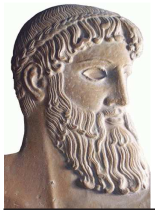
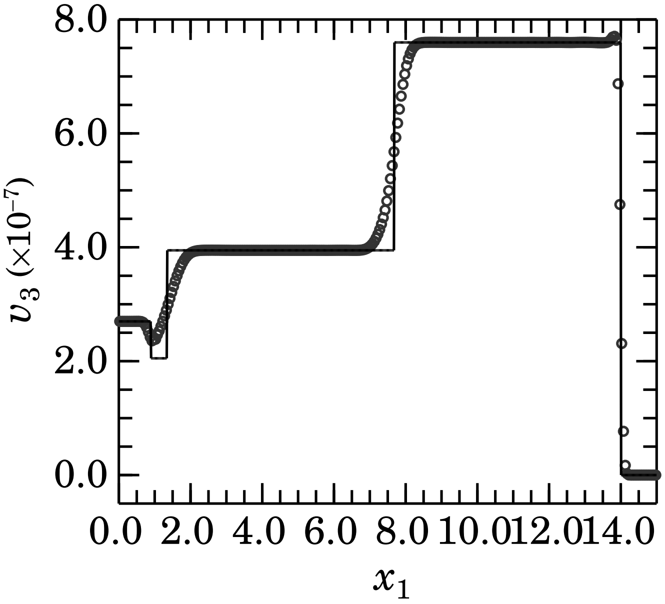
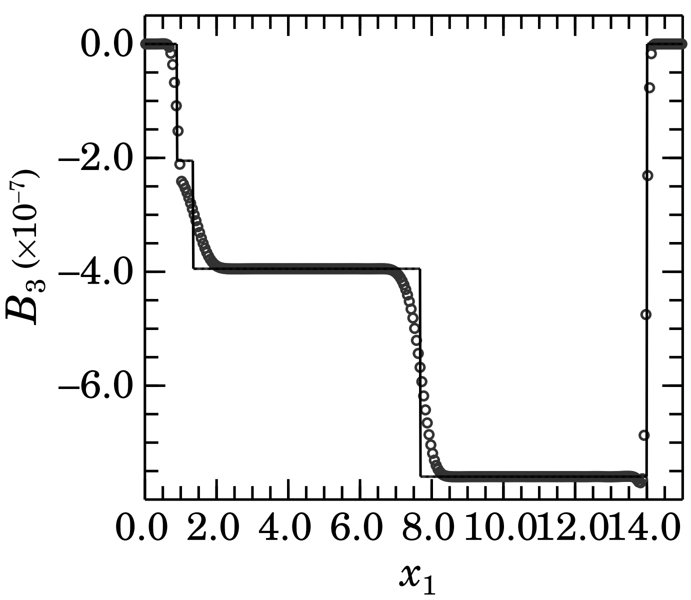


