Chapter 7: Motion in a Circle

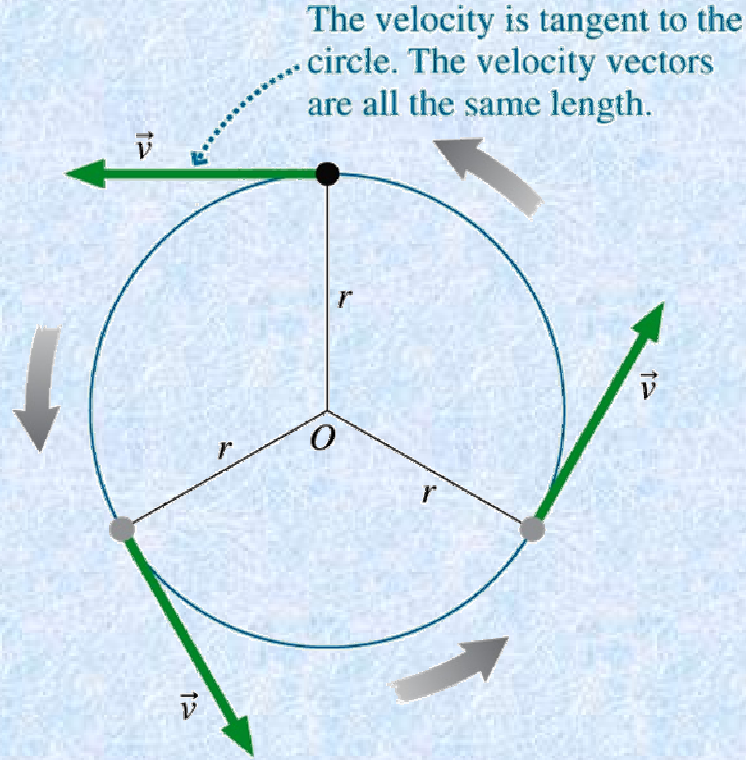
A particle undergoes uniform circular motion when it moves in a circle with a constant speed.

$v = |\vec{v}| = \text{constant}$

Because the path is a circle, $\vec{v}$ is always tangential to the path.

Period: the time it takes to go around the circle once:

$$v = \frac{2\pi r}{T}; \; T = \frac{2\pi r}{v}$$



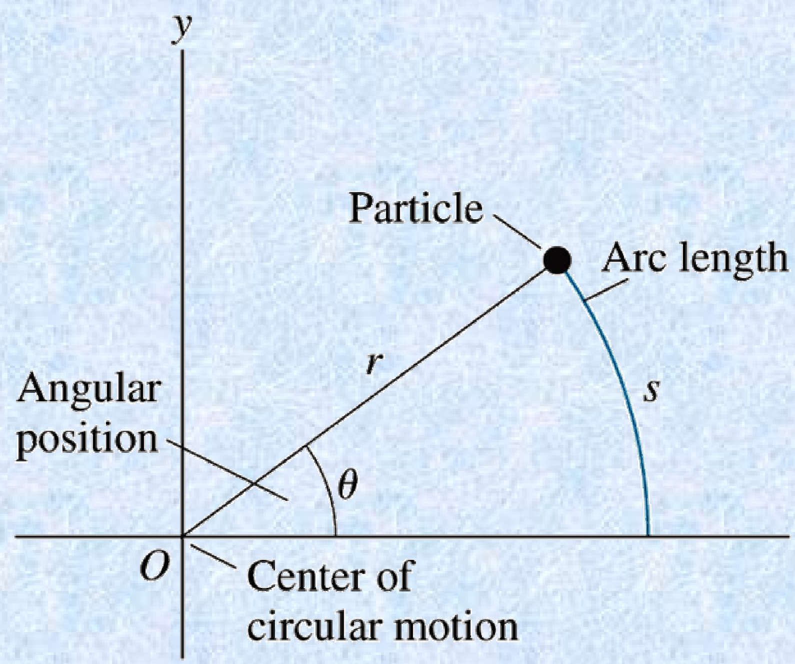
Chapter 7: Motion in a Circle

Angular position

answers the question: ***Where is the object on the circle?***

Define the angular position, θ, to be the arc length from the +x-axis to the current position, s, divided by the radius of the circle, r:

$$\theta = s/r$$



SI units of θ are *radians* (rad) which is a "unitless unit" (distance/distance)

$\theta > 0$ counterclockwise from +x-axis

$\theta < 0$ clockwise from +x-axis

***e.g.,* How many radians in a circle?**

θ_{circ} = circumference $/r$ = $2\pi r/r$ = 2π

Thus, 2π rad = 360^o.

Chapter 7: Motion in a Circle

Angular displacement

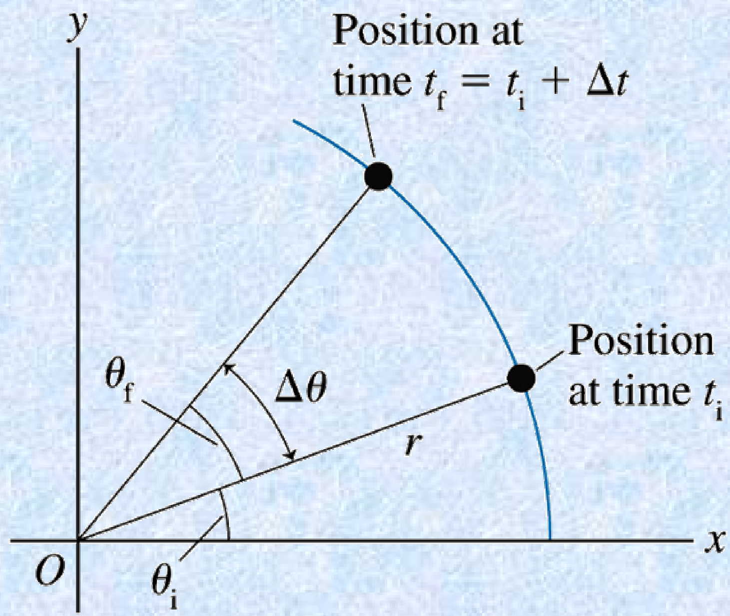
answers the question: ***How far has the object rotated?***

If a particle moves around a circle from angular position θ_i to angular position θ_f, its ***angular displacement***, $\Delta\theta$, is:

$$\Delta\theta = \theta_f - \theta_i$$

$\Delta\theta > 0 \Rightarrow$ angular displacement is counterclockwise.

$\Delta\theta < 0 \Rightarrow$ angular displacement is clockwise.



Clicker question 7.1

If you walk around a circle of diameter of 10 m for π rad, how far have you walked?

a) 5 m

b) 5π m

c) 10 m

d) 10π m

Clicker question 7.1

If you walk around a circle of diameter of 10 m for π rad, how far have you walked?

a) 5 m

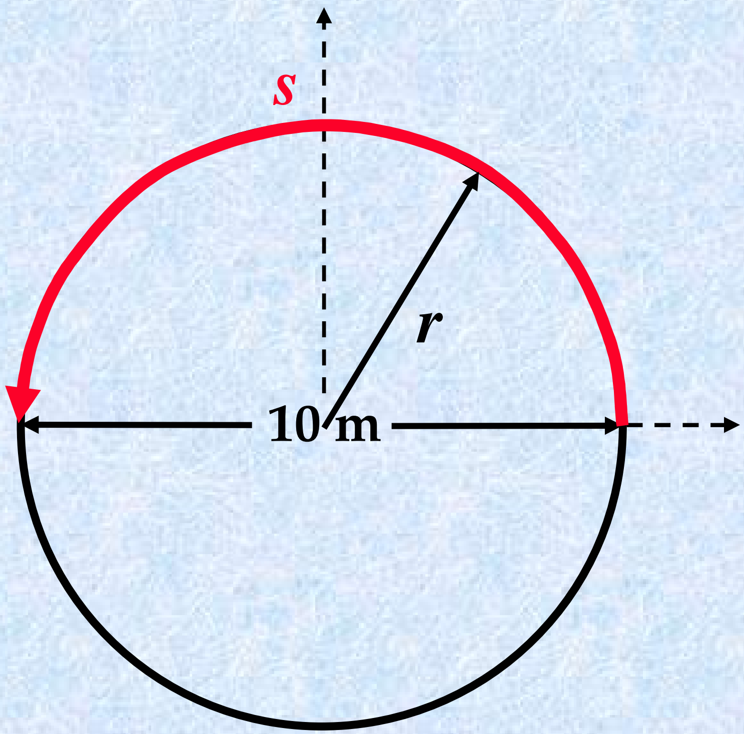
b) 5π m

c) 10 m

d) 10π m

π rad is half way around the circle of radius 5 m.

