# PHYS 3300: Classical Mechanics (fall)

*Calendar description:* This advanced course in Classical Mechanics introduces the student to the calculus of variations, constrained problems, and generalised Lagrangian and Hamiltonian dynamics. Applications are made to oscillations, the "brachistochrone problem", central force problems, rigid bodies, and the motion of tops. Additional topics include relativistic mechanics, canonical perturbation theory, and chaos.

Contents



## Overview

This is the final (terminal) course in the classical mechanics series, though specialised fourth year courses in the "300 stream" include Fluid Dynamics (PHYS 4380) and General Relativity (PHYS 4390). It should be taught near to and possibly at the level of Goldstein.

## Prerequisites

MATH 2303 Differential equations
MATH 2311 Intermediate Calculus II

Students entering PHYS 3300 are expected to have mastered the algebra of series, Taylor and binomial expansions, partial derivatives, line, surface, and volume integrals, the theorems of Gauss, Green, and Stokes, vector spaces, matrices, determinants, systems of linear equations, Dirac notation, eigenalgebra and the solution of any first order ODE or second order ODE with constant coefficients. Concurrent to this course, students should be enrolled in PHYS 3200 (Mathematical Methods in Physics I) where they cover

complex analysis in the first half of term, and the general solution to second order ODEs with variable coeffiicents in the latter half of term.

PHYS 1500 Introduction to Modern Physics

Students will have had a *brief* (three weeks) introduction to the special theory of relativity where they will have covered the concepts of time dilation, length contraction, the twin paradox, *etc.*, but will not have seen any four-vector formalism.

PHYS 2301 Analytical Mechanics

This course prepares students with the material in Part I of Marion and Thornton; namely up to but not including the Euler-Lagrange equations and the calculus of variations. The prerequisite to PHYS 2301, namely PHYS 2300 (Vibrations, Waves, and Optics), has given the student a thorough introduction to waves and oscillations but without the benefit of vector calculus.

## Dependent courses

ASTR 4600 High-Energy Astrophysics

ASTR 4600 depends on PHYS 3300 for its unit on special relativity and the level of physical intuition and sophistication imparted upon the students.

PHYS 4380 Fluid Dynamics

PHYS 4380 depends on PHYS 3300 for the way of thinking and level of sophistication imparted upon the students, rather than on any particular subject.

PHYS 4390 General Relativity

PHYS 4390 depends on PHYS 3300 for the way of thinking and level of sophistication imparted upon the students, rather than on any particular subject.

PHYS 4500 Quantum Mechanics II

PHYS 4500 depends upon PHYS 3300 for a firm foundation in the Hamiltonian formalism, the use of operators and their associated eigenvalues and eigenfunctions, and perturbation theory.

## Student Outcomes

The overall aim of the Classical Mechanics stream (300) is to develop the student into a solver of dynamic problems, and to build the student's physical intuition into a reliable tool that can be applied to any physical problem.

Students completing PHYS 3300 should have mastered the following skills:

1. understand what constitutes a "well-posed" problem, and be able to pose such questions;
2. break up a well-posed problem into its mathematical components;
3. draw meaningful physical interpretations from mathematical solutions;
4. choose and change coordinate and reference frames at will;
5. be able to exploit fully the mathematical and physical symmetry of a problem.

Typical problems students completing PHYS 3300 should be able to solve:

1. What is the path a particle should follow under the influence of gravity that would allow the particle to move between two given points in the least amount of time? (the "brachistochrone problem")
2. Show that the shortest distance between two points on a sphere (geodesics) is along the great circle.
3. Find the precession rate of a Foucault pendulum in terms of the latitude on the Earth.
4. Find the precession rate of a symmetrical top.
5. Show that the relativistic motion of a particle in an attractive inverse square law force is a precessing ellipse.
6. What are the resonant frequencies of two physical pendula coupled by a spring?

## Curriculum

1. Calculus of Variations
   - brachistochrone problem
   - constrained problems
   - Euler's equations
2. Lagrangian dynamics
   - generalised coordinates
   - Lagrange's equations (and undetermined multipliers)
   - small oscillations
3. Rigid bodies
   - coordinates and orthogonal transformations
   - inertia tensor, and principal axes

- Euler angles and Euler's equations
- a spinning top

4. Special Relativity
   - Lorentz transformations
   - covariance of four vectors
   - Lagrangian formalism of relativistic mechanics
5. Hamiltonian dynamics
   - Hamilton's equations (Hamiltonians)
   - canonical transformations
   - Poisson brackets
   - Hamilton-Jacobi theory
6. Canonical Perturbation Theory
   - time-dependent perturbation theory
   - time-independent P.T. with one and higher degrees of freedom
7. Chaos
   - fractals
   - coupled pendula

## Suggested texts

*Classical Mechanics* by Goldstein, Poole, & Safko (ISBN 0-201-65702-3)

- 4/5 stars (21 reviews) on amazon.com
- has all the right material, but is generally viewed as a graduate text. Still, many reviewers claim this is fine for a junior physics course.

Chapters 12–16 of *Classical Mechanics* by John R. Taylor (ISBN 1-891389-22-X)

- 4/5 stars (4 reviews) on amazon.com
- chapters 1–11 could be used in PHYS 2301 (Analytical Mechanics)
- would need a supplement for relativity and perturbations

Second half of *Classical Dynamics* by Thorton & Marion (ISBN 0 534 408 966)

- 2.5/5 stars (44 reviews) on amazon.com

- first half could be used for PHYS 2301 (Analytical Mechanics)
- possibly too low a level
- doesn't do much on perturbation theory

*Introduction to Dynamics* by Percival & Richards (ISBN 0 521 281 490)

- 4/5 stars (1 review) on amazon.com
- has many topics, but not relativity nor calculus of variations

## Notes to the instructor

1. This course should *start* with the calculus of variations, and then get right into the Euler-Lagrange equations and the Lagrangian formalism.
2. No where else in the student's curriculum, including the Math Methods stream, is the calculus of variations covered, and this could require a few lectures.
3. This is the first time in the curriculum where the ideas of relativistic covariance and four-vectors are used significantly, and is an important component of this course. It is discussed again in PHYS 3400 (Electrodynamics) in the context of Lorentz transformations of electric and magnetic fields in either the semester following this course, or three semesters following, depending on the cycling of PHYS 3400 with PHYS 3350 (Thermal Physics).
4. PHYS 3200 (Math Methods I) is taken concurrently with this course, and thus the instructor should be aware of when certain topics in mathematics are being covered before their use is incorporated into this course. Discussion between the instructors of PHYS 3200 and 3300 is encouraged to make optimal use of the new skills learned in both.