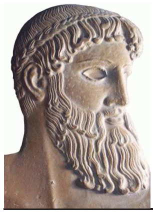
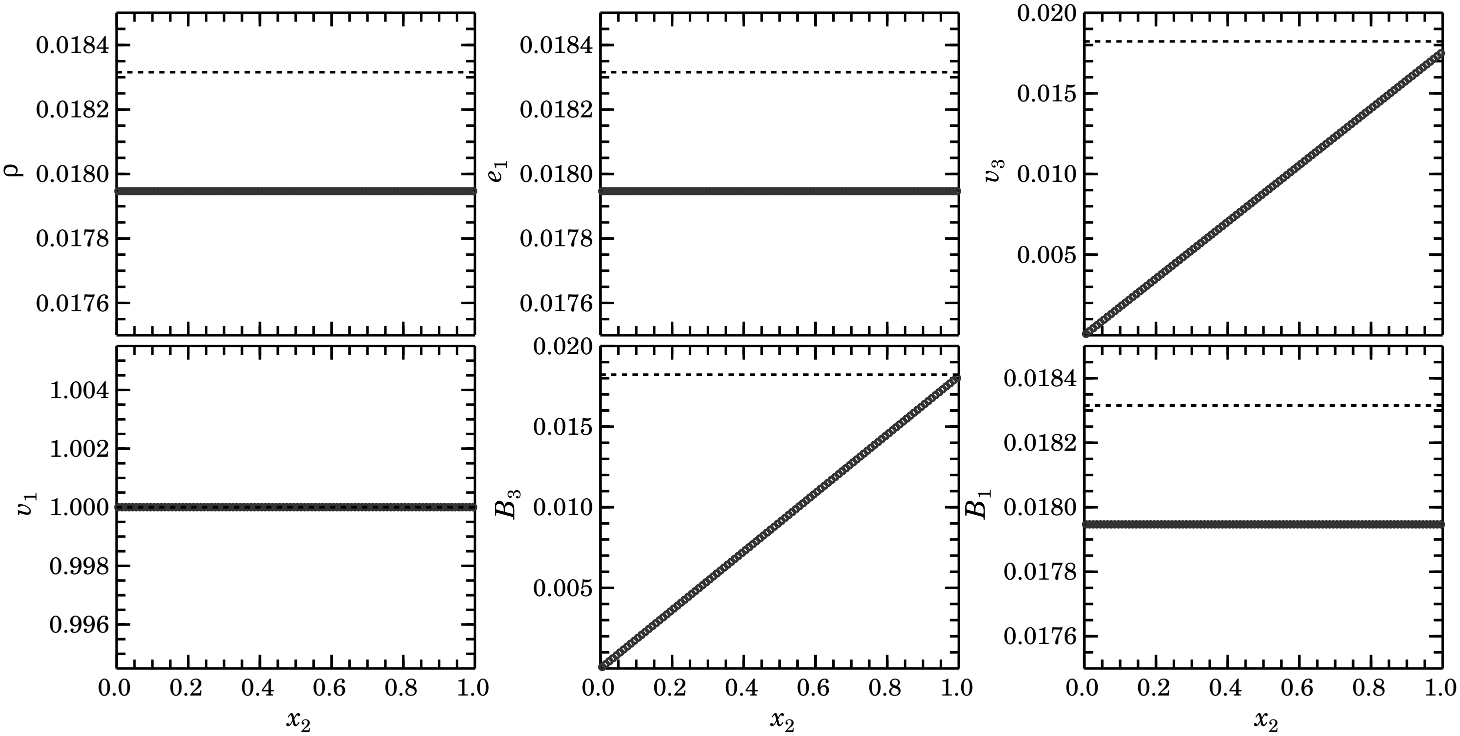


ZEUS-3D 1-D Gallery #4: Relaxation in Cylindrical Coordinates



Relaxation tests are similar to advection tests in that source terms such as ∇p and $p\nabla \cdot \vec{v}$ are set to zero, and a velocity profile is imposed in the direction of propagation. Unlike the advection tests, however, the velocity is *away* from the grid singularity ($x_2 = 0$, symmetry axis), causing material to rarefy, or "relax".

In cylindrical coordinates, scalars and axial vector components should be constant across $x_2 = 0$, whereas toroidal vector components change sign. Thus, constant profiles in scalars and axial vector components, and linear profiles in toroidal vector components should be preserved even as their values diminish. This was an important test in the early *ZEUS* development days to root out bugs in the covariant metric terms. Improper forms for the metric terms can cause significant deviations from the desired profiles particularly near the origin, causing first-order numerical "run-away errors" that can come to dominate the solution.

Initial profiles $\rho = e_1 = v_1 = B_1 = 1$ and $v_3 = B_3 = x_2$ are initialised and the profile $v_2 = x_2$ is maintained until $t = 2$. The analytical expectation of values to which all variables relax is $\sim e^{-2t}$ except for v_1 which should stay constant. Issues to be concerned with include monotonicity, the behaviour of the profiles particularly near the origin, and levels to which variables relax.

Open circles are the `dzeus36` solution using 100 zones, `CMoC`, `FIT`, no artificial viscosity, `courno=0.5`, and second order van Leer interpolation (`iord=2`). The dashed lines are the analytical levels (maximum values for linear profiles). That all profiles arrive "straight as an arrow" at the origin attests to the covariant nature of the metric terms in `dzeus36` (and, for that matter, all versions of *ZEUS* since `zeus04`.) Disagreement between numerical and analytical levels is due to second-order temporal discretisation errors, as discussed on the page for advection in spherical polar coordinates.