ASSIGNMENT 7, PHYS 2335

Assigned: Tuesday, November 14, 2006
Due: Tuesday, November 21, 2006

1. Let $A \in \mathcal{R}^{n\,n}$ and let A^{-1} exist. Using the product rule for determinants (namely, $A, B \in \mathcal{R}^{n\,n} \Rightarrow |AB| = |A||B|$.), prove that

$$|A^{-1}| = \frac{1}{|A|}.$$

2. Let A, B, $C \in \mathcal{R}^{n\,n}$ and let $B = C\,A\,C^{-1}$. That is, matrices A and B are similar.

a) Show that $\mathrm{tr}(A) = \mathrm{tr}(B)$. [Hint: Problem 1a of Tutorial 8 shows that for two matrices, P, $Q \in \mathcal{R}^{n\,n}$, $\mathrm{tr}(PQ) = \mathrm{tr}(QP)$. You may use this result here.]

b) Show that $|A| = |B|$. (Hint: Use the "product rule" for determinants.)

Thus, the determinant and trace of a matrix are invariant under similarity transformations (which we showed in class could be thought of as coordinate transformations).

3. Let A, $B \in \mathcal{R}^{n\,n}$. If $[A, B] = 0$ and A is diagonal with all diagonal elements different, show that B is also diagonal.

4. Show that the product of two orthogonal matrices is orthogonal. Hint: You should first show that $(\widetilde{AB}) = \tilde{B}\tilde{A}$.

5. Using the formula for the inverse matrix elements derived in class, namely

$$a^{-1}_{i\,j} = \frac{\mathcal{C}_{j\,i}}{|A|},$$

where $\mathcal{C}_{j\,i}$ (note the order of indices) is the co-factor for matrix element $a_{j\,i}$, evaluate the inverse of the following matrix:

$$\begin{bmatrix} 0 & 3 & -1 & 0 \\ 0 & 0 & 5 & 13 \\ 2 & 0 & 0 & 1 \\ 7 & -4 & 0 & 0 \end{bmatrix}$$

NB: You are not to use Gauss-Jordan elimination here—you'll get plenty of practise with that technique in the next assignment.