TUTORIAL 2, PHYS 2335

1. Perform a Taylor (or MacLaurin) expansion of the following functions:

a) $f(x) = \ln(x)$ about $x_0 = 1$.

b) $f(x) = 1 + 2x - 3x^2$ about $x_0 = 0$.

c) $f(x) = 1 + 2x - 3x^2$ about $x_0 = 1$.

2. Consider the potential $U(r)$ of a spherically symmetric force field (gravity, electric, magnetic, whatever). Suppose there is a local minimum in $U(r)$ located at r_0, and thus a particle at rest in this local minimum remains at rest. Show that if the position of this particle is perturbed slightly, it will undergo simple harmonic motion. [Hints: Perform a Taylor expansion about $r = r_0$, and recall that the potential for a simple harmonic oscillator is given by $U_{\text{SHO}} = k(r - r_0)^2/2$, where k is some (spring) constant.]

3. Perform a power series expansion of the following by stating explicitly the first four terms:

a) $f(x) = (1 + x)^{1/2}$. For what values of x does the expansion converge?

b) $f(x) = (x + x_0)^a$, for a convergent expansion when $|x/x_0| < 1$.

c) $f(x) = (x + x_0)^a$, for a convergent expansion when $|x/x_0| > 1$.

d) $f(y) = (1 + y)^3$. For what values of y does the expansion converge?

To do a power series expansion with a stated convergence requirement, it may be useful to do a binomial expansion on all or part of the function, since the convergence criterion of a binomial expansion is known.

4. Solve the following first-order, ordinary differential equations using separation of variables (NB: $y' \equiv dy/dx$.):

a) $xy' + 2y = 0$; b) $yy' + xy^2 - x = 0$.

5. From Kirchhoff's Law, the charge, Q, stored in the capacitor in an RC (resistance-capacitance) circuit obeys the following equation:

$$R\frac{dQ}{dt} + \frac{Q}{C} = 0,$$

where $dQ/dt = I$, the current.

a) Find $Q(t)$ and $I(t)$.

b) For a capacitance of 10^4 μF charged to 100 V and discharged through a resistance of 10^6 Ω, find the current I at $t = 0$ and $t = 100$ seconds. Note that the initial voltage is given by $V_0 = Q_0/C$.

over...

6. Solve the following second order, linear, ordinary differential equations, and apply the given boundary conditions to determine the "constants of integration":

a)
$$2y'' - y' - 3y = 0; \quad y(0) = -1, \quad y'(0) = \frac{7}{2};$$

b)
$$y'' - 2\sqrt{5}y' + 4y = 0; \quad y(0) = \sqrt{2}, \quad y'(0) = 0;$$

c)
$$4y'' - 4y' - 3y = 1; \quad y(0) = 1/2, \quad y'(0) = 1.$$