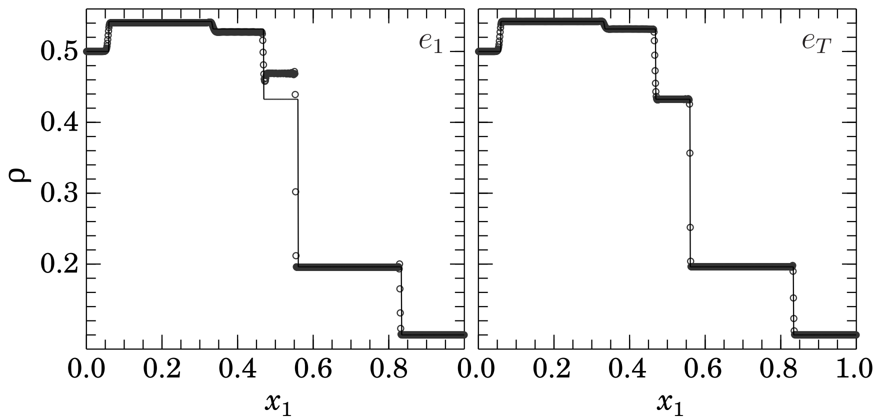
ZEUS-3D 1-D Gallery #22: Convergence and Efficiency



The two panels reproduce Fig. 6 from Falle, 2002 (ApJ, 577, L123), showing the density (ρ) from the MHD shock tube problem with left state $(\rho, v_1, v_2, v_3, B_2, B_3, p_1) = (0.5, 0, 2.0, 0, 2.5, 0, 10)$, right state $(0.1, -10.0, 0, 0, 2, 0, 0.1)$, $B_1 = 2$, and $\gamma = 5/3$ at $t = 0.06$. At $t = 0$, the discontinuity is at $x_1 = 0.5$. The left panel (e_1) shows the numerical solution using the non-conservative internal energy equation (`itote=0`), whereas the right panel (e_{T}) shows the solution using the conservative total energy equation (`itote=1`).

Both plots show from left to right: (1) fast shock, (2) slow rarefaction (at $x_1 \sim 0.33$), (3) contact discontinuity (at $x_1 \sim 0.47$), (4) slow shock (at $x_1 \sim 0.56$), and (5) fast shock. Open circles are the `dzeus36` solution using 1,000 zones, `CMoC`, `FIT`, and second-order interpolation. `dzeus36` parameters controlling the time step and artificial viscosity are: `courno=0.75`, `qcon=2.0`, and `qlin=0.2`. Lines are the results from the non-linear Riemann solver described in Ryu & Jones (1995, ApJ, 442, 228).