$s = r\theta = 5\pi$ m



Chapter 7: Motion in a Circle

Angular velocity

answers the question: ***How fast is the object rotating?***

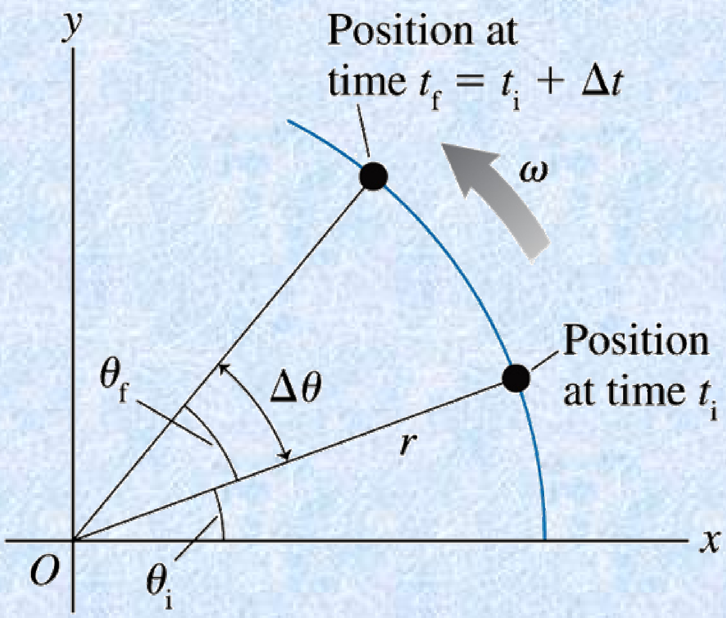
Then the ***average angular velocity*** is the angular displacement divided by the time interval:

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

The ***instantaneous angular velocity*** is:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

SI units for ω are rad s^{-1}



Clicker question 7.2

An bug crawls along the edge of a pizza of radius 20 cm from a starting point (**S**) to a final point (**F**) exactly half around the pizza. If it takes the bug 10 s to complete the journey, what was its average **angular velocity**?

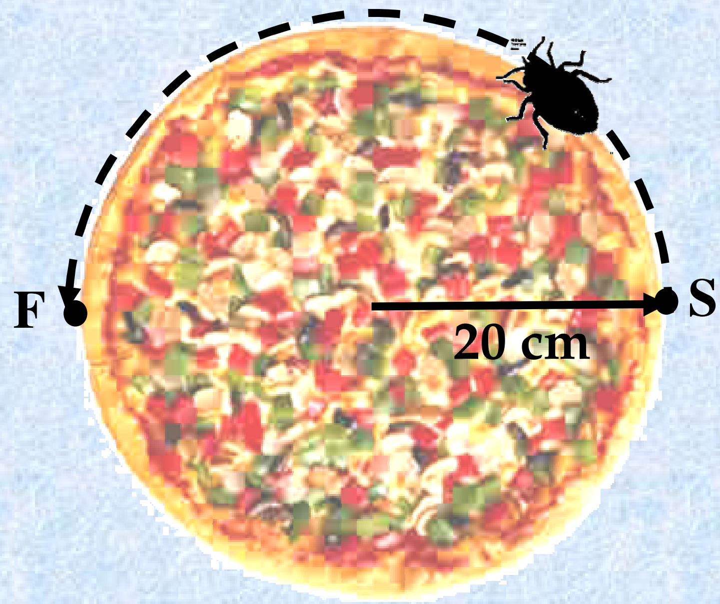
a) π rad/s

b) **20 π** cm/s

c) **π /10** rad/s

d) **40** cm/s

e) ***pizza* π**



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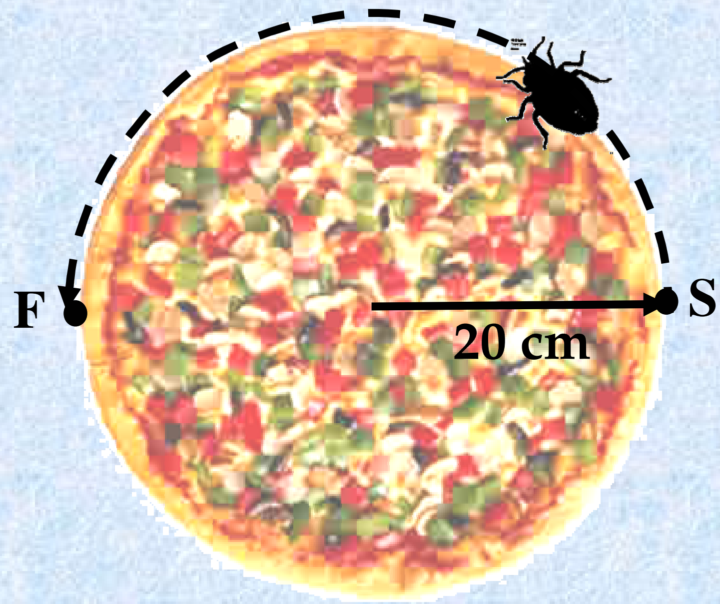
e) pizza **π**



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Chapter 7: Motion in a Circle

Uniform circular motion

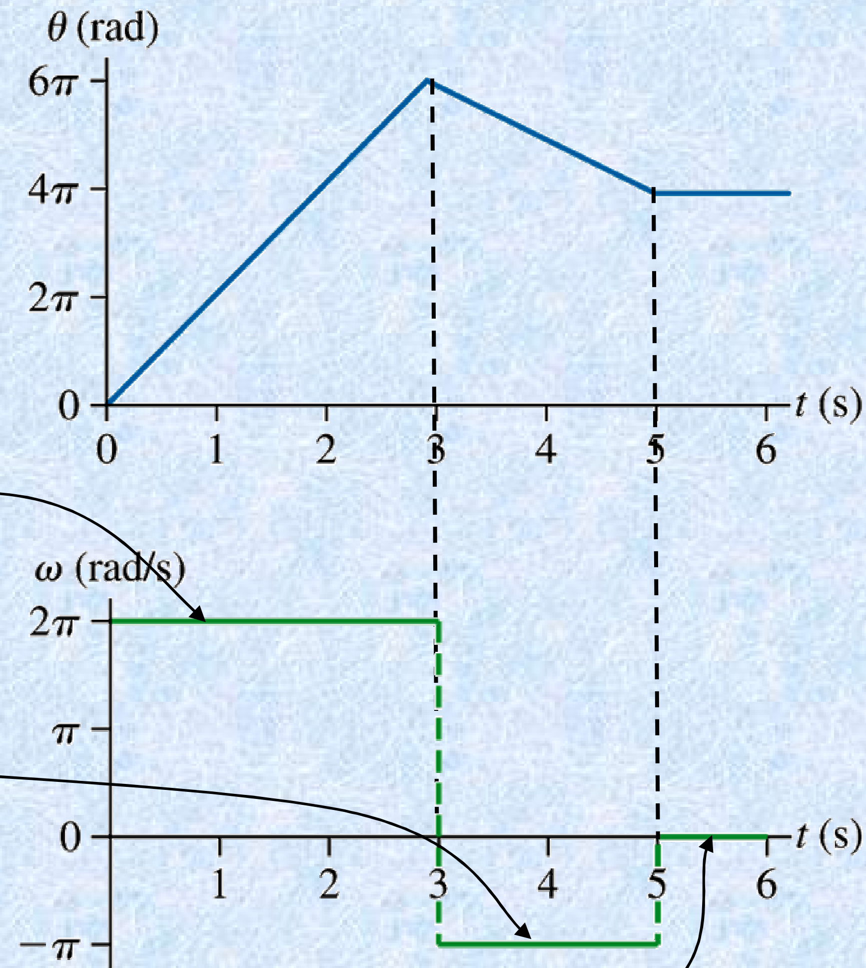
- means angular velocity is constant
- $\theta_f = \theta_i + \omega \Delta t$
- $\omega = 2\pi/T$; $T = 2\pi/\omega$ (period and angular velocity for constant angular velocity)

Uniform circular motion is completely analogous to **1-D motion with zero acceleration**, with s (x or y) replaced by θ, and v replaced by ω. Nothing new!

Similarly, graphical representation same as 1-D motion with zero acceleration.

Chapter 7: Motion in a Circle

Example: For the given $\boldsymbol{\theta}$ *vs*. $\boldsymbol{t}$ graph, construct an $\boldsymbol{\omega}$ *vs*. $\boldsymbol{t}$ graph.



