TUTORIAL 4, PHYS 2335

1. Evaluate all partial derivatives of the following functions:

a)
$$f(x,y) = 3x^3 + 2x^2y - 5xy^2 - y^3$$

b)
$$g(x,t) = Ae^{i(kx-\omega t)} + Be^{i(kx+\omega t)}$$

c)
$$f(\vec{r}) = \frac{Ae^{i\vec{k}\cdot\vec{r}}}{r}$$

where $\vec{r} = (x,y,z)$, $r = \sqrt{x^2+y^2+z^2}$, and $\vec{k} = (k_x, k_y, k_z)$.

2. Let

$$g(x) = \ln x + \frac{1}{x}.$$

If $h(x) = g(f(x))$, use the chain rule to find $h'(x)$ for each of the following functions, f.

a) $f(x) = \sin x$, b) $f(x) = e^x$, c) $f(x) = 1/x$.

3. Consider rotating the x-y plane through an angle θ to the x'-y' plane according to the coordinate transformation discussed in class:

$$x' = x\cos\theta + y\sin\theta; \qquad y' = -x\sin\theta + y\cos\theta$$

a) Let $f(x,y) = xy$. Thus, the partial derivatives are given by $\partial_x f = y$, and $\partial_y f = x$. (Note that ∂_ξ is a common short form for $\partial/\partial\xi$, where ξ is an independent variable.) Using the chain rule, find what the slopes of the function are relative to the x'-y' axes. That is, let $g(x',y') = f(x(x',y'), y(x',y'))$, and use the multivariate chain rule to find $\partial_{x'}g$ and $\partial_{y'}g$ in terms of x' and y'.

b) Repeat part a) for $f(x,y) = x^2 + y^2$.

c) You should have found that as functions, $\partial_x f$ and $\partial_y f$ are different from $\partial_{x'}g$ and $\partial_{y'}g$ respectively for part a), but the same for part b). Can you explain in words why you might have expected this?

4. Using the chain rule, show formally (using limits) that a small change $d\phi$ in a trivariate function $\phi(x,y,z)$ is given by:

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$$

5. a) If a vector function, $\vec{F}$, depends explicitly on all spatial coordinates, (x,y,z), and on time, t, show that

$$d\vec{F} = (d\vec{r}\cdot\nabla)\vec{F} + \frac{\partial\vec{F}}{\partial t}dt.$$

where ∇ is the dual vector $(\partial_x, \partial_y, \partial_z)$.

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b) Next, assume that

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$$

(and therefore x, y, and z are functions of t), and show that

$$\frac{d\vec{F}}{dt} = \frac{\partial \vec{F}}{\partial t} + (\vec{v} \cdot \nabla)\vec{F}$$

Explain the physical significance of each of the two terms on the right hand side.