## Typical magnetic field strengths

<table>
<thead>
<tr>
<th>source of magnetism</th>
<th>Tesla</th>
</tr>
</thead>
<tbody>
<tr>
<td>surface of a neutron star</td>
<td>$10^8$</td>
</tr>
<tr>
<td>a superconducting magnet</td>
<td>10</td>
</tr>
<tr>
<td>a laboratory electromagnet</td>
<td>0.1 - 1.5</td>
</tr>
<tr>
<td>a refrigerator magnet</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>surface of the Earth</td>
<td>$5 \times 10^{-4}$</td>
</tr>
<tr>
<td>interstellar space</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>in a magnetically shielded room</td>
<td>$10^{-14}$</td>
</tr>
</tbody>
</table>
Magnetic dipole field lines and the Hall effect

magnetic dipole

the Hall effect
Larmor motion and force on a wire

positive charge orbits clockwise within a positive $\vec{B}$

negative charge orbits counterclockwise within a positive $\vec{B}$

a length of wire, $L$, carrying a current, $i$, is embedded in an external magnetic field, $\vec{B}$. 
Current loop and galvanometer

\[ \mathbf{F}_{B_2} = \mathbf{F}_{B_4} = 0 \quad (\mathbf{L} \parallel \mathbf{B}) \]

Torque on a square current loop immersed in a uniform magnetic field.

Schematic diagram of a galvanometer.
The Law of Biot and Savart

magnetic field from a general piece of wire

magnetic field at the centre of curvature of a circular arc of wire

magnetic field around a long, straight wire
Force on (anti)parallel wires

parallel wires

antiparallel wires
Combined magnetic field from antiparallel currents
Clicker question 1

The magnetic field at point P points:

a) up  b) down  c) left

d) right  e) into the page  f) out of the page
Clicker question 1

The magnetic field at point P points:

- a) up
- b) down
- c) left
- d) right
- e) into the page
- f) out of the page  ✓
Clicker question 2

A positive charge moves straight out of the page.

The magnetic field at point P points:

a) up  b) down  c) left

d) right  e) into the page  f) out of the page
A positive charge moves straight out of the page.

The magnetic field at point P points:

a) up  

\(\text{b) down}\)  

\(\checkmark\)  

c) left  

d) right  

e) into the page  

f) out of the page
Clicker question 3

Considering only the force exerted on the wires by the background magnetic field, \( \vec{B} \), which of the following statements is true?

- a) Force on wire 1 is \( iaB \) and points in the \(-y\)-direction
- b) Force on wire 2 is \( ibB \) and points in the \(+x\)-direction
- c) Force on wire 3 is \( iaB \) and points in the \(-z\)-direction
- d) Force on wire 3 is \( iaB \) and points in the \(-y\)-direction
- e) None are true.
- f) All are true

Magnetic force on a wire: \( \vec{F}_B = i \vec{L} \times \vec{B} \)

where \( \vec{L} \) is a vector with length of the wire and pointing in the direction of the current.
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- d) Force on wire 3 is \( iaB \) and points in the \(-y\)-direction
- e) None are true.
- f) All are true
Clicker question 4

Which current segment, if any, produces no magnetic field at point \( P \)?

Law of Biot and Savart:

\[
\frac{d\vec{B}}{dB} = \frac{\mu_0}{4\pi} \frac{i \, ds \times \hat{r}}{r^2}
\]

d) None of the current segments produce a magnetic field at \( P \).

e) All current segments produce a magnetic field at \( P \).
Clicker question 4

Which current segment, if any, produces no magnetic field at point $P$?

Law of Biot and Savart: $d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{i \, ds \times \hat{r}}{r^2}$

- a) $ds$
- b) $i \, ds$
- c) $ds$
- d) None of the current segments produce a magnetic field at $P$.
- e) All current segments produce a magnetic field at $P$. 

PHYS 1101, Winter 2009, Prof. Clarke
Ampère’s law

Direction around loop is given by the right-hand-rule: Place the right thumb in the direction of $i_{\text{encl}}$, fingers then curl in the direction to traverse loop.

As with Gauss’ Law for $\vec{E}$, Ampère’s Law can be used to calculate $\vec{B}$ only for highly symmetric problems.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{encl}}$$
Magnetic field around and within a long straight wire

Definition of current density

\[ i = \int_A \mathbf{J} \cdot d\mathbf{A} \]
The solenoid

(a) A single loop

(b) A stack of three loops

The fields here nearly cancel.