$0 < t < 3,\ \Delta\boldsymbol{\theta} = 6\pi$

$\Rightarrow \boldsymbol{\omega} = 6\pi/3 = 2\pi$ rad s^{-1}

$3 < t < 5,\ \Delta\boldsymbol{\theta} = -2\pi$

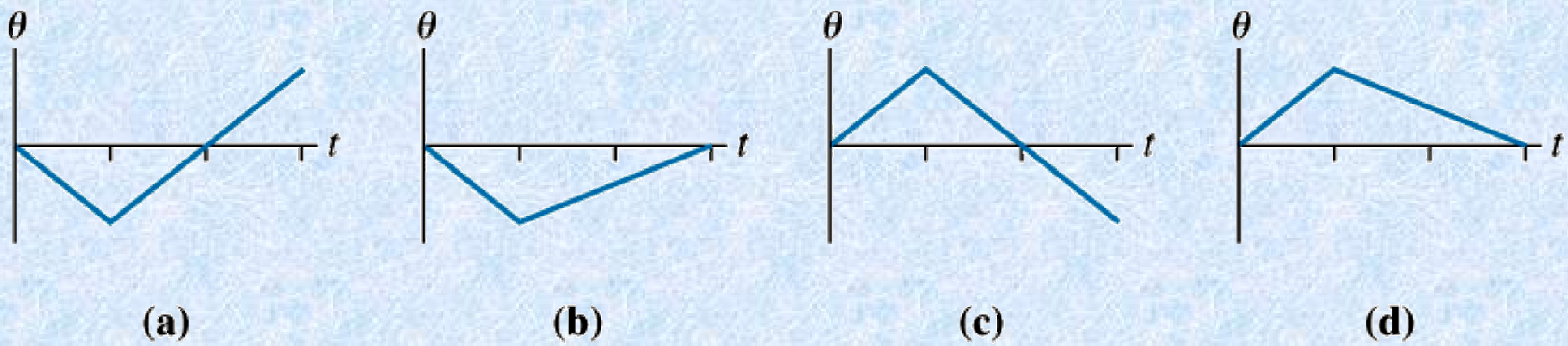
$\Rightarrow \boldsymbol{\omega} = -2\pi/2 = -\pi$ rad s^{-1}

$5 < t < 6,\ \Delta\boldsymbol{\theta} = 0$

$\Rightarrow \boldsymbol{\omega} = 0$ rad s^{-1}

Clicker question 7.3

A particle moves around a circle clockwise at a constant speed for 2.0 s. It then reverses direction and moves counter-clockwise at half the original speed until it has traveled through the same angle. Which is the particle’s angle-versus-time graph?



Chapter 7: Motion in a Circle

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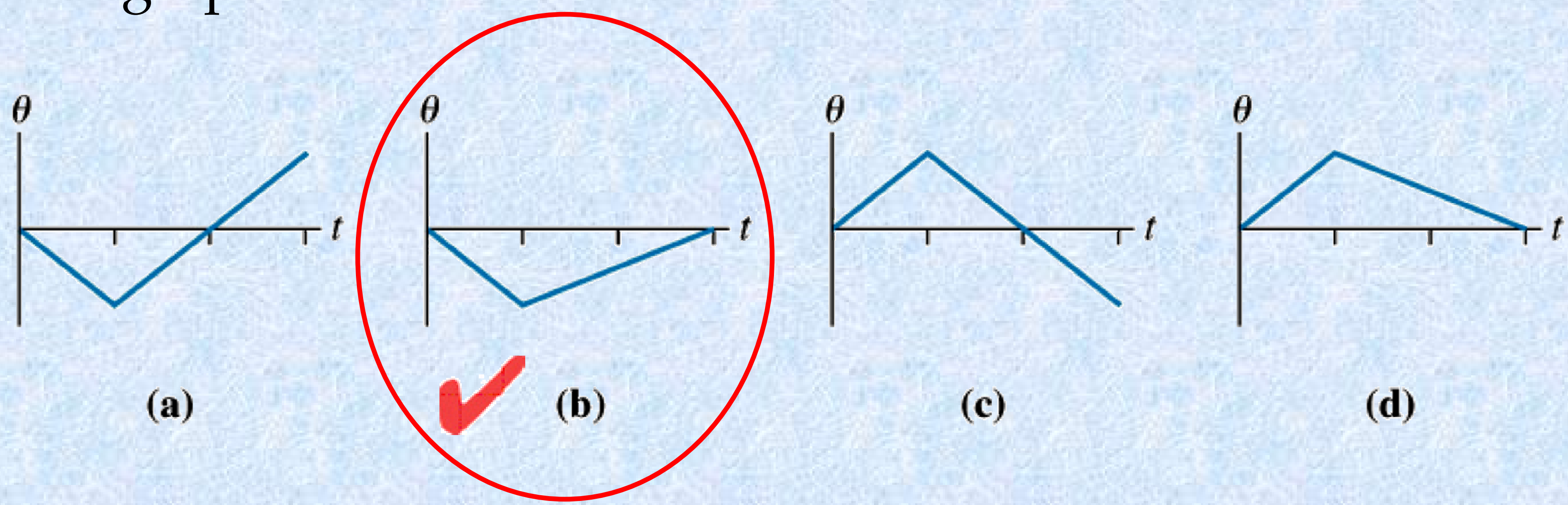
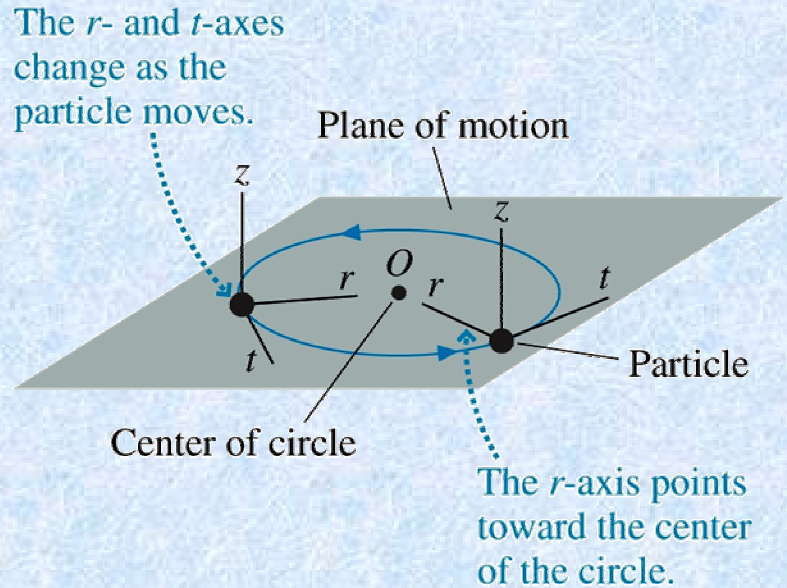


Chapter 7: Motion in a Circle

7.2 Velocity and acceleration in uniform circular motion



The *r-t-z* coordinate system

To a particle moving around a circle, affix a coordinate system with:

1. **the radial axis (*r*) always pointing toward the centre of the circle;**
2. **the tangential axis (*t*) always pointing tangent to the circle; and**
3. **the *z*-axis always pointing ⊥ to the plane defined by the *r*-*t* axes.**

All coordinates will be perpendicular to each other.

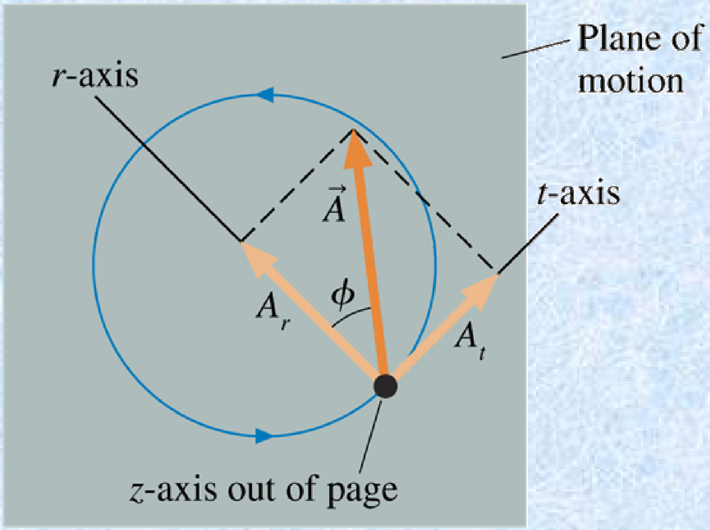
$\hat{z}$ will be fixed in direction, $\hat{r}$ and $\hat{t}$ will change as particle moves around circle.

