PHYS 3500: Quantum Mechanics I (fall)

Calendar description: This course builds on the foundation set in PHYS 1500.1 (.2). Topics in this course include the (time-independent) Schrödinger equation, one-dimensional potentials, barriers and tunnelling, the Heisenberg Uncertainty Principle, Dirac notation, expectation values, the three-dimensional Schrödinger equation, single-electron atoms, spin, and identical particles.

Contents

Overview

This is the first of three courses in Quantum Mechanics, with the third (PHYS 4501) being taught only occasionally. Students will have seen the basic concepts such as Heisenberg, de Broglie, and Pauli, as well as the Bohr atom and even the infinite square well problem, but two semesters and two summers previously in PHYS 1500 (Introduction to Modern Physics). Thus, these elementary ideas may have to be reviewed quickly before new material is introduced.

Prerequisites

MATH 2301 Linear Algebra (indirectly)
MATH 2303 Differential equations
MATH 2311 Intermediate Calculus II

The instructor can expect students to have a solid background in vector calculus, including applications of the theorems of Gauss and Stokes as well as line and surface integration. Students will also be very familiar with all types of first order ODEs, second order ODEs with constant coefficients, solutions to systems of equations, matrix theory, Dirac notation, eigenalgebra, Taylor and binomial expansions, and the algebra of series.

Second order ODEs with variable coefficients, and Sturm-Liouville Theory (expressing differential equations as differential operators acting on their eigenfunctions) is covered in PHYS 3200 (Mathematical Methods in Physics I), which most students will be taking concurrently to PHYS 3500. Should the instructor need these concepts in this course, (s)he is encouraged to discuss timing of these subjects with the instructor of PHYS 3200.

PHYS 1500 Introduction to Modern Physics

PHYS 3500 builds on the elementary ideas in quantum mechanics introduced in PHYS 1500. These include the idea of wave-particle duality, the Bohr model of the atom, and very simple applications of Schrödinger's wave equation, such as the allowed energy levels and wave functions in an infinite square well in which the SWE reduces easily to quadratures. It is beyond the scope of PHYS 1500 to do any boundary condition matching, and thus the finite square well is not covered until PHYS 3500.

The instructor should note that PHYS 3500 starts two semesters and two summers after PHYS 1500 ends, and thus *some* review might be useful. It is the intent, however, that PHYS 3500 should be able to *start* with the Schrödinger Wave Equation (*i.e.*, no need to review the "old quantum theory"), and work toward the ultimate goals of this course, namely the one-electron atom, spin, angular momentum, and identical particles.

Dependent courses

PHYS 3210 Computational Methods in Physics

Familiarity with quantum mechanics is necessary for PHYS 3210 because a number of examples used in class stem from quantum mechanics (*e.g.*, finding the allowed energy states in a finite square well require using a root-finder, which is developed in PHYS 3210), and many of the computational projects students perform are based on finding numerical solutions to Schrödinger's equation.

PHYS 3350 Thermal Physics

PHYS 3350 covers both thermodynamics and statistical mechanics, the latter of which is traditionally approached from the quantum mechanical point of view. Thus, statistical mechanics need not and probably should not be part of PHYS 3500.

PHYS 4370 Philosophy of Physics

This course, part of the Philosophy/Physics double major and honours programmes, is co-taught by faculty in the Department of Astronomy and Physics and the Department of Philosophy. It is a discussion-oriented course that depends on PHYS 2400 (Electricity and Magnetism) and PHYS 3500 for an overall level of knowledge in physics so that discussion can be based on sound physical principles. The problem-solving skills gained in PHYS 3500 are less germane.

PHYS 4500 Quantum Mechanics II

It is the intent that PHYS 4500 be a natural follow-on to PHYS 3500. To cover the curriculum for PHYS 4500, that instructor will count on PHYS 3500 having covered up to and including the one-electron atom, spin, and the quantisation of angular momentum, including their associated quantum numbers, l and s.

Student Outcomes

Students completing PHYS 3500 should begin to master the following skills:

1. Develop an intuition for "hard to see" science; testing ideas that can't be experimentally checked in a simple way
2. "Divorce" oneself from "classical thinking"
3. Develop interpretations based solely on mathematics
4. Solve a second-order differential equation, including matching boundary conditions. This is something many students find very difficult to master.

