TUTORIAL 10, PHYS 2335

1. Let $A \in \mathcal{R}^{n\,n}$ be orthogonal. Show that $|A| = \pm 1$.

2. In class, we introduced the *elementary row operators* (ERO) to represent the Gauss-Jordan row operations as matrices. One of the important results from this formalism was Theorem 3.10 which proved that if $A,\ B \in \mathcal{R}^{n\,n}$ and if $AB = I$, then A is non-singular and $B = A^{-1}$. Since B is identified as the inverse, then it followed that $BA = I$ as well. In essence, we used all the "fire-power" of the ERO formalism to prove that $AB = I \Rightarrow BA = I$.

Consider the following "alternate proof" for the same result:

$$AB = I \;\Rightarrow\; B(AB) = BI \;\Rightarrow\; (BA)B = B = IB \;\Rightarrow\; BA = I.$$

Surely this is a much simpler proof than the one developed in class. It is, however, logically flawed. Can you spot the problem(s) with this proof?

3. Using the formula for the inverse matrix elements derived in class, namely

$$a^{-1}_{i\,j} = \frac{\mathcal{C}_{j\,i}}{|A|},$$

where $\mathcal{C}_{j\,i}$ (note the order of indices) is the co-factor for matrix element $a_{j\,i}$, evaluate the inverses of the following matrices:

$$\text{a)}\quad \begin{bmatrix} 7 & 4 \\ -2 & 3 \end{bmatrix} \qquad\qquad \text{b)}\quad \begin{bmatrix} -3 & 5 & 1 \\ 2 & 0 & 3 \\ 1 & 1 & 3 \end{bmatrix}$$

4. Using the Gauss-Jordan elimination scheme, find the inverses of the following matrices, if they exist:

$$\text{a)}\quad \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 4 \end{bmatrix} \qquad \text{b)}\quad \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & -2 \\ 3 & -2 & -2 \end{bmatrix} \qquad \text{c)}\quad \begin{bmatrix} 3 & -1 & 0 \\ 5 & 2 & -1 \\ 1 & -4 & 1 \end{bmatrix}$$

5. Using the Gauss-Jordan elimination scheme, solve the following system of linear equations:

$$\begin{aligned} 2x_1 - \ x_2 + \ x_3 &= 2 \\ 3x_1 + 3x_2 - \ x_3 &= 0 \\ -\ x_1 - 2x_2 + 3x_3 &= -4. \end{aligned}$$

$\frac{2}{19}(11,\ -18,\ -21)$