The fields here reinforce each other.
The solenoid as a magnetic dipole
The toroid

pictorial representation

top view, cut-away
Lenz’ Law

When magnetic flux through a loop increases, induced current in the loop is such that induced magnetic field opposes the increasing flux.

When magnetic flux through a loop decreases, induced current in the loop is such that induced magnetic field adds to the decreasing flux.

Induced flux works to minimise the rate of change of flux through the loop.
Examples of time-varying magnetic fluxes

A loop spinning at a constant angular speed, $\omega$, in a uniform $\vec{B}$ sets up a sinusoidally varying $\Phi_B$.

A rail sliding at a constant speed, $v$, in a uniform $\vec{B}$ sets up a linearly varying $\Phi_B$.

A time-varying $i$ creates a time-varying $\vec{B}$ and thus $\Phi_B$. 
Clicker question 5

At the moment shown in the figure, the loop is spinning so that the angle, $\theta$, between the area vector of the loop, $\vec{A}$, and the background magnetic field, $\vec{B}$, is decreasing with time. Which of the following statements is true?

a) The induced current in the loop is clockwise and the induced magnetic field is parallel to $\vec{A}$.

b) The induced current in the loop is counterclockwise and the induced magnetic field is parallel to $\vec{A}$.

c) The induced current in the loop is clockwise and the induced magnetic field is antiparallel to $\vec{A}$.

d) The induced current in the loop is counterclockwise and the induced magnetic field is antiparallel to $\vec{A}$.
Clicker question 5

At the moment shown in the figure, the loop is spinning so that the angle, \( \theta \), between the area vector of the loop, \( \vec{A} \), and the background magnetic field, \( \vec{B} \) is \textit{decreasing} with time. Which of the following statements is true?

a) The induced current in the loop is clockwise and the induced magnetic field is parallel to \( \vec{A} \).

b) The induced current in the loop is counterclockwise and the induced magnetic field is parallel to \( \vec{A} \).

c) The induced current in the loop is clockwise and the induced magnetic field is \textit{antiparallel} to \( \vec{A} \).

d) The induced current in the loop is counterclockwise and the induced magnetic field is antiparallel to \( \vec{A} \).

As \( \theta \) decreases, \( \Phi_{B} \) increases so \( \vec{B}_{\text{ind}} \) is set up to offset that increase.
Clicker question 6

At the moment shown in the figure, the rail is sliding to the right so that the area of the loop is *decreasing* with time. The rail and loop are conductors and electrically connected. Which of the following statements is true?

a) The induced current in the loop is clockwise and the induced magnetic field is parallel to $\vec{B}$.

b) The induced current in the loop is *counterclockwise* and the induced magnetic field is parallel to $\vec{B}$.

c) The induced current in the loop is clockwise and the induced magnetic field is *antiparallel* to $\vec{B}$.

d) The induced current in the loop is *counterclockwise* and the induced magnetic field is *antiparallel* to $\vec{B}$.
Clicker question 6

At the moment shown in the figure, the rail is sliding to the right so that the area of the loop is \textit{decreasing} with time. The rail and loop are conductors and electrically connected. Which of the following statements is true?

- a) The induced current in the loop is clockwise and the induced magnetic field is parallel to $\vec{B}$.
- b) The induced current in the loop is \textit{counterclockwise} and the induced magnetic field is parallel to $\vec{B}$.
- c) The induced current in the loop is clockwise and the induced magnetic field is \textit{antiparallel} to $\vec{B}$.
- d) The induced current in the loop is counterclockwise and the induced magnetic field is antiparallel to $\vec{B}$.

Flux $\Phi_B$ decreases as the area of the loop decreases, so $\vec{B}_{\text{ind}}$ is set up to offset that decrease.
Clicker question 7

Match ‘em up!!
Which of the loops, A, B, C, D, should replace the grey loops in 1, 2, 3, 4?

- a) 1-C, 2-A, 3-B, 4-D
- b) 1-A, 2-C, 3-D, 4-B
- c) 1-B, 2-B, 3-C, 4-C
- d) 1-D, 2-A, 3-A, 4-D
- e) 1-C, 2-C, 3-B, 4-B
- f) 1-B, 2-C, 3-C, 4-B
Clicker question 7

Match ‘em up!!

