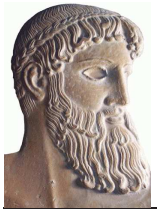
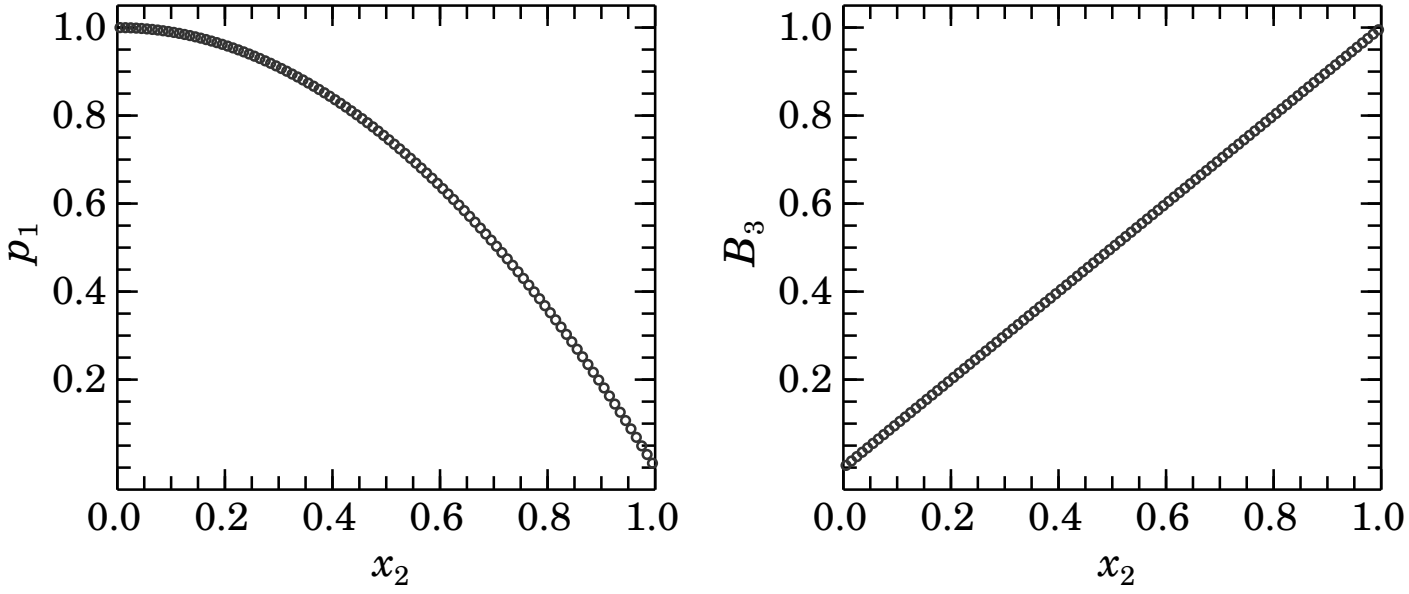


ZEUS-3D 1-D Gallery #6: "Bennett pinch"



The "Bennett pinch" (*a.k.a.* the "Z-pinch"), is a problem from plasma physics. In cylindrical coordinates with z and ϕ symmetric, a toroidal magnetic field proportional to the radial coordinate, $B_\phi = br$ (which, to first order, a toroidal field must do as $r \to 0$), generates an axial current density:

$$\vec{J} = \nabla \times \vec{B} = \frac{1}{r}\frac{\partial (rB_\phi)}{\partial r}\hat{e}_z = 2b\hat{e}_z,$$

that points along the axis in the $+z$ direction. Thus, the generated Lorentz force is:

$$\vec{F}_{\rm L} = \vec{J} \times \vec{B} = (2b\hat{e}_z) \times (br\hat{e}_\phi) = -2b^2 r\hat{e}_r,$$

that points inward toward the symmetry axis, whence the descriptor "pinch". Left to its own devices, such a setup would drive all matter on the grid toward the symmetry axis.

On the other hand, simultaneously setting up the appropriate pressure gradient should arrest this collapse. Evidently,

$$\vec{F}_p = -\nabla p(r) = -\frac{dp}{dr}\hat{e}_r = -\vec{F}_{\rm L} = 2b^2 r\hat{e}_r \quad \Rightarrow \quad p(r) = p_0 - b^2 r^2,$$

where p_0 is a constant of integration. Setting $p(1) = 0$, $p_0 = b^2$ which we take to be 1. Thus the following initialisations should be numerically stable:

$$B_\phi = B_3 = r = x_2; \qquad p = 1 - r^2 = 1 - x_2^2.$$

The panels above show the `dzeus36` solutions over a domain $0 \le x_2 \le 1$ with 100 radial zones at $t = 10$ (several magnetosonic crossing times) and ∼1,300 MHD steps. Values of p and B_3 have remained constant to within machine round-off error.

This test was critical in the very early days of developing *ZEUS*. In particular, how one differences the equations especially as one approaches a grid singularity can be tricky, and if not done correctly can produce numerically driven artefacts that can dominate the physical solution. The details of how `dzeus36` was written in "covariant form" (so that cylindrical and spherical polar coordinates can be used as seamlessly as Cartesian coordinates) had much to do with tests such as the "Z-pinch".