TUTORIAL 9, PHYS 2335

1. Let A, $B \in \mathcal{R}^{nn}$ be diagonal matrices. Show that $[A, B] = 0$.

2. Let A, $B \in \mathcal{R}^{nn}$, let A be symmetric, and let B be antisymmetric. Show that

$$\mathrm{tr}(AB) = 0.$$

You may use the result of problem 1a of tutorial 8, namely, $\mathrm{tr}(AB) = \mathrm{tr}(BA)$ even if $AB \neq BA$.

3. Let A, $B \in \mathcal{R}^{nn}$ be nonsingular matrices (*i.e.*, A^{-1} and B^{-1} exist). If A and B anticommute ($AB = -BA$), show that $\mathrm{tr}(A) = \mathrm{tr}(B) = 0$.

4. Consider two general 2×2 matrices

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \qquad B = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}.$$

a) Show by direct computation that $|AB| = |A||B|$ but $|A + B| \neq |A| + |B|$.

b) Show by direct computation that $\mathrm{tr}(A + B) = \mathrm{tr}(A) + \mathrm{tr}(B)$ but $\mathrm{tr}(AB) \neq \mathrm{tr}(A)\,\mathrm{tr}(B)$.

5. Let $A \in \mathcal{R}^{nn}$. Prove if any row of A is a linear combination of any other row(s) in A, $|A| = 0$. This is Corollary 3.2 from class and to prove it, you may invoke the results of Lemmas 3.1 and 3.2.

6. Let A, $B \in \mathcal{R}^{nn}$. Let B be the same as A except for row i which is given by α times row j of A plus row i of A, where $\alpha \in \mathcal{R}$ and $i \neq j$. Prove that $|B| = |A|$. This is Corollary 3.3 from class.

7. Evaluate

$$\text{a)} \quad \begin{vmatrix} -1 & 2 & 7 \\ 3 & 1 & 1 \\ 4 & -2 & 9 \end{vmatrix} \qquad \text{b)} \quad \begin{vmatrix} 2 & 1 & -9 \\ 5 & 3 & 0 \\ -7 & -1 & 2 \end{vmatrix} \qquad \text{c)} \quad \begin{vmatrix} -3 & 2 & 1 & 0 \\ 5 & 3 & -2 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 8 & 0 & 2 \end{vmatrix}$$