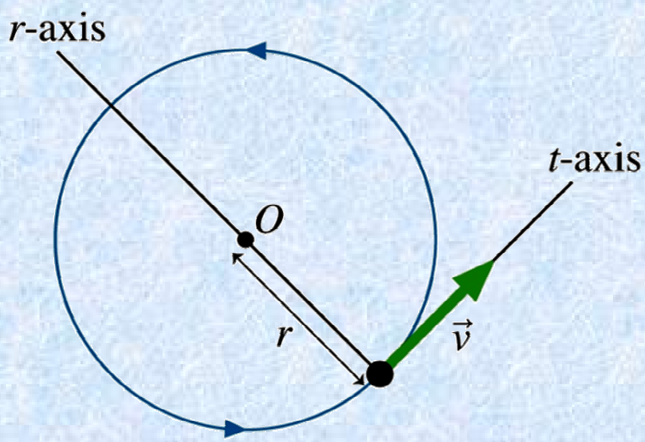
Chapter 7: Motion in a Circle

Any vector $\vec{A}$ with magnitude A can be decomposed into its radial and tangential components. In the figure to the right,

$$A_r = A\cos\phi\,; \quad A_t = A\sin\phi$$

$$A^2 = A_r^2 + A_t^2$$





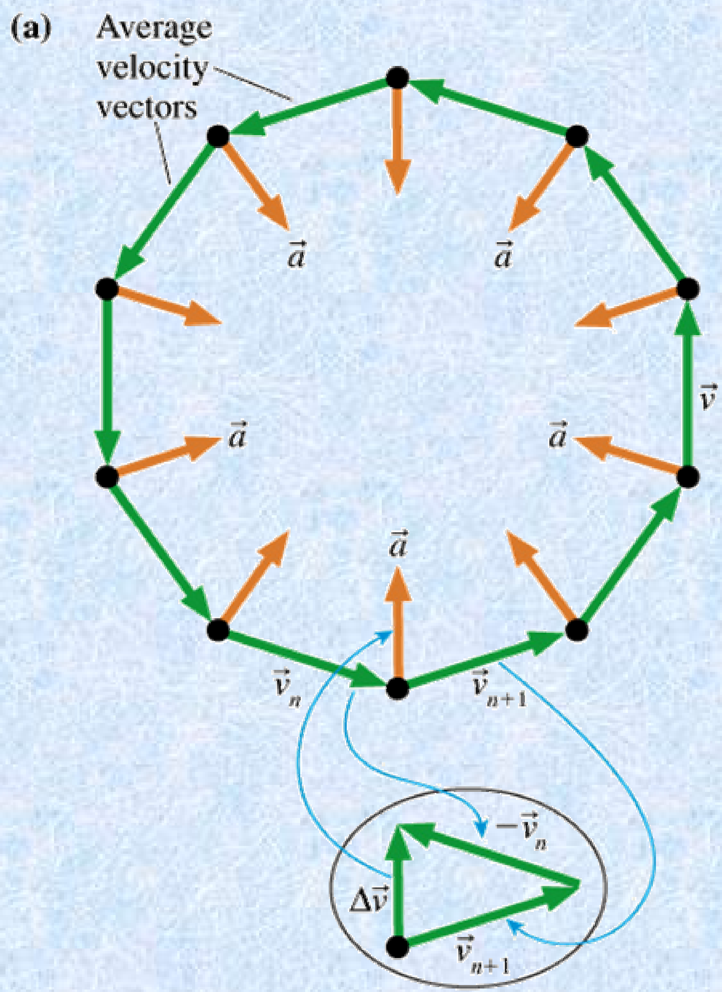
In circular motion, velocity only has a tangential component.

$$v_t = \frac{ds}{dt} = \frac{d(r\theta)}{dt} = r\,\frac{d\theta}{dt} = \omega r$$

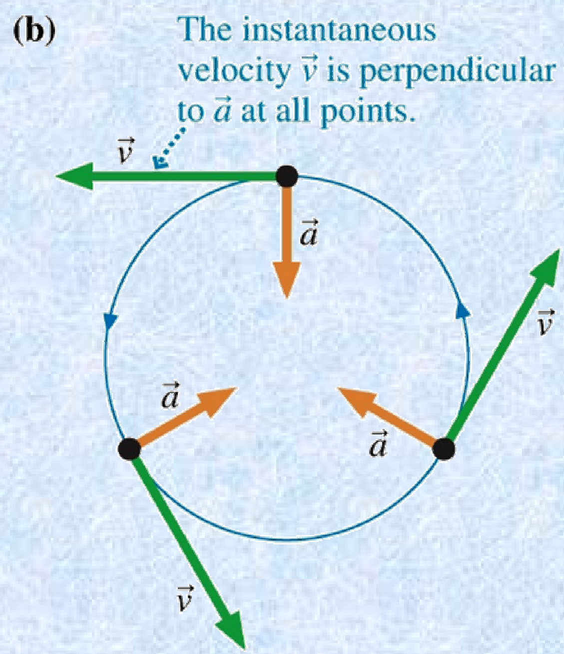
where ω has units rad s^{-1}

v_t has units m s^{-1}

Centripetal (*centre-seeking*) acceleration



For uniform circular motion, (a) shows that acceleration points to the centre, whence *centripetal acceleration*. Since instantaneous velocity is tangential to the circle, $\vec{v} \perp \vec{a}$, as shown in (b)

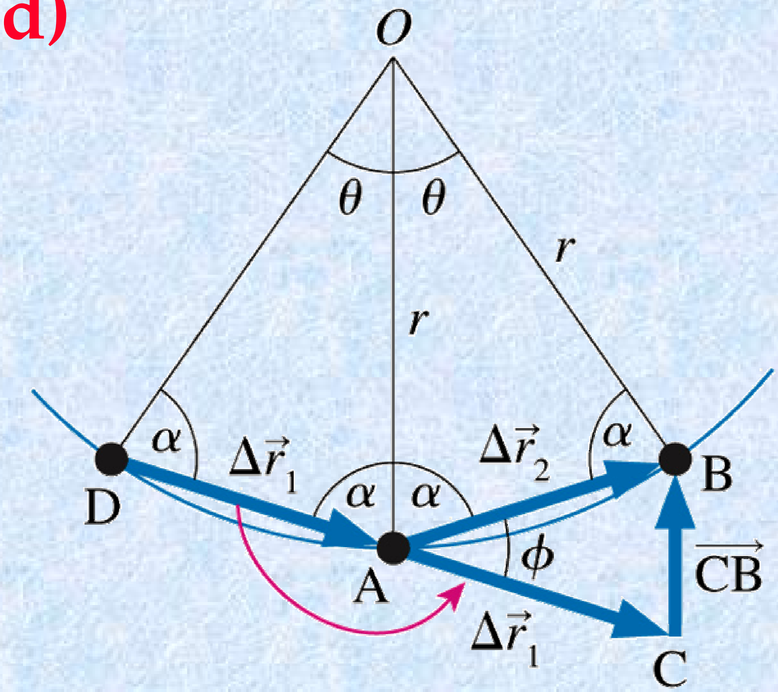


Centripetal acceleration (cont'd)

To find the magnitude of $\vec{a}_{\text{cent}}$:

$$\overrightarrow{CB} = \Delta\vec{r}_2 - \Delta\vec{r}_1 = \vec{v}_2\Delta t - \vec{v}_2\Delta t$$

$$= (\vec{v}_2 - \vec{v}_1)\,\Delta t = \Delta\vec{v}\,\Delta t$$



ΔOAB and ΔABC are isosceles and similar ($\theta + 2\alpha = \phi + 2\alpha = 180^{\circ} \Rightarrow \theta = \phi$). Thus:

$$\frac{CB}{AB} = \frac{AB}{AO} \Rightarrow \frac{\Delta v\,\cancel{\Delta t}}{v\,\cancel{\Delta t}} = \frac{v\,\Delta t}{r} \Rightarrow \frac{\Delta v}{\Delta t} = a_{\text{cent}} = \frac{v^2}{r} = \omega^2 r$$

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Summary:

for a particle in circular motion: $\boldsymbol{v}_\mathbf{r} = 0$; $\boldsymbol{v}_\mathbf{t} = \boldsymbol{\omega r}$; $\boldsymbol{v}_\mathbf{z} = 0$.

acceleration of uniform circular motion: $\boldsymbol{a}_\mathbf{r} = \boldsymbol{\omega}^2\boldsymbol{r}$; $\boldsymbol{a}_\mathbf{t} = 0$; $\boldsymbol{a}_\mathbf{z} = 0$

