TUTORIAL 1, PHYS 2335

1. Test the following series for convergence.

$$\text{a)} \quad \sum_{n=1}^{\infty} \frac{1}{2n(2n-1)}, \qquad \text{b)} \quad \sum_{n=1}^{\infty} \frac{2^n}{n^{10^6}}, \qquad \text{c)} \quad \sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}.$$

2. Which of the following series is different, and what is the most minor change necessary to make it the same, but still appear different than the other four?

$$\text{a)} \ \sum_{i=1}^{\infty} a_i, \qquad \text{b)} \ \sum_{j=1}^{\infty} a_j, \qquad \text{c)} \ \sum_{i=-\infty}^{-1} a_{-i}, \qquad \text{d)} \ \sum_{k=0}^{\infty} a_{k+1}, \qquad \text{e)} \ \sum_{k=0}^{\infty} a_{k-1}.$$

3. Let $b_k = 0, \ \forall \ k > N$. Which of the following series is different, and what is the most minor change necessary to make it the same, but still appear different than the other four?

$$\text{a)} \ \sum_{i=1}^{\infty} b_i, \qquad \text{b)} \ \sum_{i=1}^{N} b_i, \qquad \text{c)} \ \sum_{i=2}^{N+1} b_{i-1}, \qquad \text{d)} \ \left(\sum_{j=0}^{\infty} b_j\right) - b_0, \qquad \text{e)} \ \sum_{k=1}^{[N/2]} b_{2k},$$

where in sum e), $[N/2] = N/2$ for N even, $(N-1)/2$ for N odd.

4. Let $c_q = 0 \ \forall \ q$ even. What must j be to make each sum the same as sum S:?

$$S = \sum_{i=0}^{\infty} c_i \qquad \text{a)} \ \sum_{i=1}^{\infty} c_{i+j}, \qquad \text{b)} \ \sum_{i=0}^{\infty} c_{2i+j}, \qquad \text{c)} \ \sum_{i=1}^{\infty} c_{2i+j}, \qquad \text{d)} \ \sum_{i=j}^{\infty} c_{2i-3}.$$

5. Collect all like powers of x; *i.e.*, manipulate the sum indices so that you end up with something of the form: $\sum_m x^m \sum_n a_{mn}$.

$$\text{a)} \ \sum_{i=0}^{\infty} a_i x^i \sum_{j=0}^{\infty} a_j x^j, \qquad \text{b)} \ \sum_{\alpha=1}^{\infty} \sum_{\beta=1}^{\infty} a_{\alpha\beta} x^{\beta},$$

$$\text{c)} \ \sum_{a=1}^{\infty} \sum_{b=1}^{\infty} A_{ab}(x^a + x^b), \quad \text{assuming } A \text{ is symmetric; } \ i.e., \ A_{ab} = A_{ba}.$$

6. Perform a Taylor (or MacLaurin) expansion of the following functions:

a) $f(x) = \cos(x)$ about $x_0 = 0$.

b) $f(x) = \cos(x)$ about $x_0 = \pi/2$.

c) $f(x) = \sin(x)$ about $x_0 = \pi/2$.