TUTORIAL 3, PHYS 2335

1. Another way to express the concept of *linear independence* of vectors is the following. Suppose:

$$a\vec{A} + b\vec{B} + c\vec{C} = 0. \tag{1}$$

If the *only* way equation (1) can be true is for $a = b = c = 0$, then we say that the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are *linearly independent*. Note that $a = b = c = 0$ is always a possible solution to equation (1); the point is for *linear independence*, $a = b = c = 0$ must be the *only* solution. Thus, if there exist non-zero values for a, b, and c that solve equation (1), then the three vectors are said to be *linearly dependent*.

For each of the three sets of vectors below, determine if the three vectors are linearly independent. If not, how many possible solutions are there for a, b, and c?

a) $\vec{A} = (1, 2, 0)$; $\vec{B} = (-1, 2, -2)$; $\vec{C} = (-1, -4, 0)$.

b) $\vec{A} = (1, 2, 0)$; $\vec{B} = (-1, 2, -2)$; $\vec{C} = (-1, -4, 1)$.

c) $\vec{A} = (1, 2, 0)$; $\vec{B} = (-1, 2, -2)$; $\vec{C} = (-2, -4, 0)$.

2. Given three vectors $\vec{A} = (-1, 0, 3)$, $\vec{B} = (2, 1, 1)$, and $\vec{C} = (0, 1, z)$, what must z be for these vectors to be linearly dependent? You should find there is one and only one value of z that makes this triple linearly dependent.

3. Consider a force $\vec{F} = F_x\hat{x} + F_y\hat{y}$ resolved on an (x, y) coordinate system. Now consider the same force in an (x', y') system which is rotated by an angle ϕ counterclockwise from the $+x$-axis. Using the physical fact that forces can be resolved into components, show that $\vec{F}$ is indeed a true vector. Thus, show that the force coordinates, (F_x, F_y) transform in the same fashion as the coordinates of the displacement vector, namely:

$$F'_x = F_x \cos\phi + F_y \sin\phi;$$
$$F'_y = -F_x \sin\phi + F_y \cos\phi.$$

4. Let $\vec{A} = (1, -1, 2)$, $\vec{B} = (-2, 0, 2)$.

a) Using the scalar (dot) product, find a vector $\vec{C}$ that is orthogonal to both $\vec{A}$ and $\vec{B}$.

b) Using the vector (cross) product, find a vector $\vec{C}$ that is orthogonal to both $\vec{A}$ and $\vec{B}$.

c) Using $\vec{C}$ from part b), evaluate $\vec{C} \cdot (\vec{A} \times \vec{B})$.

d) Using $\vec{C}$ from part b), evaluate $\vec{C} \times (\vec{A} \times \vec{B})$.

e) Using $\vec{C}$ from part b), evaluate $\vec{A} \times (\vec{B} \times \vec{C})$. Your answer should be different from part d).

f) Using $\vec{C}$ from part b), evaluate $(\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$.

g) Using $\vec{C}$ from part b), evaluate $(\vec{C} \cdot \vec{B})\vec{A} - (\vec{C} \cdot \vec{A})\vec{B}$.

h) The answers to parts d) and e) match which of the answers to parts f) and g)? Verify your answer from the vector identity for the triple cross product derived in class.

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5. a) From the definition of the vector product (namely the direction of the vector product is given by the right hand rule), argue that $\vec{A} \times \vec{A} = 0$.

b) Now interpreting the vector product as the area of a parallelogram, argue that $\vec{A} \times \vec{A} = 0$.

c) Now, from the component representation of the cross product, namely

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{x} + (A_z B_x - A_x B_z)\hat{y} + (A_x B_y - A_y B_x)\hat{z},$$

show rigorously that $\vec{A} \times \vec{A} = 0$.

d) If $\vec{A}$ and $\vec{B}$ are perpendicular, show that

$$\vec{A} \times (\vec{A} \times \vec{B}) = -A^2 \vec{B},$$

where $A^2 = \vec{A} \cdot \vec{A}$.