ASSIGNMENT 2, PHYS 2335

Assigned: Tuesday, September 26, 2006
Due: Tuesday, October 3, 2006

1. Consider the vector equation:

$$a\vec{A} + b\vec{B} + c\vec{C} = 0 \tag{1}$$

If the *only* way equation (1) can be true is for $a = b = c = 0$, then we say that the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are *linearly independent*. Thus, if there exist non-zero values for a, b, and c that solve equation (1), then the three vectors are said to be linearly *dependent*.

For each of the three sets of vectors below, determine if the three vectors are linearly independent. If not, how many possible solutions are there for a, b, and c?

a) $\vec{A} = (3, 0, 1)$; $\vec{B} = (1, 0, 2)$; $\vec{C} = (0, 1, 1)$

b) $\vec{A} = (1, 2, 3)$; $\vec{B} = (0, 1, 2)$; $\vec{C} = (-1, 0, 1)$

c) $\vec{A} = (1, -1, 0)$; $\vec{B} = (2, 0, 1)$; $\vec{C} = (0, -2, -1)$

2. In class we discussed the rotation of a coordinate system, and showed that the length of the displacement vector is independent of the rotation of the coordinate system used to describe the vector.

Now consider a *translation*. Consider the coordinate system (x', y') whose axes are parallel to the (x, y) axes, but whose origin is located at the point (x_0, y_0) in the (x, y) system.

a) Draw the two coordinate systems, showing clearly the fact that the origin in the (x', y') system is displaced relative to the origin of the (x, y) system by a displacement (x_0, y_0).

b) Write down expressions to give x' in terms of x and x_0, and y' in terms of y and y_0.

c) Consider a vector whose end points in the (x, y) system have coordinates (x_1, y_1) and (x_2, y_2). What are the coordinates of the end points of the same vector in the same absolute position in the (x', y') coordinate system?

d) While the coordinates of the end points of the same vector are different in the two coordinate systems, show that the length of the vector does not depend on which coordinate system is used.

3. Let $\vec{A} = (2, 0, 1)$, $\vec{B} = (1, -1, -1)$, and $\vec{C} = (3, 1, 0)$

a) Evaluate $\vec{A} \cdot \vec{B}$, $\vec{B} \cdot \vec{C}$, and $\vec{C} \cdot \vec{A}$.

b) Evaluate $\vec{A} \times \vec{B}$, $\vec{B} \times \vec{C}$, and $\vec{C} \times \vec{A}$.

c) Evaluate $\vec{C} \cdot (\vec{A} \times \vec{B})$, $\vec{A} \cdot (\vec{B} \times \vec{C})$, and $\vec{B} \cdot (\vec{C} \times \vec{A})$.

d) Evaluate $\vec{C} \times (\vec{A} \times \vec{B})$, $\vec{A} \times (\vec{B} \times \vec{C})$, and $\vec{B} \times (\vec{C} \times \vec{A})$.

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4. Show, by expanding in Cartesian components, that

$$(\vec{A} \times \vec{B}) \cdot (\vec{A} \times \vec{B}) = (AB)^2 - (\vec{A} \cdot \vec{B})^2,$$

where $A = |\vec{A}|$ and $B = |\vec{B}|$.

5. a) Find the solution, $y(x)$, for

$$3y''(x) + 2y'(x) - 5y(x) = 1; \quad y(0) = 0,\ y'(0) = -1.$$

b) For a second order homogeneous ODE with constant coefficients:

$$y''(x) + py'(x) + qy(x) = 0 \qquad (2),$$

what condition on the coefficients p and q must be satisfied in order for the characteristic equation to yield two distinct roots, and thus for the method discussed in class to work?

c) When the roots of the characteristic equation are degenerate, show by direct substitution into equation (2) that,

$$y_2(x) = xe^{ax},$$

is also a solution to the homogeneous equation, where a is the degenerate root. Further show that $y_2(x)$ is linearly independent of the first solution, namely:

$$y_1(x) = e^{ax}.$$

d) Thus, the most general solution to a second order homogeneous ODE with constant coefficients and degenerate roots is:

$$y(x) = A_1 e^{ax} + A_2 x e^{ax}.$$

Use this equation to find the solution for

$$y''(x) + 6y'(x) + 9y(x) = -3; \quad y(0) = 1,\ y'(0) = 0.$$