ASSIGNMENT 5, PHYS 2335

Assigned: Tuesday, October 17, 2006
Due: Tuesday, October 31, 2006 (two weeks)

1. If the gradient of a scalar function ϕ is given by:

$$\nabla\phi(x,y,z) = \left(6xy - z\sin 2x - ye^{-x},\ 3x^2 + e^{-x} - \frac{z^2}{y^2},\ \frac{1}{2}\cos 2x + 2\frac{z}{y}\right),$$

find $\phi(x,y,z)$ to within a constant of integration.

2. The force field acting on a two-dimensional linear oscillator may be described by

$$\vec{F} = \hat{x}kx + \hat{y}ky.$$

By evaluating the integral

$$W = \int_{(1,1)}^{(4,4)} \vec{F}\cdot d\vec{r},$$

compare the work done moving against this force field when going from (1, 1) to (4, 4) by the following straight line paths:

a) (1, 1) $\rightarrow$ (4, 1) $\rightarrow$ (4, 4);

b) (1, 1) $\rightarrow$ (1, 4) $\rightarrow$ (4, 4);

c) (1, 1) $\rightarrow$ (4, 4) along $y = x$.

3. The integral form for Gauss' Law (for an electric field as learned in PHYS 1211) is given by

$$\Phi_E = \oint_S \vec{E}\cdot d\vec{\sigma} = \frac{q_{\text{encl}}}{\epsilon_0}.$$

From this, derive Gauss' Law in differential form, namely

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0},$$

where ρ is the charge density, and related to q_{encl} in the usual way, namely

$$q_{\text{encl}} = \int_V \rho dV.$$

4. Show that

$$\frac{1}{3}\oint_S \vec{r}\cdot d\vec{\sigma} = V,$$

where V is the volume enclosed by the closed surface S.

over...

5. The calculation of the magnetic moment of a current loop confined within the x-y plane leads to the line integral

$$\oint_C \vec{r} \times d\vec{r}.$$

a) Integrate around the perimeter of a current loop C (in the x-y plane) and show that the scalar magnitude of this line integral is twice the area of the enclosed surface.

b) The perimeter of an ellipse is described by $\vec{r} = \hat{x}a\cos\theta + \hat{y}b\sin\theta$. From part a), show that the area of an ellipse is πab.

MIDTERM: Thursday, November 2, 2:30pm – 3:45pm

All material up to and including Stoke's Theorem (§1.12) will be covered.

Bring three sharp pencils, an eraser, a simple calculator, and a clear mind. All required paper and formulae will be provided. No crib sheets, notes, or texts permitted.