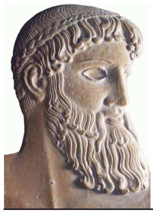
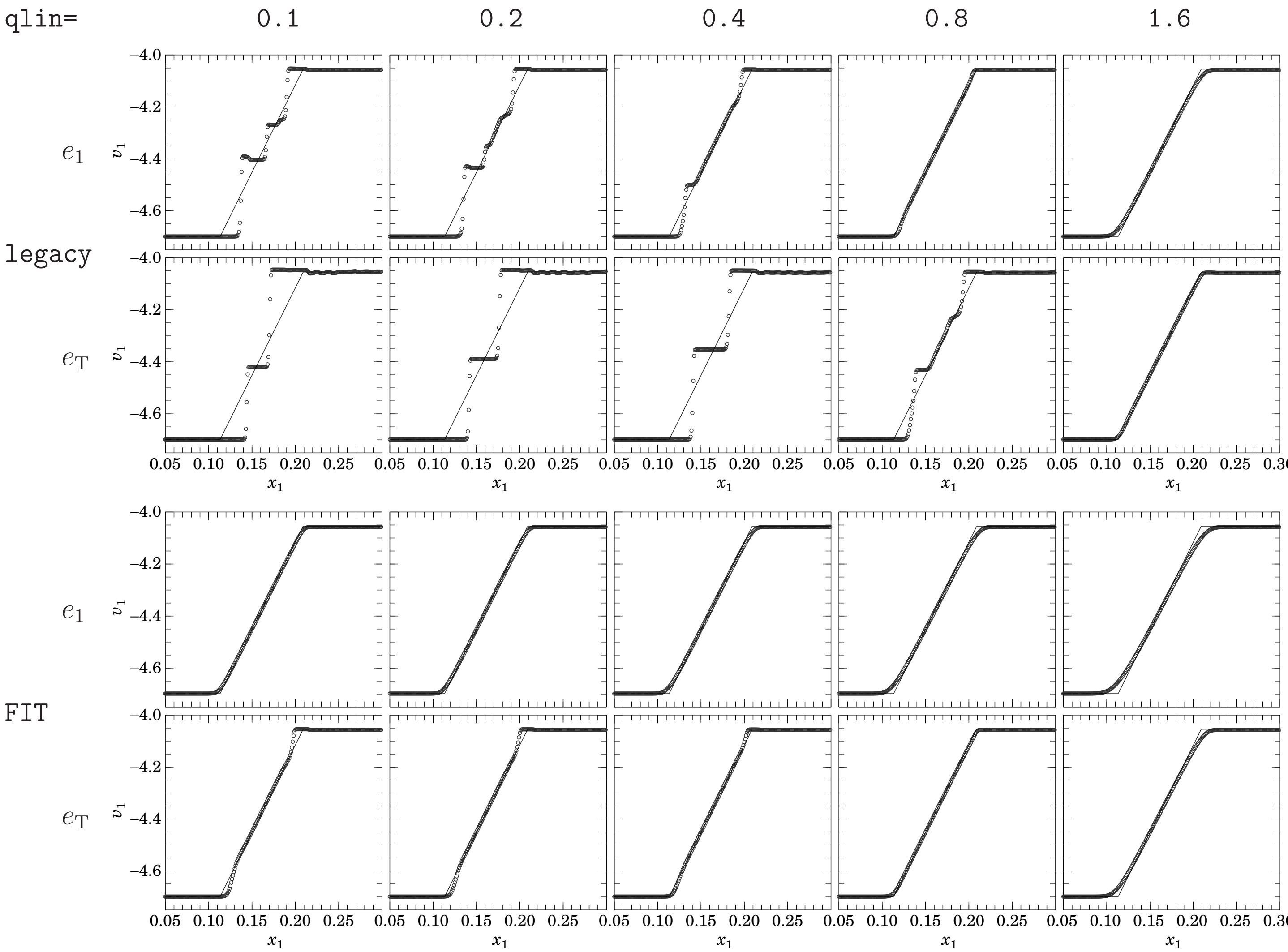


ZEUS-3D 1-D Gallery #21: Rarefaction Shocks



Falle (2002, ApJ, 577, L123) pointed out that for certain 1-D shock tube problems, the transport algorithm in the version of *ZEUS* available at the time could break up a rarefaction fan into discrete steps, as seen in some of the panels above. Falle referred to these unphysical features as *rarefaction shocks*, and attributed them to *ZEUS*' operator split momentum transport scheme whose design led to an imbalance in the order of accuracy: second-order in space; first-order in time. Typically, rarefaction shocks arise in problems where the ratio between the propagation speed of the rarefaction through the grid and the drop in speed across the fan itself is large. In the problem depicted above, this ratio is about seven.

The panels above are based on Falle's Fig. 2, in which an MHD shock tube (domain $0 \le x_1 \le 1$) is set with a left state $(\rho, v_1, v_2, v_3, B_2, B_3, p_1) = (1, -4.6985, -1.085146, 0, 1.9680, 0, 0.2327)$, right state $(0.7270, -4.0577, -0.8349, 0, 1.355, 0, 0.1368)$, $B_1 = -0.7$, $\gamma = 5/3$, and the discontinuity at $x_1 = 0.8$. The panels show only the portion of the domain containing the rarefaction fan at $t = 0.1$. Open circles are the `dzeus36` solution for v_1 using 1,000 zones across the entire domain, `CMoC`, and second-order interpolation, with a different transport algorithm for each row. `dzeus36` parameters controlling the time step and artificial viscosity are: `courno=0.5`, `qcon=0.0`, and `qlin` as indicated at the top of each column of panels. Lines are the results from the non-linear Riemann solver described in Ryu & Jones (1995, ApJ, 442, 228).

The top two rows show results using what I now refer to as *legacy transport* (`trnvrsn=0`), the coarsely operator-split transport algorithm used in all versions of *ZEUS* up to and including my own version 3.5. The first row shows the results when the non-conservative internal energy (e_1) equation is solved (`itote=0`), and the second row when the conservative total energy ($e_{\rm T}$) equation is solved instead (`itote=1`). While the rarefaction shocks are still problematic in the second row, these results are considerably "cleaner" than those presented with the `dzeus35` solution, owing to modifications made to the total energy algorithm between versions 3.5 and 3.6. As noted on the version 3.5 page, the rarefaction shocks could be attenuated by higher linear artificial viscosity (as demonstrated in the panels above), by resolving the rarefaction fan with far fewer zones, or by performing the appropriate Galilean transformation so that the propagation speed of the fan through the grid is comparable to the drop in speed across the fan.

As serendipity would have it, the new (to version 3.6) *Finely Interleaved Transport* algorithm (`FIT`; `trnvrsn=1`)—designed to eliminate transverse striping of wave forms transported diagonally across a 2-D grid—seems to address rarefaction shocks as well. The cure appears to be *complete* when the internal energy equation is used, as panels in the third row show no sign of rarefaction shocks even for `qlin=0.1`. Indeed, `FIT` with e_1 seems to be CFL limited, as computing the third row with `courno=0.9` instead of `courno=0.5` gives virtually identical results for all values of `qlin`.

On the other hand, `FIT` as implemented in version 3.6 doesn't seem to completely cure rarefaction shocks when the total energy equation is used. As the fourth row shows, there still remain vestiges of rarefaction shocks that do not disappear until `qlin>0.5`. Evidently, there remains some degree of freedom in the sequencing and/or modularisation of the total energy equation algorithm not yet exploited that would give solutions as clean as those provided by the internal energy algorithm. Completing the fine interleaving of the total energy equation algorithm remains an open task.