ASSIGNMENT 1, PHYS 2335

Assigned: Tuesday, September 19, 2006
Due: Tuesday, September 26, 2006

1. Test the following series for convergence:

$$\text{a)} \sum_{n=1}^{\infty} \frac{1}{n2^n}, \qquad \text{b)} \sum_{n=1}^{\infty} [n(n+1)]^{-1/2}, \qquad \text{c)} \sum_{n=0}^{\infty} \frac{1}{2n+1}.$$

2. Let

$$S = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_{ij}.$$

Make the substitutions $j = n \geq 0$ and $i = m - 2n \geq 0$ and show that

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_{ij} = \sum_{m=0}^{\infty} \sum_{n=0}^{[m/2]} a_{m-2n,n}.$$

where $[m/2] = m/2$ for m even, $(m-1)/2$ for m odd.

3. What is the binomial expansion for the functions:

a) $f(x) = 1 + x + x^2$;

b) $f(x) = 1/x$ about $x = 1$;

c) $f(x) = 1/x(x-1)$ about $x = 1$?

4. a) Let $x = \sin y$. Show (using the chain rule) that $dy/dx = 1/\sqrt{1-x^2}$, and thus

$$\sin^{-1} x = \int_0^x \frac{dt}{(1-t^2)^{1/2}}.$$

b) Obtain a series expansion for the arcsine by first doing a binomial expansion on the integrand, then integrate the series term by term. Do this for enough terms (four, say) so you can write down, by inspection, a closed form for the summation.

Answer:

$$\sin^{-1}(x) = \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{2n+1},$$

where the "double factorial", $n!!$, is given by $n(n-2)(n-4)\ldots$ with the last factor being either 2 or 1 depending on whether n is even or odd, and where $1!! = 0!! = (-1)!! \equiv 1$.

5. a) Suppose a first order differential equation has the form

$$\frac{dy}{dx} = g(y/x),$$

over. . .

where g is some arbitrary function of the *ratio* of y and x. Show that by substituting $u = y/x$, the differential equation becomes separable in u and x.

b) Solve the first order ordinary differential equation:

$$2xyy' + x^2 + y^2 = 0.$$

If you find this equation doesn't separate easily, try substituting $y = vx$, and solve for $v(x)$ instead. Once you have $v(x)$, multiply it by x to get $y(x)$.