TUTORIAL 7, PHYS 2335

1. Evaluate

$$\oint_C \vec{r} \cdot d\vec{r}.$$

HINT: You just can't write $\vec{r} \cdot d\vec{r} = rdr$, then integrate to $r^2/2$. First of all, what limits would you apply? Second, $\vec{r}$ and $d\vec{r}$ don't need to be parallel. You must think of this as a *vector* integral, not a *scalar* integral. Therefore, is the vector $\vec{r}$ irrotational? If so, develop the integral as an integral of a perfect differential, then argue that such an integral around a closed loop must always be zero.

2. Evaluate the surface integral of $\vec{E} = E_x\hat{x}$ over a hemisphere of radius r whose axis of symmetry is the $+x$ axis.

3. Prove the last of the three forms of Gauss' theorem, namely

$$\oint_S \vec{A} \times d\vec{\sigma} = -\int_V \nabla \times \vec{A}\, dV.$$

4. Prove the second form of Green's theorem, namely:

$$\oint_S (u\nabla v - v\nabla u) \cdot d\vec{\sigma} = \int_V (u\nabla^2 v - v\nabla^2 u)\, dV.$$

5. Using Gauss' Theorem, prove

$$\oint_S d\vec{\sigma} = 0.$$

6. If $\vec{B} = \nabla \times \vec{A}$, prove that

$$\oint_S \vec{B} \cdot d\vec{\sigma} = 0.$$

7. Ampère's Law in differential form (one of Maxwell's four equations on electrodynamics) states:

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J},$$

where $\vec{H} = \mu_0 \vec{B}$ is the magnetic field (strictly speaking, $\vec{B}$ is the *magnetic induction*), $\vec{E}$ is the electric field, and $\vec{J}$ is the current density. If the electric field, $\vec{E}$, has no explicit time dependence, show that

$$\oint_C \vec{H} \cdot d\vec{l} = I$$

where $I = \int_S \vec{J} \cdot d\vec{\sigma}$ is the total current passing through a surface S bounded by a loop C.

over...

8. A magnetic induction $\vec{B}$ is generated by an electric current in a ring of radius R. Show that the *magnitude* of the vector potential $\vec{A}$ (where $\vec{B} = \nabla \times \vec{A}$) at the ring is:

$$A = \frac{\varphi}{2\pi R}$$

where φ is the total magnetic flux passing through the ring. Hint: $\vec{A}$ is everywhere tangential to the ring.