ASSIGNMENT 4, PHYS 2335

Assigned: Tuesday, October 10, 2006
Due: Tuesday, October 17, 2006

1. Demonstrate the validity of the "product rule" for gradients, namely

$$\nabla(\phi\theta) = \phi\nabla\theta + \theta\nabla\phi,$$

where θ and ϕ are differentiable scalar functions of $\vec{r} = (x, y, z)$. You should do this by expanding $\nabla(\phi\theta)$ into its Cartesian components, and then use the usual product rule from univariate calculus on each term.

2. Demonstrate the validity of the identity:

$$\nabla(\vec{A}\cdot\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} + (\vec{A}\cdot\nabla)\vec{B} + \vec{B}\times(\nabla\times\vec{A}) + \vec{A}\times(\nabla\times\vec{B}),$$

by showing it to be true when expanded out in Cartesian components. Here, $\vec{A}$ and $\vec{B}$ are differentiable vector functions of $\vec{r} = (x, y, z)$.

For problems 3 to 5, you must prove the assertions using the ten vector identities given in class. You are *not* to expand these out into their Cartesian components as you did for the first two problems. You should find that by using the identities, each problem is a two-liner.

3. Prove that $\vec{u}\times\vec{v}$ is solenoidal (*i.e.*, has zero divergence) if both $\vec{u}(\vec{r})$ and $\vec{v}(\vec{r})$ are irrotational (*i.e.*, have zero curl).

4. Prove that the vector function $\phi\nabla\phi$ is irrotational, where $\phi(\vec{r})$ is an arbitrary scalar function.

5. If $\phi(\vec{r})$ and $\psi(\vec{r})$ are two arbitrary scalar functions, prove that the vector function $\nabla\phi\times\nabla\psi$ is solenoidal.