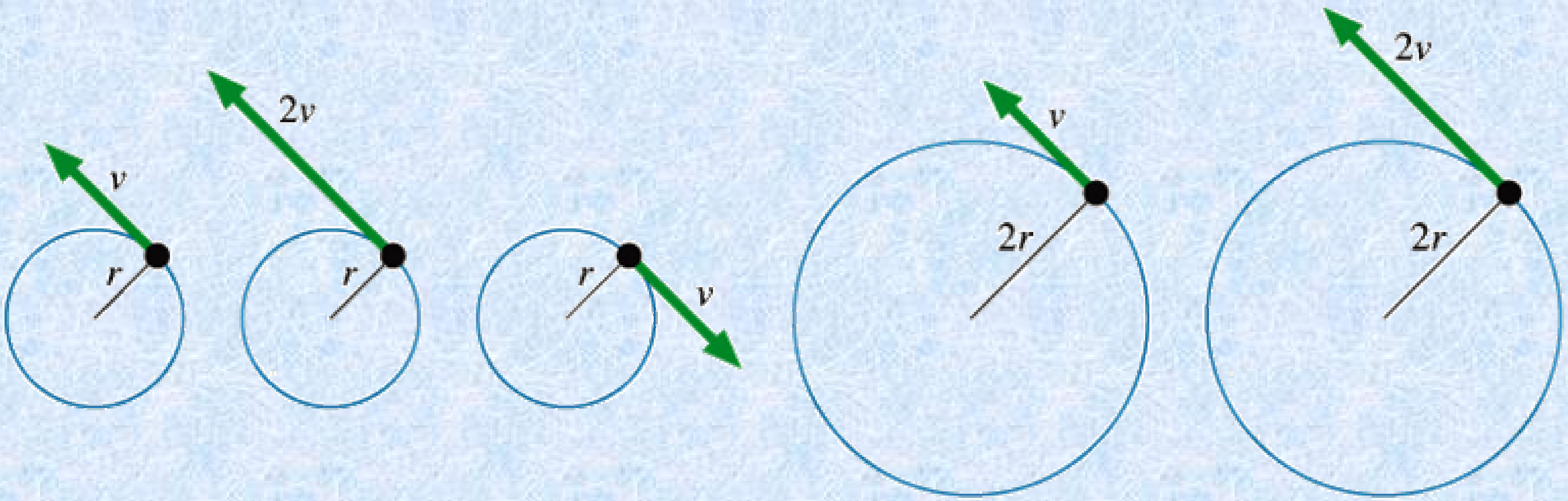
Centripetal acceleration is **not** a new acceleration. We are simply **choosing to label** an acceleration caused by **real forces** (**never** centrifugal force!) that always **points to the centre** of a circular path as ***centripetal*** (centre-seeking).

e.g., In the Bohr model of the atom, an electron orbits the nucleus much like a planet orbits the sun. If $\boldsymbol{r} = 5.3 \times 10^{-11}$ m and $\boldsymbol{T} = 1.5 \times 10^{-16}$ s, what is a_{cent}?

$a_{\text{cent}} = \omega^2 r = (2\pi/T)^2 r = 9.3 \times 10^{22}\ \text{ms}^{-2}$!!!

Clicker question 7.4

In which diagram is the centripetal acceleration the greatest?

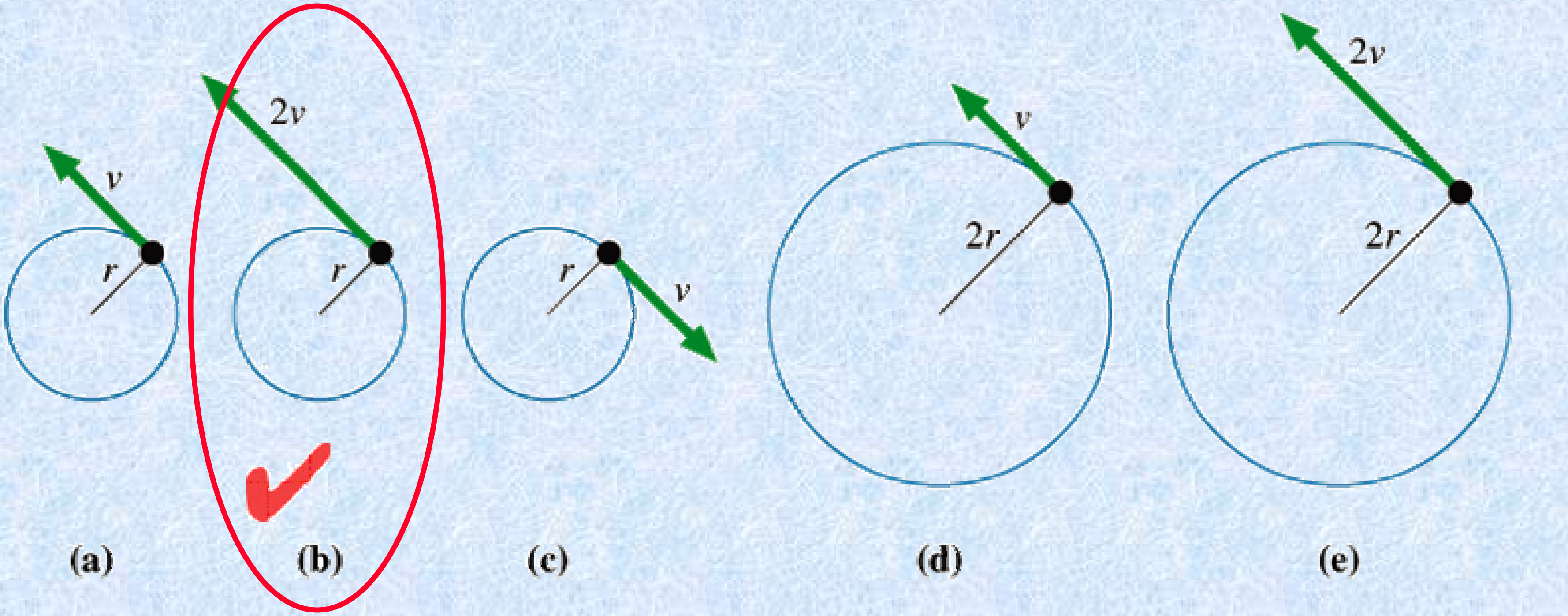


(a) (b) (c) (d) (e)

Clicker question 7.4

In which diagram is the centripetal acceleration the greatest?

v^2/r $\quad 4v^2/r$ $\quad v^2/r$ $\quad v^2/2r$ $\quad 2v^2/r$



Chapter 7: Motion in a Circle

7.3 Dynamics of uniform circular motion

When a particle is undergoing uniform circular motion, the net force acting on the particle must point towards the centre.

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cent}} = \left(\frac{mv^2}{r}, \text{toward centre of circle}\right)$$

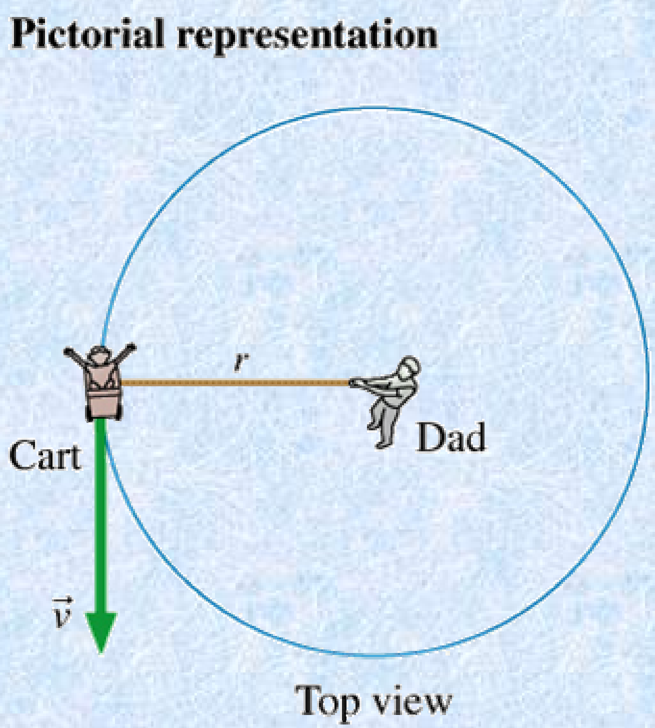
A force that always points to the centre of a circular path is called the **centripetal** (centre-seeking) **force**.

A **centripetal force** is ***not*** a new force. It will always be one or the sum of the forces found in the force catalogue (Chapter 4).

The adjective **centripetal** is a ***label*** we apply to the force(s) that point toward the centre of a circular path.

Chapter 7: Motion in a Circle

Example 1. A man spins a child in her cart (total mass 25 kg) in circles using a 2-m-long rope always holding the rope parallel to the ground. If the tension in the rope is 100 N, what is the angular speed of the cart? (Neglect friction.)