Which of the loops, A, B, C, D, should replace the grey loops in 1, 2, 3, 4?

a) 1-C, 2-A, 3-B, 4-D
b) 1-A, 2-C, 3-D, 4-B
c) 1-B, 2-B, 3-C, 4-C
d) 1-D, 2-A, 3-A, 4-D
e) 1-C, 2-C, 3-B, 4-B
f) 1-B, 2-C, 3-C, 4-B
A square conducting loop is dragged at a constant velocity, $\vec{v}$, along a path taking it through a region of uniform magnetic field, $\vec{B}$, as shown. Considering the four “snapshots” shown in the figure, which statement is correct?

a) $F_2 = F_4 > F_3 = F_1$

b) $F_1 = F_2 = F_3 = F_4$

c) $F_3 > F_4 > F_2 > F_1$

d) $F_3 > F_4 = F_2 > F_1$
Clicker question 8

A square conducting loop is dragged at a constant velocity, $\vec{v}$, along a path taking it through a region of uniform magnetic field, $\vec{B}$, as shown. Considering the four “snapshots” shown in the figure, which statement is correct?

(a) $F_2 = F_4 > F_3 = F_1$  
(b) $F_1 = F_2 = F_3 = F_4$
(c) $F_3 > F_4 > F_2 > F_1$  
(d) $F_3 > F_4 = F_2 > F_1$
Clicker question 9

A conducting loop is incompletely embedded in a region of uniform magnetic field, $\vec{B}$. If $\vec{B}$ begins to increase rapidly in strength, what happens to the loop?

a) The loop is pulled to the right, deeper into the magnetised region.
b) The loop is pushed to the left and expelled from the magnetised region.
c) The tension in the sides of the loop increase, but the loop doesn’t move.
d) The loop is pushed upward.
e) The loop is pushed downward.
f) Nothing.
A conducting loop is incompletely embedded in a region of uniform magnetic field, $\vec{B}$. If $\vec{B}$ begins to increase rapidly in strength, what happens to the loop?

a) The loop is pulled to the right, deeper into the magnetised region.

b) The loop is pushed to the left and expelled from the magnetised region.  

(Answer)

Note that expelling the loop helps to counter the increasing flux, which is consistent with Lenz’ Law.
Static and induced electric fields

A simple $i$-$R$ circuit with a seat of emf, $\varepsilon$, that sets up a static electric field, $\vec{E}_{\text{stat}}$

$\vec{E}_{\text{stat}}$ is conservative

A simple $i$-$R$ circuit embedded in a time-varying magnetic flux, $\Phi_B$, that induces an electric field, $\vec{E}_{\text{ind}}$

$\vec{E}_{\text{ind}}$ is not conservative
Faraday’s law

The time-rate-of-change of the magnetic flux, $\Phi_B$, passing through an arbitrary surface of area $A$ is equal to the induced emf, $\varepsilon_{\text{ind}}$, about the circumference, $C$, of that loop which in turn is equal to the closed line integral of the induced electric field, $E_{\text{ind}}$, about $C$.

$$\frac{d\Phi_B}{dt} = \frac{d}{dt} \int_A \vec{B} \cdot d\vec{A} = -\varepsilon_{\text{ind}} = -\oint_C \vec{E}_{\text{ind}} \cdot d\vec{s}$$

Lenz’ Law
Applications of Faraday’s law

A rectangular loop is pulled from a region with a uniform magnetic field, $\vec{B}_{\text{ext}}$.

A semi-circular portion of the circuit is rotated within a uniform magnetic field, $\vec{B}_{\text{ext}}$. 

The applied force $\vec{F}$ is given by the integral of the magnetic field $B$ across the area of the loop.

The induced current $i_{\text{ind}}$ is related to the change in magnetic flux through the loop.

The equations for the forces are:

1. $\vec{F}_1 = i_{\text{ind}} \vec{B}_{\text{ext}}$
2. $\vec{F}_2 = i_{\text{ind}} \vec{B}_{\text{ind}}$
3. $\vec{F}_3$ is perpendicular to $\vec{B}_{\text{ext}}$.
Maxwell’s Equations (in integral form)

“To anyone who is motivated by anything beyond the most narrowly practical, it is worth while to understand Maxwell’s equations simply for the good of the soul.”

J. R. Pierce, 1956, in *Electrons, Waves, and Messages* (Hanover House)

Gauss’ Law for electric field
\[ \oint_A \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0} \]

Gauss’ Law for magnetic field
\[ \oint_A \vec{B} \cdot d\vec{A} = 0 \]

Faraday’s Law
\[ \oint_C \vec{E} \cdot d\vec{s} = \frac{d\Phi_B}{dt} \]

Ampère-Maxwell Law
\[ \oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \]

the “displacement current”, \( i_d \)