Typical problems students completing PHYS 3500 should be able to solve:

1. Find the energy levels and allowed wave functions of a particle trapped in a finite square well.

   This problem is easy to gloss over as it seems like such an easy problem to those used to this kind of mathematics. However, this is the first time most students will have seen such a problem in which two solutions exist in different realms, and where the final solution is determined by matching boundary conditions. Indeed, many students never really "get" this problem and cannot apply what we hope they learn from it to other similar situations. It would therefore do the entire programme a great service if the instructor of PHYS 3500 could make sure that every student can reliably solve this problem.

   Note that this problem is often used in PHYS 3210 (Computational Methods in Physics) taken in the semester following PHYS 3500 as an application of the root-finding program students develop there, and is thus reviewed in that class again.

2. Give a cohesive account to a non-expert of the "Schrödinger's cat" paradox.
3. Find the probability that a particle will penetrate a finite width and height barrier whose potential is greater than the kinetic energy of the particle.
4. Find the first few energy levels and the allowed wave functions of the one-electron atom.

Curriculum

In parentheses is the approximate number of lectures spent on each topic.

1. Conceptual review (1)
    - classical waves
    - wave-particle duality
2. The wave function (2)
    - "Derivation" of the Schrödinger wave equation
    - statistical interpretation, probability, normalisation
    - momentum
    - The Heisenberg Uncertainty Principle, and "Schrödinger's cat"
3. Time-independent Schrödinger equation (6)
    - stationary states
    - the infinite square well (review from PHYS 1500)
    - finite square well
    - the harmonic oscillator (and zero-point energy)
    - the free particle
    - delta-function potential
    - barriers and tunneling
4. Formalism (4)
    - Hilbert space (review)
    - observables
    - eigenfunctions of a Hermitian operator
    - generalised statistical interpretation
    - the Uncertainty Principle
    - Dirac Notation (review from MATH 2301)
5. Quantum mechanics in three dimensions (6)
    - Schrödinger equation in spherical coordinates
    - the hydrogen atom
    - angular momentum and spin
6. Identical particles (4)

- Two-particle systems
- atoms
- solids

Note: Quantum statistical mechanics is covered in detail in PHYS 3350 (Thermal Physics).

Suggested texts

For the most part, the curriculum committee gives names of texts merely as suggestions. However, in this stream the committee strongly urges the instructor to use *Introduction to Quantum Mechanics* by Griffiths, which can serve both this course and its follow-on, PHYS 4500. Students and instructors universally praise both of Griffiths' texts (this, and *Introduction to Electrodynamics*) as the most enjoyable and comprehensible treatment of both subjects; there simply is no equal.

The text by Eisberg and Resnick is offered as a possible supplement for problems and examples, and alternative approaches.

Part I of *Introduction to Quantum Mechanics*, by David Griffiths (ISBN 0-131-24405-1)

- universally praised by students and instructors as the best text on the market for a two-semester upper-year course in quantum mechanics.
- second half of the text ideal for PHYS 4500.
- covers all the topics required
- includes many problems of varying degrees of difficulty, with many worked examples

Quantum Physics of Atoms, Molecules,... by Roberts, Eisberg, and Resnick (ISBN 0-471-87373-X)

- used previously at SMU, is an excellent text for PHYS 3500, but not suitable for PHYS 4500.

Notes to the instructor

1. The instructor should spend little, if any, time reviewing ideas in mathematics such as linear algebra, Dirac notation, vector calculus, *etc.*; these should have been adequately covered in the MATH prerequisites. If not, the instructoer is asked to bring this to the attention of the Curriculum Committee, so that we may raise these possible deficiencies with our colleagues in the math department.

2. After collectively trying many texts for PHYS 3500 and PHYS 4500 (Quantum Mechanics II), this faculty has determined that hands down, David Griffiths' text (*Introduction to Quantum Mechanics*) is exactly the right text for our students; a sentiment

endorsed enthusiastically by *every* student asked. We have made a similar choice for our E&M stream (PHYS 2400 and PHYS 3400). In both cases, Griffiths has just the right amount of material for two courses—one introductory, one intermediate—at exactly the right level for a complete undergraduate curriculum in each subject.

3. As soon as the instructor for PHYS 4500 (Quantum Mechanics II) for the following year is known (usually sometime in March), it is strongly advised that the instructor of PHYS 3500 discuss with the instructor of PHYS 4500 what material was covered and what, if any, weaknesses in the students' preparedness has been noted.