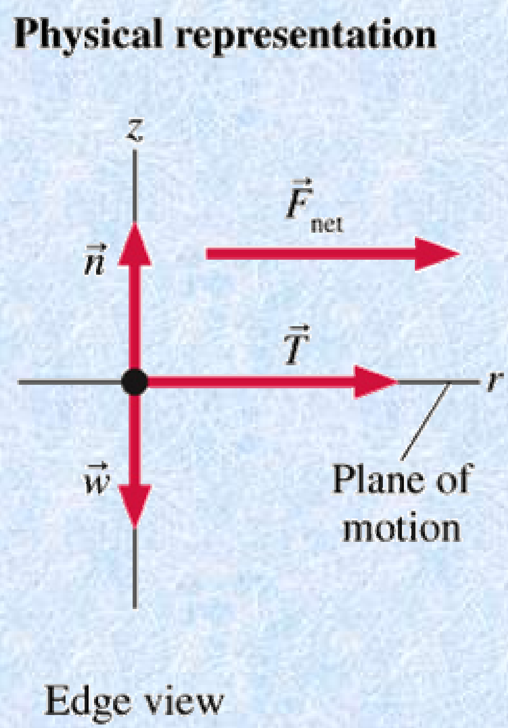


Known

$m = 25$ kg
$r = 2$ m
$T = 100$ N

Find

ω



Chapter 7: Motion in a Circle

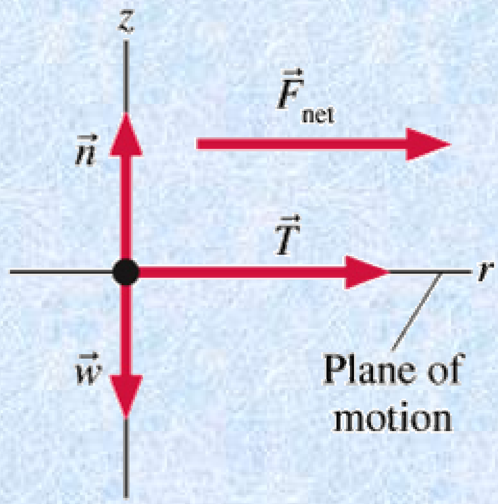
solution…

Known

$m = 25$ kg
$r = 2$ m
$T = 100$ N

Find

ω



$$z/\quad \sum F_z = n - w = 0$$

$$r/\quad \sum F_r = T = ma_r = \frac{mv^2}{r} = m\omega^2 r$$

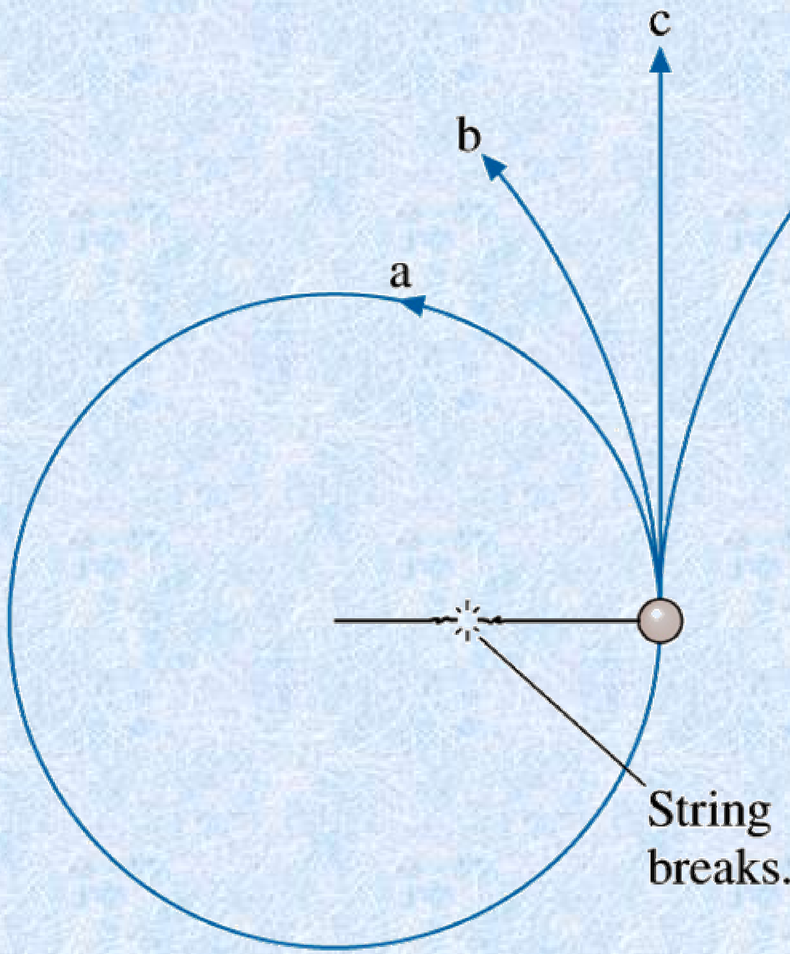
The *z*-equation is irrelevant for this problem. The ***r***-equation gives us:

$$\omega^2 = \frac{T}{mr} \quad \Rightarrow \quad \omega = \sqrt{\frac{T}{mr}} = \underline{\underline{1.41 \text{ rad s}^{-1}}}$$

Clicker question 7.5

A ball is swung around by a string in a horizontal circle on the surface of a table.

If the string breaks, which way does the ball continue?



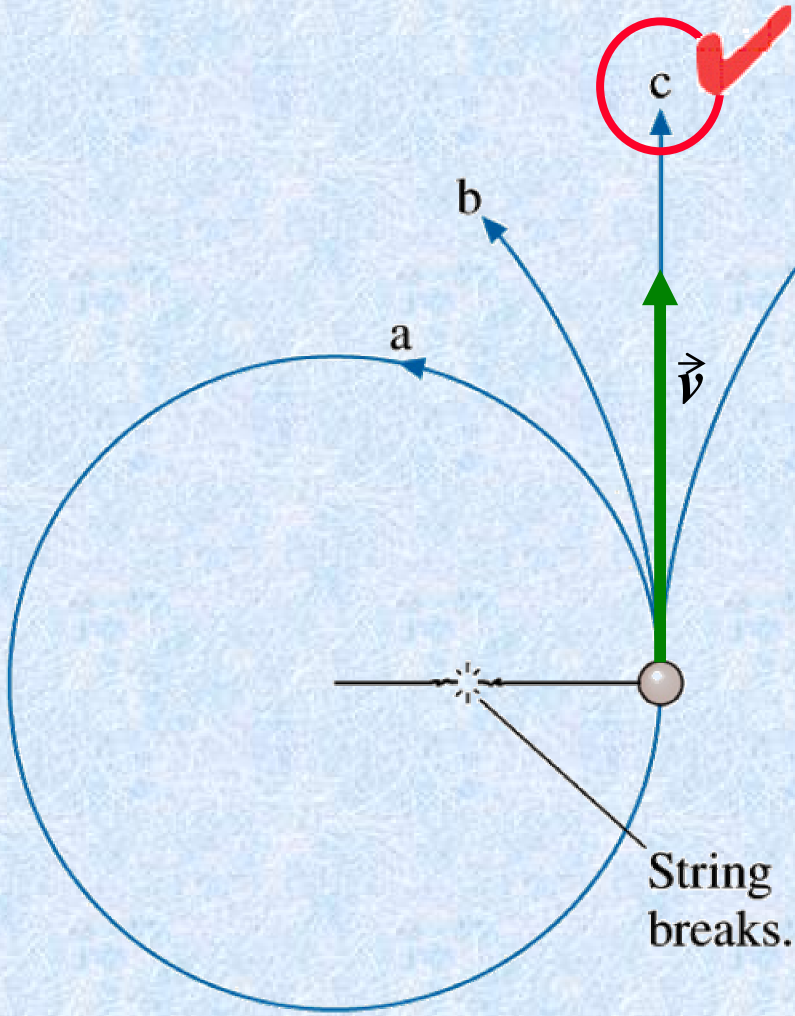
Chapter 7: Motion in a Circle

Clicker question 7.5

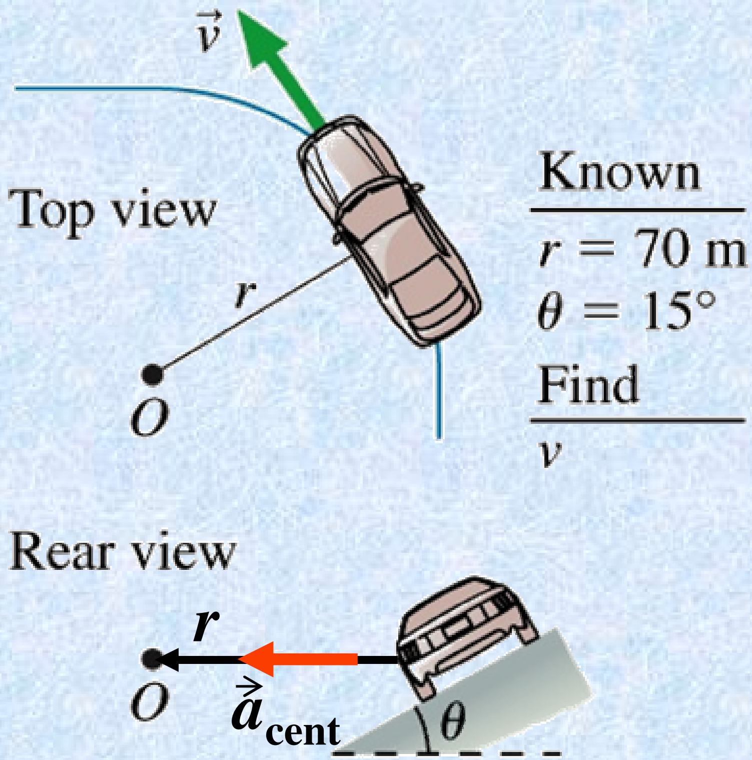
A ball is swung around by a string in a horizontal circle on the surface of a table.

