PHYS 3200: Mathematical Methods in Physics I (fall)

Calendar description: Students are provided with the mathematics needed to solve problems in advanced physics courses. Topics include separation of variables, the method of Frobenius, the Wronskian integral, Greens functions, Dirac notation, eigenfunctions and eigenkets, Hermitian operators, properties of analytic functions, Cauchys Integral Theorem, Laurent expansions and the calculus of residues.

Note: Credit for PHYS 3200 cannot be given to students who have completed MATH 4436 and MATH 3406.

Contents

Overview

The "Methods (200) Stream" is specifically designed to provide the mathematics and computing skills needed in concurrent and subsequent physics courses. This course and PHYS 3210 (Computational Methods in Physics) are not intended to teach mathematics and computing for their own sakes, but for the purpose of relieving the instructors of the physics courses from having to teach mathematics in support of their curriculum. Thus, the curriculum of the 200-stream courses should be sensitive and altered to suit the mathematical and computational needs of the dependent physics courses.

The two mathematical methods courses (this course and the follow-on PHYS 4200) are part of the programme because:

1. the material they contain cannot be found in any one, and often any two MATH courses; and
2. the math department doesn't cover the needed material in time for when the physics courses are taught.

That said, students in the MATH/PHYS double major/honours programmes take MATH courses instead of the math methods courses.

The "equivalent" MATH courses to this course are MATH 3406 and MATH 4436. Together, these courses cover in more detail about 75% of the topics covered in this course. The main omission is Sturm-Liouville theory, which students will get in PHYS 3500.

Prerequisites

PHYS 2303 Mechanics II

Examples from physics may be drawn from assumed previous and concurrent physics courses, and in particular PHYS 2303. The mathematical and physical maturity gained by taking PHYS 2303 is also a key asset.

MATH 2301 Applied Linear Algebra

Students learn about matrix algebra including determinants, solving linear systems of equations, and eigenvalues/eigenvectors in MATH 2301; all prerequisite material for this course.

MATH 2303 Differential Equations I

This course deals mostly with second order differential equations with variable coefficients, and thus will rely on students having mastered first order ODEs and second order ODEs with constant coefficients from MATH 2303.

MATH 2311 Intermediate Calculus

The Calendar entry for MATH 2311 says its topics include line, surface, and volume integrals as well as an introduction vector calculus, including the the nabla operator and the theorems of Gauss, Green, and Stokes. However, in practise the latter three theorems are rarely covered and will therefore need to be reviewed/introduced in PHYS 3200.

Dependent courses

PHYS 3210 Computational Methods in Physics

This course benefits from the mathematical sophistication derived from PHYS 3200, more than the specific topics taught.

PHYS 3300 Classical Mechanics

This course is taken concurrently with PHYS 3200. The unit on solving second order differential equations is needed later in the term, and the instructors of PHYS 3200 and 3300 might coordinate their lectures to make the best use of the mathematics developed in PHYS 3200.

PHYS 3500 Quantum Mechanics I

This course is taken concurrently with PHYS 3200, and students will need to be able to solve the Schrödinger equation using series expansion (Frobenius) methods by the end of the term and well after the unit on series solutions is complete in PHYS 3200. Sturm-Liouville (operator) theory, taught in midterm in PHYS 3200, is also taught in PHYS 3500, usually after students have seen it in PHYS 3200. This is one of the rare cases where a topic in mathematics is covered both in the 200 stream and the physics course most dependent upon it. Sturm-Liouville theory is particularly difficult for most students to master, and seeing it twice in reasonably rapid succession has shown to be beneficial.

Dirac notation, also important to PHYS 3500, is introduced in PHYS 3200. The university does not allow for "co-requisites". If ever there were a case for one, the relationship between PHYS 3200 and PHYS 3500 would be it.

PHYS 3510 Statistical Mechanics

This course depends more specifically on the mathematical skill sets learned in MATH 2303/2311 (*e.g.*, partial derivatives and vector calculus), but also depends on the mathematical sophistication garnered in PHYS 3200 including familiarity with complex numbers and the abilty to solve simple ODEs.