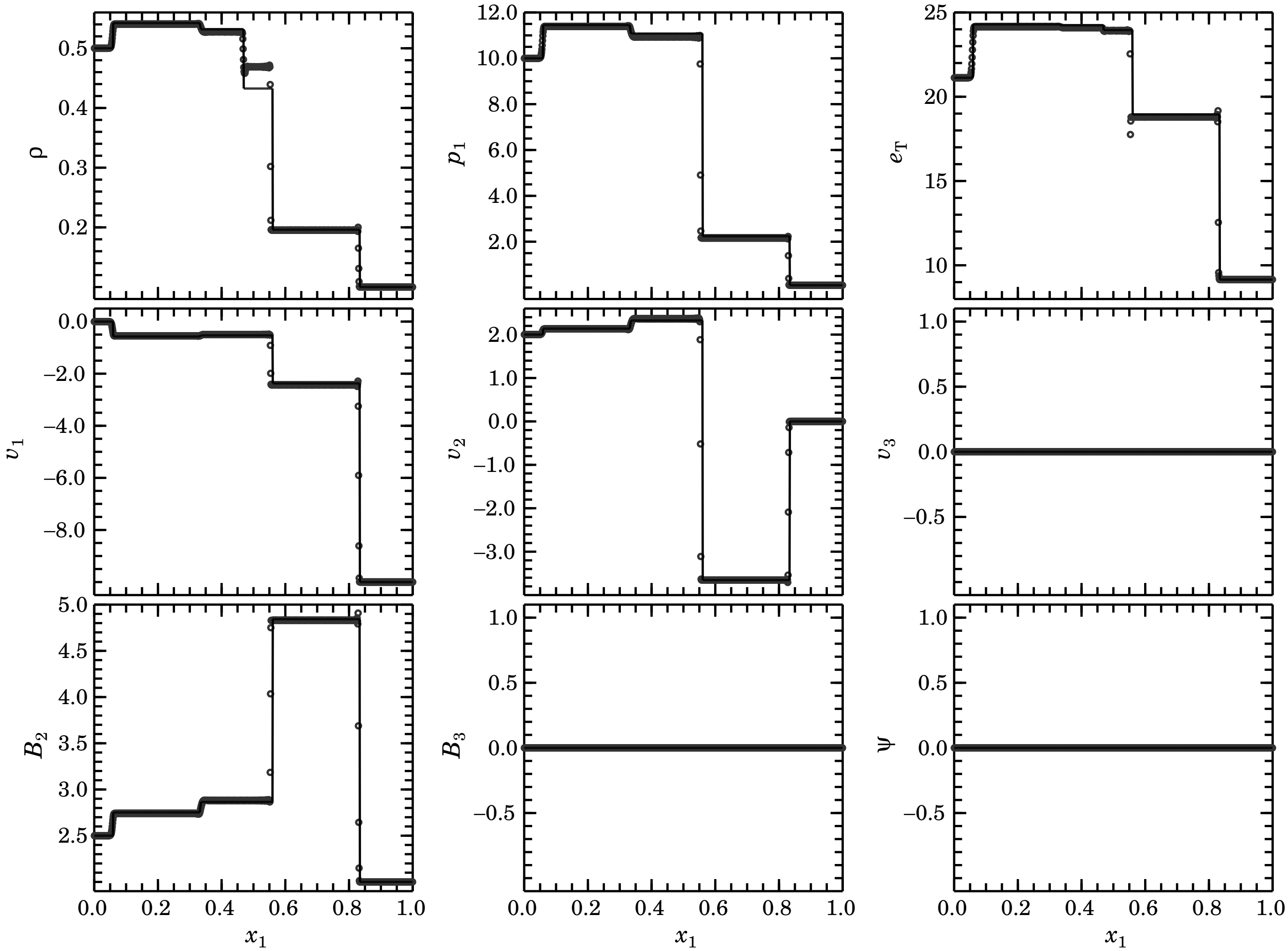
The `dzeus36` solution shows a markedly shallower undershoot at the base of the contact than the `dzeus35` solution; otherwise, the two versions produce very similar solutions. The levels in the non-conservative solution (left) differ from those in the conservative (right) and analytical (overlays) solutions (which do agree), particularly between the contact and slow shock. Further, the wave positions in the non-conservative solution differ slightly from those in the conservative and analytical solutions.

This test problem reveals a qualitative difference between the non-conservative and conservative algorithms. The former has the advantage of positive-definite pressures, while the latter has the advantage of strict conservation of energy. However, there is no single transport algorithm within `dzeus36` that will deliver both, and the user needs to be aware of which consideration—positive-definite pressure or strict conservation of energy—is more critical to the integrity of the simulation.

As for efficiency, Falle's (2002) claim that *ZEUS* is 24 times slower and requires 8 times more memory than an upwinded scheme is based primarily upon the assertion that *ZEUS* requires twice as many zones to resolve the same features as an upwinded code. The results in either this or the version3.5 1-D Gallery do not support this assertion. For example, counting the number of zones embedded in all shocks and contacts in all twelve of Ryu & Jones' (1995) test problems (Problems 9–20 in the 1-D Gallery), one finds that in aggregate, TVD (the algorithm used by Ryu & Jones) requires ∼200 zones to resolve 34 features, whereas `dzeus36` requires ∼140 zones. Much, but not all, of this difference is attributable to the contact steepener in `dzeus36` which Ryu & Jones do not use. In Falle's (2002) Fig. 6, his upwinded scheme requires 16 zones to resolve the four discontinuities, whereas `dzeus36` requires 20 zones without the contact steepener. For the test problems in the 1-D Gallery, the claimed factor of two is simply not there and, zone for zone, *ZEUS* provides as crisp a shock-tube solution at a given resolution as most upwinded schemes.