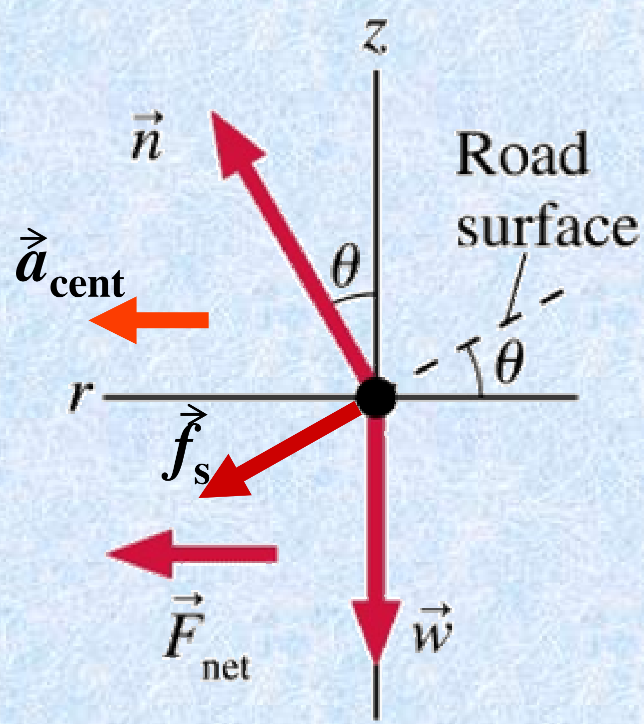
If the string breaks, which way does the ball continue?

Without the tension force, the ball continues moving in direction of $\vec{v}$.



Example 2. **A car approaches a highway curve with a radius 70 m and banked at 15° relative to the horizontal. If the coefficient of static friction between the tires and the highway is 1.0, what is the maximum constant speed the car can take this curve without sliding?**





Chapter 7: Motion in a Circle

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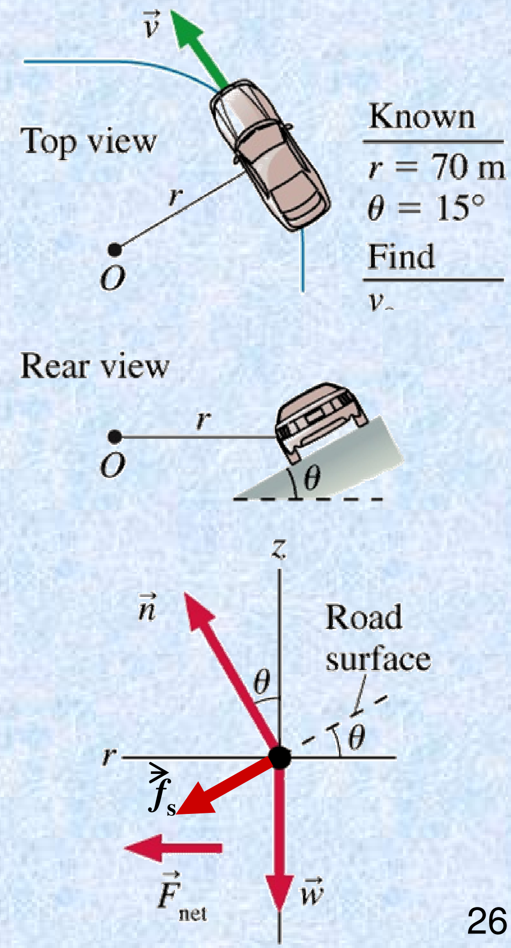

$$z/\quad \sum F_z = n\cos\theta - f_s\sin\theta - w = 0$$

$$r/\quad \sum F_r = n\sin\theta + f_s\cos\theta = \frac{mv^2}{r}$$

Speed is limited because of friction. Thus, take f_s to be its maximum: $\mu_s n$

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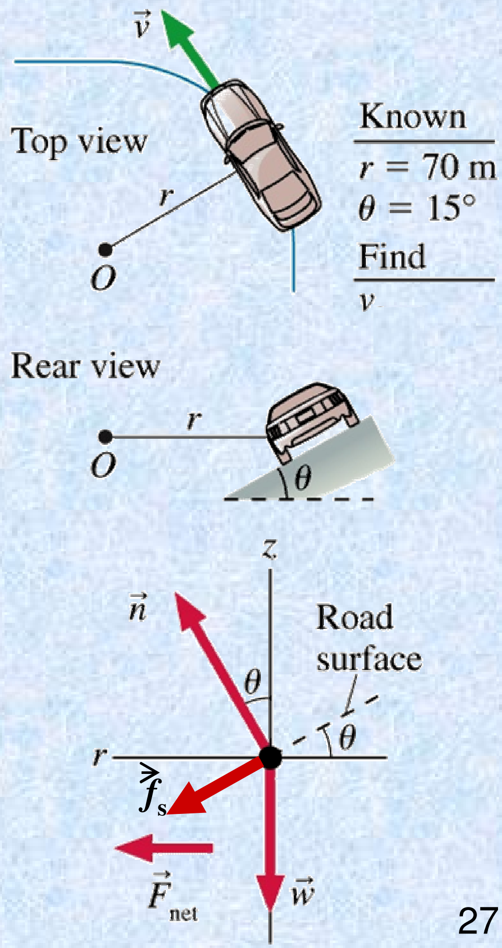

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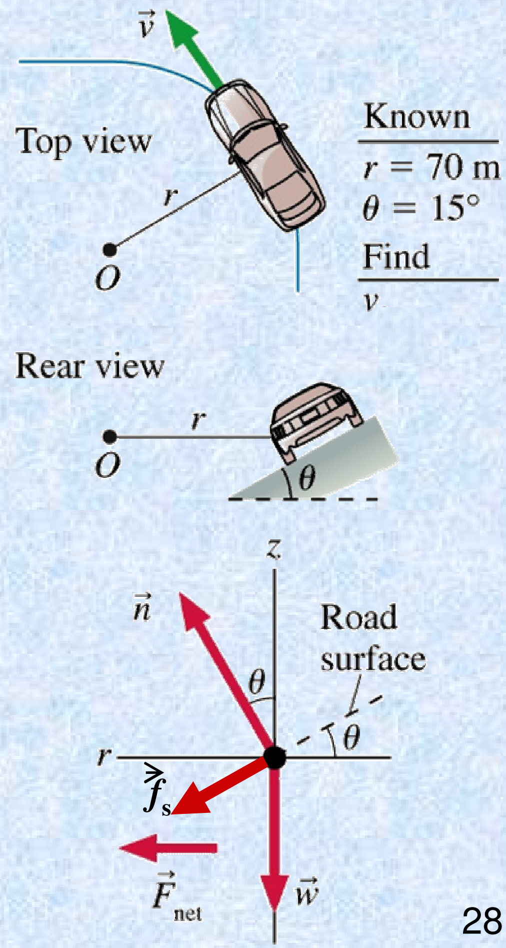
$$r/ \quad \sum F_r = n\sin\theta + f_s \cos\theta = \frac{mv^2}{r}$$

Speed is limited because of friction. Thus, take f_s to be its maximum: $\mu_s n$. From the z-equation:

$$n\cos\theta - \mu_s n \sin\theta = mg \quad \Rightarrow \quad n = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

Example 2. A car approaches a highway curve with a radius 70 m and banked at 15° relative to the horizontal. If the coefficient of static friction between the tires and the highway is 1.0, what is the maximum constant speed the car can take this curve without sliding?



$$z/\quad \sum F_z = n\cos\theta - f_s\sin\theta - w = 0$$

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Speed is limited because of friction. Thus, take f_s to be its maximum: $\mu_s n$. From the z-equation:

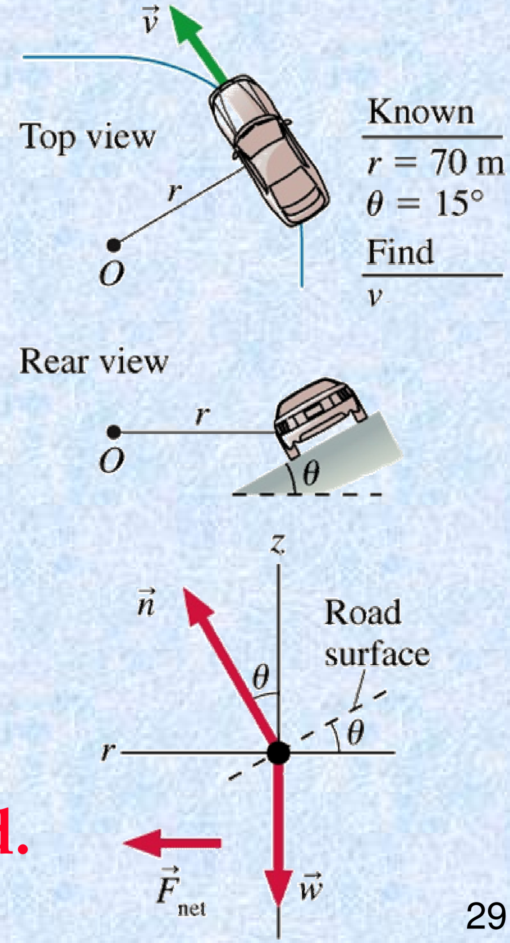
$$n\cos\theta - \mu_s n\sin\theta = mg \quad\Rightarrow\quad n = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

and from the r-equation: $$n = \frac{mv^2}{r(\sin\theta + \mu_s\cos\theta)}$$

$$\Rightarrow \quad \boxed{v = \sqrt{rg\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}}} = 34.5\ \text{ms}^{-1}\ (124\ \text{kph})$$

Chapter 7: Motion in a Circle

Example 3. Same situation as example 2, but now it's January and black ice is everywhere (thus, $\mu_s = 0$). What is the maximum speed the car can take the bank without slipping now?



