TUTORIAL 6, PHYS 2335

1. A vector function $\vec{f}(x,y,z)$ is not irrotational, but the product of $\vec{f}$ and a scalar function $g(x,y,z)$ is. Show that $\vec{f}\cdot\nabla\times\vec{f}=0$.

2. Let $\vec{f}$ and $\vec{g}$ be vector functions of (x,y,z). If $\nabla\times\vec{f}=\nabla\times\vec{g}$, show that

$$\vec{f}-\vec{g}=\nabla\phi$$

where ϕ is an arbitrary scalar function of x, y, and z.

3. Suppose ϕ solves Laplace's equation, namely $\nabla^2\phi=0$. Prove that the (dual) vector $\nabla\phi$ is both solenoidal and irrotational.

4. In a non-rotating isolated mass such as a star, the condition for *hydrostatic equilibrium* is

$$\nabla P+\rho\nabla\phi=0$$

where P is the thermal pressure, ρ the density, and ϕ the gravitational potential. Show that a surface of constant pressure is also a surface of constant gravitational potential (*i.e.*, that the two surfaces are everywhere parallel).

5. Evaluate $\phi(x,y,z)$ given the following gradient:

$$\nabla\phi=\left(6xe^{y/z},\frac{3x^2}{z}e^{y/z},-\frac{3x^2y}{z^2}e^{y/z}\right)$$

6. Evaluate the following line integrals (and draw the paths of integration on an x-y plot):

a) $\int_c xy\,d\vec{r}$, where $d\vec{r}=\hat{x}dx+\hat{y}dy$, along the path $y=x$ between (0,0) and (1,1).

b) $\int_c xy\,d\vec{r}$ along the path $y=x^2$ between (0,0) and (1,1).

c) $\oint_c xy\,d\vec{r}$ around the unit circle.

d) $\oint_c x\,d\vec{r}$ around the unit circle.

7. Find the work done by the force $\vec{F}=-y\hat{x}+x\hat{y}$, around a unit circle in the x-y plane

a) going counterclockwise from 0 to π, and

b) going clockwise from 0 to $-\pi$.

c) Evaluate $\nabla\times\vec{F}$. Is the force conservative?

8. Repeat problem 7 for the force $\vec{F}=y\hat{x}+x\hat{y}$.

9. Consider the force field

$$\vec{F} = \frac{-y}{x^2+y^2}\hat{x} + \frac{x}{x^2+y^2}\hat{y}.$$

a) Evaluate $\nabla \times \vec{F}$.

b) Evaluate the work done by $\vec{F}$ along the upper half ($0 \leq \theta \leq \pi$) and lower half ($0 \geq \theta \geq -\pi$) of the unit circle, as done for problems 7 and 8. (HINT: the denominator of $\vec{F}$ is just 1 all along the unit circle.) Can you explain why the work done along the two paths should be different, when the results of part a) would suggest the force should be conservative?