PHYS 4200 (formerly 3201) Mathematical Methods in Physics II

This course is a follow-on course to PHYS 3200, and will continue to build up the student's mathematical sophistication and skill sets.

PHYS 4380 Fluid Dynamics

In addition to the firm foundation on vector calculus afforded by MATH 2311 and the theorems of Gauss and Stokes (more likely from PHYS 3200), this course depends on the student's ability to solve second order differential equations, as well as some complex analysis. Eigenvalues and eigenvectors (eigenfunctions) are also critical in PHYS 4380, much of which comes from MATH 2301. However, the reinforcement afforded by the unit on Sturm-Liouville theory in PHYS 3200 is critical as most students coming from MATH 2301 have very little idea on how important "eigenalgebra" is to physics.

PHYS 4390 General Relativity

This course benefits from the mathematical sophistication obtained from PHYS 3200 (and to some extent its follow-on PHYS 4200), and will then introduce its own topics in mathematics (e.g., Tensors, p-forms, etc.) to pursue Einstein's equations. At this time, it is suggested that tensor theory *not* be covered in any of the courses in the 200 (methods) series.

PHYS 4410 Electrodynamics

This course very much depends on the students' ability to solve any ordinary second order differential equation, to solve certain equations with Maple or Mathematica, and to use the results of Complex Analysis.

PHYS 4500 Quantum Mechanics II

This course depends heavily students being able to solve second order differential equations, and to apply Sturm-Liouville (operator) theory. In particular, the student's ability to think of solutions to second order differential equations as eigenfunctions of operators and evaluating their associated eigenvalues (expectation values) is critical.

Student Outcomes

Students completing PHYS 3200 should begin to master the following skills:

1. Pose and analyse a physical problem mathematically without undue hindrance from a lack of appropriate mathematical skills;
2. Prepare and present a mathematical physics problem in a coherent fashion, easily understood by another student.
3. Solve problems and display results with commonly available computational packages such as Maple, Wolfram, etc.

Curriculum

1. differential equations
   - PDEs, wave equation in 3-D, separation of variables
   - second order ODEs
     - singular/ordinary points
     - Method of Frobenius (series expansion)
     - Wronskians, second solutions
     - inhomogeneous equations
2. Sturm-Liouville Theory
   - Hermetian operators
   - eigenfunctions and eigenvalues
   - Gram-Schmidt orthogonality
   - Dirac delta function
   - Green's functions
3. complex analysis

- complex numbers and functions
- analytic functions and the Cauchy-Riemann conditions
- Cauchy theorem and integrals
- singularities and Laurent expansions
- calculus of residues

Suggested texts

Mathematical Methods for Physicists by Arfken (ISBN 0-12-059826-4)

- has all the material required at an appropriate level for third year students, and can be used for PHYS 4200 as well.
- acknowledged by most professors as the best text available for Mathematical Physics

Mathematical Methods of Physics by Mathews and Walker (ISBN 0-8053-7002-1)

- while used at some institutions for an undergraduate course, this text is widely acknowledged as a graduate text on the subject.
- Much denser than Arfken.

Mathematical Physics by Butkov (ISBN 0-201-00724-4)

- a viable alternate to Arfken, though much drier and considerably denser.

Notes to the instructor

1. PHYS 3200 is taken concurrently with PHYS 3300 (Classical Mechanics) and PHYS 3500 (Quantum Mechanics I). To a large extent, the topics covered in this course and the order in which they appear are determined by the needs of these courses. In particular, it is essential that differential equations be covered first and that Sturm-Liouville theory follow that. The unit on complex analysis can be safely taught in the last third of the semester.
2. This course is the only place where students will learn how to solve variable co-efficient second order differential equations, and be exposed to Complex Analysis.
3. There is no reason not to use the same text for PHYS 3200 and PHYS 4200, and the two instructors of these courses should agree on a common text.