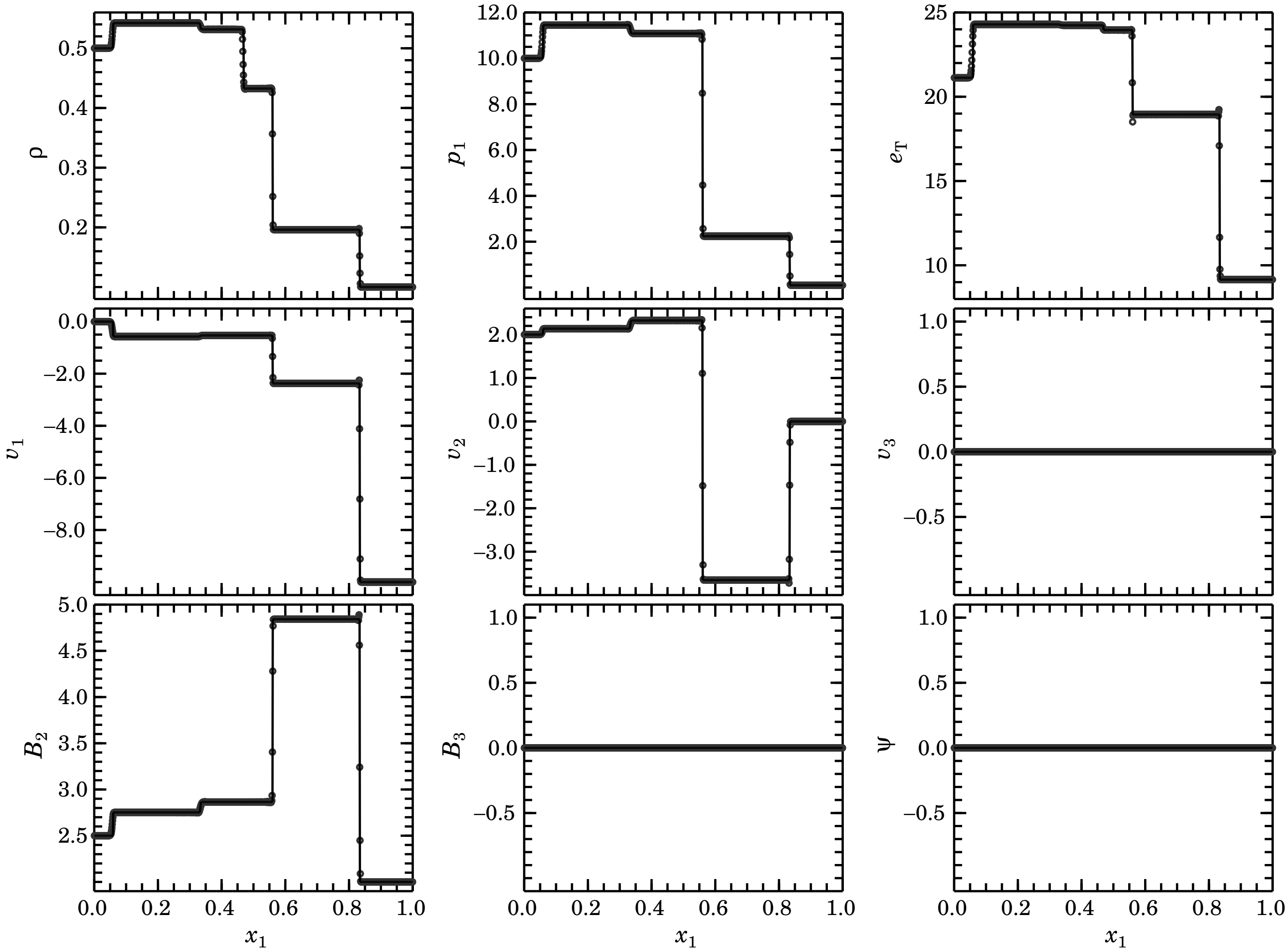
Complete non-conservative and conservative solutions for Falle's Riemann problem follow.

Non-conservative **dzeus36** solutions with conservative analytical overlays



Left state $(\rho, v_1, v_2, v_3, B_2, B_3, p_1) = (0.5, 0, 2.0, 0, 2.5, 0, 10)$ and the right state $(0.1, -10.0, 0, 0, 2, 0, 0.1)$ with $B_1 = 2$, $\gamma = 5/3$ at time $t = 0.06$. Open circles are the `dzeus36` solution using 1,000 zones, `CMoC`, `FIT`, the internal energy equation, and second-order interpolation. `dzeus36` parameters controlling the time step and artificial viscosity are: `courno=0.75`, `qcon=2.0`, and `qlin=0.2`. Lines are the results from the non-linear Riemann solver described in Ryu & Jones (1995).

Conservative `dzeus36` solutions with conservative analytical overlays



Left state $(\rho, v_1, v_2, v_3, B_2, B_3, p_1) = (0.5, 0, 2.0, 0, 2.5, 0, 10)$ and the right state $(0.1, -10.0, 0, 0, 2, 0, 0.1)$ with $B_1 = 2$, $\gamma = 5/3$ at time $t = 0.06$. Open circles are the `dzeus36` solution using 1,000 zones, `CMoC`, `FIT`, the total energy equation, and second-order interpolation. `dzeus36` parameters controlling the time step and artificial viscosity are: `courno=0.75`, `qcon=2.0`, and `qlin=0.2`. Lines are the results from the non-linear Riemann solver described in Ryu & Jones (1995).