ASSIGNMENT 9, PHYS 2335

Assigned: Tuesday, November 28, 2006
Due: Friday, December 1, 2006
(will be accepted until Tuesday, December 5 with no late penalty)

1. a) If $A \in \mathcal{R}^{2\,2}$, show that its eigenvalues λ satisfy the equation

$$\lambda^2 - \lambda\,\mathrm{tr}(A) + |A| = 0. \tag{1}$$

b) If equation (1) yields two degenerate eigenvalues, what relationship must exist between the trace and determinant of A?

2. Let $A \in \mathcal{R}^{n\,n}$ be nonsingular. If A has eigenvalues λ_i and corresponding eigenkets $|\vec{x}_i\rangle$, show that A^{-1} has the same eigenkets but with eigenvalues λ_i^{-1}.

3. a) Let A, B, $C \in \mathcal{R}^{n\,n}$, and let $B = C\,A\,C^{-1}$ (thus A and B are similar). Show that A and B have the same eigenvalues.

b) If, in addition to what is stated in part a), B is diagonal, show that

$$\sum_{i=1}^{n} \lambda_i = \mathrm{tr}(A) \qquad \text{and} \qquad \prod_{i=1}^{n} \lambda_i = |A|,$$

where λ_i, $i = 1 \ldots n$ are the eignevalues and where the symbol $\prod_{i=1}^{n}$ means take the *product* instead of the *sum*, as indicated by the symbol $\sum_{i=1}^{n}$. (Hint: use the result of problem 2 of Assignment 7.)

Note this result means that if a matrix can be rendered diagonal by a similarity transformation, then the sum and the product of its eignevalues are equal to the its trace and determinant respectively.

4. Find the eigenvalues and the corresponding normalised eigenkets for the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

5. Find the eigenvalues and the corresponding normalised eigenkets for the matrix

$$A = \begin{bmatrix} 1 & \sqrt{8} & 0 \\ \sqrt{8} & 1 & \sqrt{8} \\ 0 & \sqrt{8} & 1 \end{bmatrix}.$$