PHYS 4200 (formerly 3201): Mathematical Methods in Physics II (winter)

Calendar description: In this continuation of PHYS 3200, students cover additional topics in mathematical physics, including special functions (Gamma, Beta, Digamma, Bessel, Neumann, spherical Bessel), Fourier series and transforms (both discrete and continuous), and an introduction to Group theory including Lie groups culminating in the classification of baryons.

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Overview

The renumbering of this course in 2019 to a 4000 level is perhaps unfortunate, as it may encourage students to postpone it to their fourth year. This course is designed to be taught to third year students and can and should be taken in the second semester of the third year following PHYS 3200.

As a "terminal course" to the "Methods (200) Stream", no other courses list it as a prerequisite. This may be an oversight as it is the only place in the curriculum where students see formally Fourier series and transforms (and thus useful for PHYS 4410, Electrodynamics) or the origin of Stirling's series and approximation (useful for PHYS 3510, Statistical Mechanics). In addition to Fourier series and transforms, this course provides introductions to higher mathematics that physics students probably should be aware of and will not likely see elsewhere unless they continue on to graduate school. This includes properties of the Gamma, Beta, Bessel, and Neumann functions, as well as a cursory introduction to Group theory.

The two mathematical methods courses (this course and its prerequisite PHYS 3200) are part of the programme because:

1. the material they contain cannot be found in any one, and often any two MATH courses; and

2. the math department doesn't cover the needed material in time for when the physics courses are taught.

That said, students in the MATH/PHYS double major/honours programmes take MATH courses instead of the math methods courses.

There is no practical equivalency in the Math department for this course. That department certainly includes units on group theory, Fourier series and analysis, and special functions, but these are scattered among several courses they offer.

Prerequisites

PHYS 3200 Mathematical Methods in Physics I

This course is a continuation of PHYS 3200, and mastery of many ideas covered there is assumed. Students should be able to solve any ordinary second order differential equation, be proficient with Green's functions, eigenvalues and eigenfunctions (eigenvectors), and contour integration.

Alternatively, MATH 3405 and MATH 4436 serve as prerequisites for PHYS 4200.

Dependent courses

None (but see the Overview).

Student Outcomes

Students completing PHYS 4200 should have mastered the following skills:

1. Associate with a physical problem certain special functions that will aid in finding a solution;
2. Prepare and present a mathematical physics problem in a coherent fashion, easily understood by another student.
3. Analyse functions as expansions of other functions, and appreciate the meaning of the terms revealed.
4. Learn how to acquire additional math skills for problems students may uncover in their physics careers.

Curriculum

1. special functions
    - gamma and beta functions
    - digamma and polygamma functions; stirling series

  - Bessel, Neumann, Hankel functions
  - spherical Bessel functions.

2. Fourier series

  - Fourier expansions of functions; relevance of Sturm-Liouville theory
  - evaluating sums

3. Fourier transforms

  - discrete Fourier transforms
  - continuous transforms and solving PDEs
  - convolution theorem; Parseval relations

4. Group theory

  - basic axioms and theorems
  - discrete and continuous groups
  - group representations
  - Lie groups, generators, structure constants
  - SU(3) and the classification of baryons

Suggested texts

Mathematical Methods for Physicists by Arfken (ISBN 0-12-059826-4)

- has all the material required at an appropriate level for third year students, and can be used for PHYS 3200 as well.

- acknowledged by most professors as the best text available for Mathematical Physics

Mathematical Methods of Physics by Mathews and Walker (ISBN 0-8053-7002-1)

- while used at some institutions for an undergraduate course, this text is widely acknowledged as a graduate text on the subject.

- Much denser than Arfken.

Mathematical Physics by Butkov (ISBN 0-201-00724-4)

- a viable alternate to Arfken, though much drier and considerably denser.

Notes to the instructor

1. As a "terminal course", this is the place for an instructor to teach students some higher forms of mathematics for the sake of the math, and to try to convey to students a little of how a mathematician *thinks*. Of all courses, the curriculum suggested here is the most flexible. Instructors may wish to replace group theory with a little tensor analysis, choose other special functions to discuss, or maybe delve into integro-differential equations, Laplace transforms (covered to some extent in MATH 2303) and the Bromwich integral. All of these topics have been covered by one instructor of this course or another in the past and all serve the purpose.

2. This course is the only place where students will see formally Fourier series and analysis; this should remain as the one fixed portion of the curriculum.

3. There is no reason not to use the same text for PHYS 3200 and PHYS 4200, and the two instructors of these courses should agree on a common text.