ZEUS-3D 1-D Gallery #8: MHD Rotor



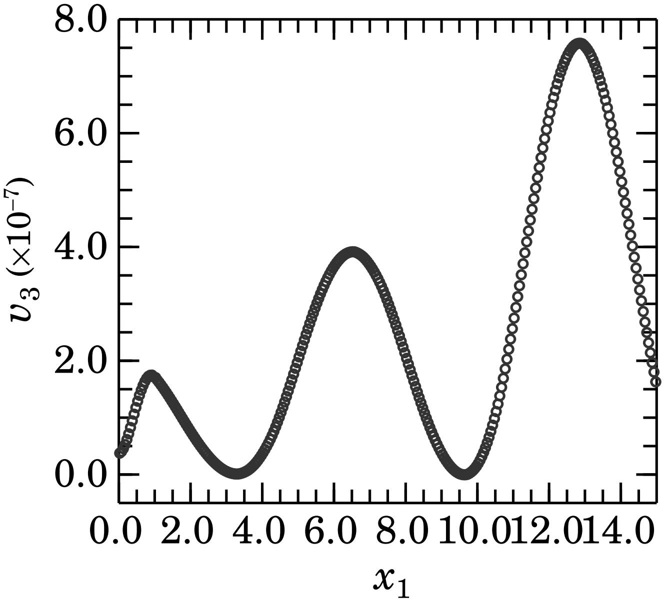


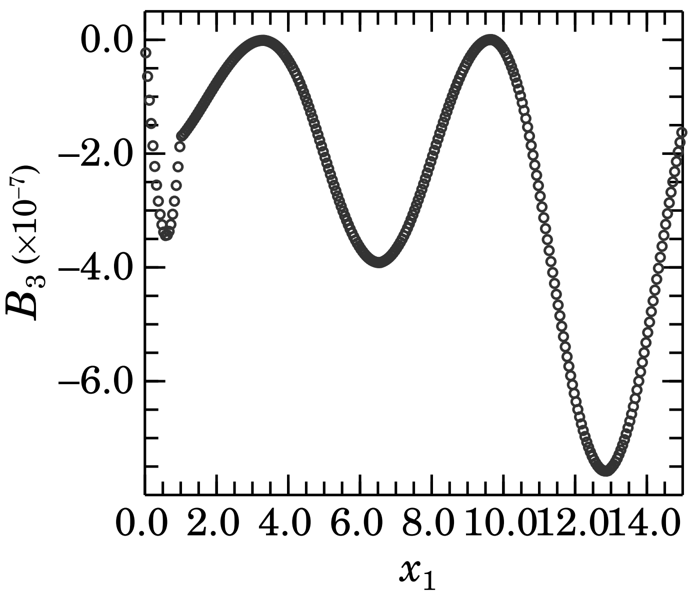
The "MHD rotor" test problem was introduced into *ZEUS-2D* by Stone & Norman (1992, ApJS, 80, 791; SN92) and is based on the analytical solution by Mouschovias & Paleologou (1980 ApJ, 237, 877; MP80) of an aligned rotor that is being braked magnetically. It comprises of a cylindrically symmetric rotor of arbitrary radius and height h with a density contrast η relative to the external medium above. Both disc and external medium are threaded by a uniform magnetic field, B_z. At $t = 0$, the rotor is set to rotate with angular speed ω_0 relative to the external medium, causing shear Alfvén waves to propagate along the magnetic field lines. For $\eta \neq 1$, Alfvén waves are partially reflected from the disc surface back toward the origin, which then reflect off the $z = x_1 = 0$ surface to make their way back to the surface of the cylinder, part of which is again reflected, and so on.

The culminated back reaction of the Alfvén waves and their reflections/transmissions slows the rotor down whilst spinning up the external medium. This was an important test in the development of the Method of Characteristics (SN92), as it is a problem impossible for any algorithm to do that is not properly upwinded in the Alfvén speed.

Open circles are the `dzeus36` solution on a cylindrical grid (with $r = x_2$ and $\phi = x_3$ the ignorable coordinates) using 300 axial zones, `CMoC`, the internal energy equation, and second-order interpolation. Flow variables are initialised at: $e_1 = 0.9$; $B_z = B_1 = 1$; $\rho = 10$ for $z \leq h = 1$, 1 elsewhere; $v_\phi = v_3 = \omega_0 r = 1.0 \times 10^{-7} \ll a_{\rm Alf}$ for $z \leq 1$, 0 elsewhere. The small value of v_3 is to assure that a principle assumption of MP80 remains valid. For this problem, `dzeus36` is CFL limited and, with no artificial viscosity, the maximum Courant number is $\sqrt{2}$. This problem was run with `courno=1.4` to $t = 13$, just before the leading Alfvén wave at $x_1 = 14$ leaves the grid. Solid lines are the analytical solutions, taken from MP80.

The quality of this solution has not changed appreciably since the release of *ZEUS-2D* in 1992. `dzeus36` gets the horizontal levels correct, but there is considerable diffusion at the discontinuities, owing to the numerous reflections and superpositions of waves that make up the solution presented.





A variation of the rotor test is one with a continuous profile in $v_\phi = v_3$. The setup for this test is *identical* to the previous test, with the single exception:

$$v_3 = \begin{cases} \frac{1}{2}\omega_0 r\bigl(1+\cos(\pi z/h)\bigr) & z \le h; \\ 0 & z > h, \end{cases}$$

instead of the discontinuous v_3 at $z = h$. Here, $\omega_0 r$ is taken again to be 1.0×10^{-7}.

This test was run until $t = 15$ deliberately to allow the leading Alfvén wave to leave the grid. This is to demonstrate that the family of `MoC` algorithms, of which `CMoC` is a member, produce perfectly transparent outflow boundary conditions for incident Alfvén waves.

The analytic solution also exists for this problem (MP80), and is coded up in Stone *et al.* (1992, ApJ, 388, 415). This particular solution is lengthy, and I just haven't imported it into `dzeus36` yet. Were it on the plot, every open circle for both v_3 and B_3 would lie directly on the line.