TUTORIAL 11, PHYS 2335

1. Find the eigenvalues and the corresponding normalised eigenkets for the matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{bmatrix}. \qquad \text{(eigenvalues} = -1,\ 1,\ 2)$$

2. Find the eigenvalues and the corresponding normalised eigenkets for the matrix:

$$B = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}. \qquad \text{(eigenvalues} = 1,\ 1,\ 6)$$

3. Find the eigenvalues and the corresponding normalised eigenkets for the matrix:

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \qquad \text{(eigenvalues} = 0,\ 0,\ 3)$$

4. Let $A \in \mathcal{R}^{22}$ have eigenvalues 1 and -1, and let the eigenbra corresponding to $\lambda = 1$ be $(1,\ 0)$ and that corresponding to $\lambda = -1$ be $(0,\ 1)$. Find A.

5. Prove that $A \in \mathcal{R}^{nn}$ is singular $\Leftrightarrow \exists$ at least one non-zero ket $|\vec{v}\,\rangle$ such that $A|\vec{v}\,\rangle = 0$.

(HINT: Look for the eigenvalues of A, and show that one of the eigenvalues for a singular matrix must be zero.)

6. Along a frictionless linear air track, attach two equal masses, m, with a spring of spring constant k, and place the assembly in between two immovable supports attached to the track. Next, between each support and the adjacent mass, attach a spring of spring constant K. Find the natural oscillation frequencies and normal modes of the system.