There is no need to do this problem again from Newton's 2nd law, **except for practice**! So this is left as an exercise.

Here, we just use the result from example 2 and set $\mu_s = 0$:

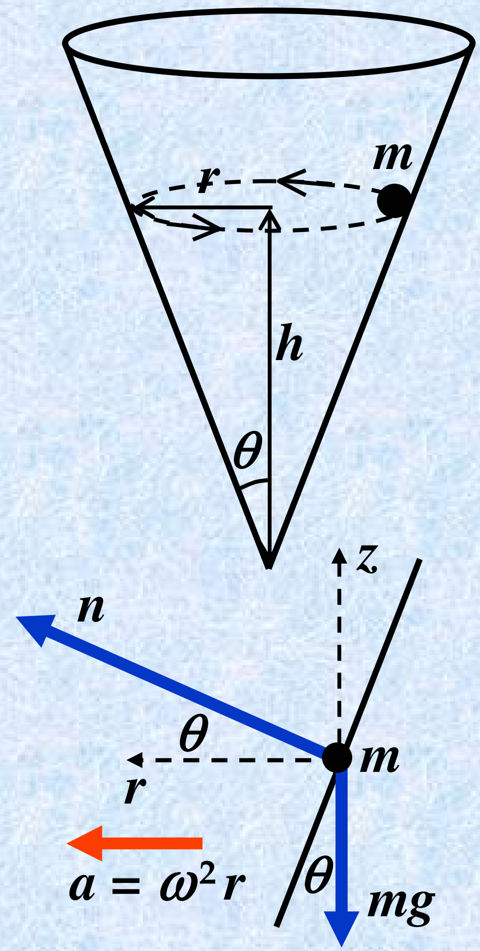
$$v = \sqrt{rg\frac{\sin\theta}{\cos\theta}} = \sqrt{rg\tan\theta}$$

$$= 13.6\ \text{ms}^{-1}\ \ (48.8\ \text{kph})$$

Note that in neither case was the mass needed.

Chapter 7: Motion in a Circle

Example 4. A marble spinning inside a funnel (right cone) of half-opening angle $\boldsymbol{\theta}$ has a period of revolution $\boldsymbol{T}$. Find its height, $\boldsymbol{h}$, above the apex of the funnel.



$$z/\quad \sum F_z = n\sin\theta - mg = 0 \;\Rightarrow\; n = \frac{mg}{\sin\theta}$$

$$r/\quad \sum F_r = n\cos\theta = m\omega^2 r$$

$$\omega = \frac{2\pi}{T};\quad \tan\theta = \frac{r}{h} \;\Rightarrow\; r = h\tan\theta$$

Thus, the r-equation $\Rightarrow \cancel{m}g\dfrac{\cos\theta}{\sin\theta} = \cancel{m}\dfrac{4\pi^2}{T^2}h\tan\theta$

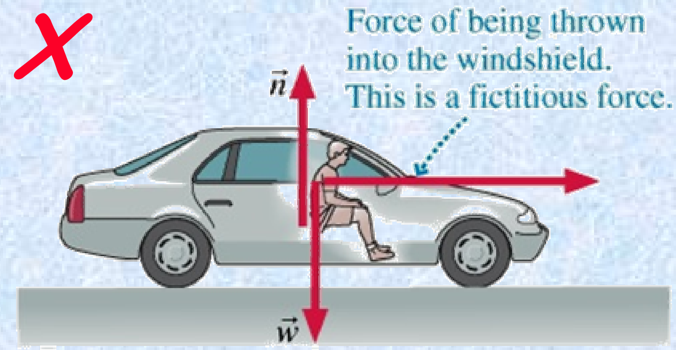
$$\Rightarrow\; \frac{g}{\tan^2\theta} = h\frac{4\pi^2}{T^2} \;\Rightarrow\; \boxed{h = g\left(\frac{T}{2\pi\tan\theta}\right)^2}$$

e.g., for $T = 0.25$ s, $\theta = 20^{\circ}$, $h = 0.12$ m

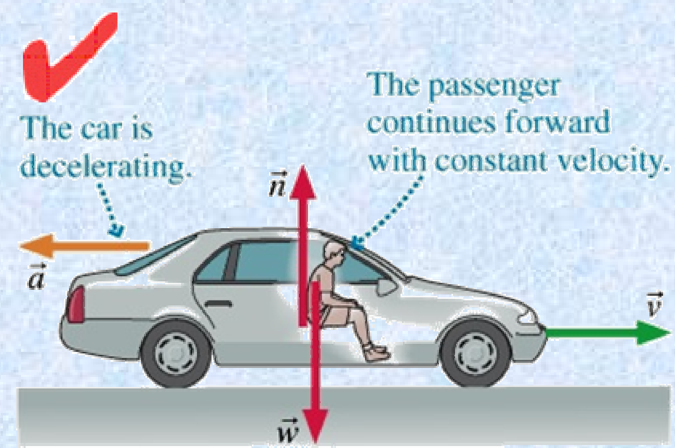
7.5 Fictitious forces

If the brakes are suddenly applied, the passenger feels as though he is being thrown forward by some force. This perceived force is a *fictitious force*. **There is no actual force pushing the passenger forward!**

Viewed from an **inertial frame of reference**, the car is accelerating backward. Since the passenger is not part of the car, he continues to move forward with the velocity the car had before the brakes were applied.



Noninertial reference frame of passenger



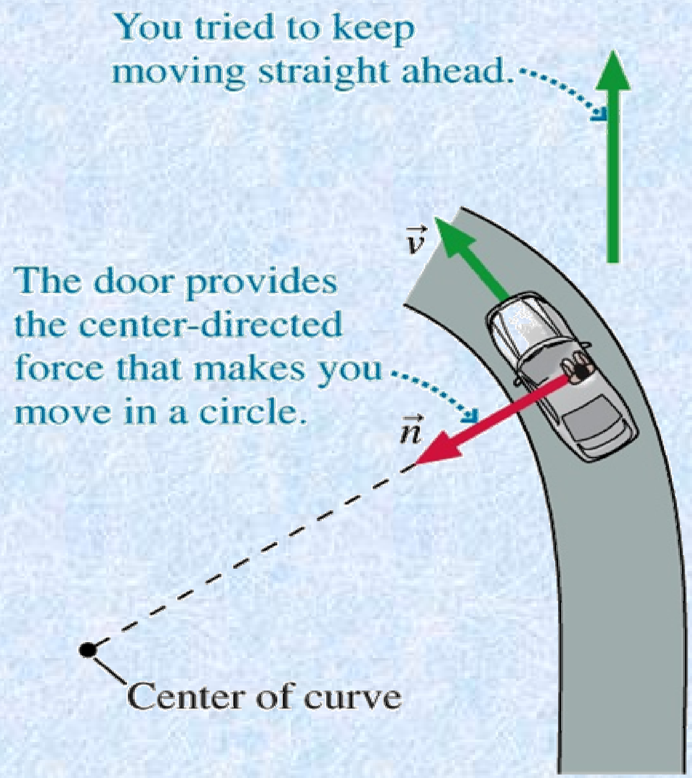
Inertial reference frame of the ground

Chapter 7: Motion in a Circle

"Centrifugal force": **no such thing!!!!**

The sensation of a *"centrifugal force"* is also felt only in an **accelerating frame of reference**. Turning a sharp corner, one feels pushed against the car door by this **fictitious force.**

Viewed from an **inertial frame**, the passenger is not part of the car and tries to continue moving in a straight line. The car door provides the normal force necessary to cause the passenger to turn the circle too.



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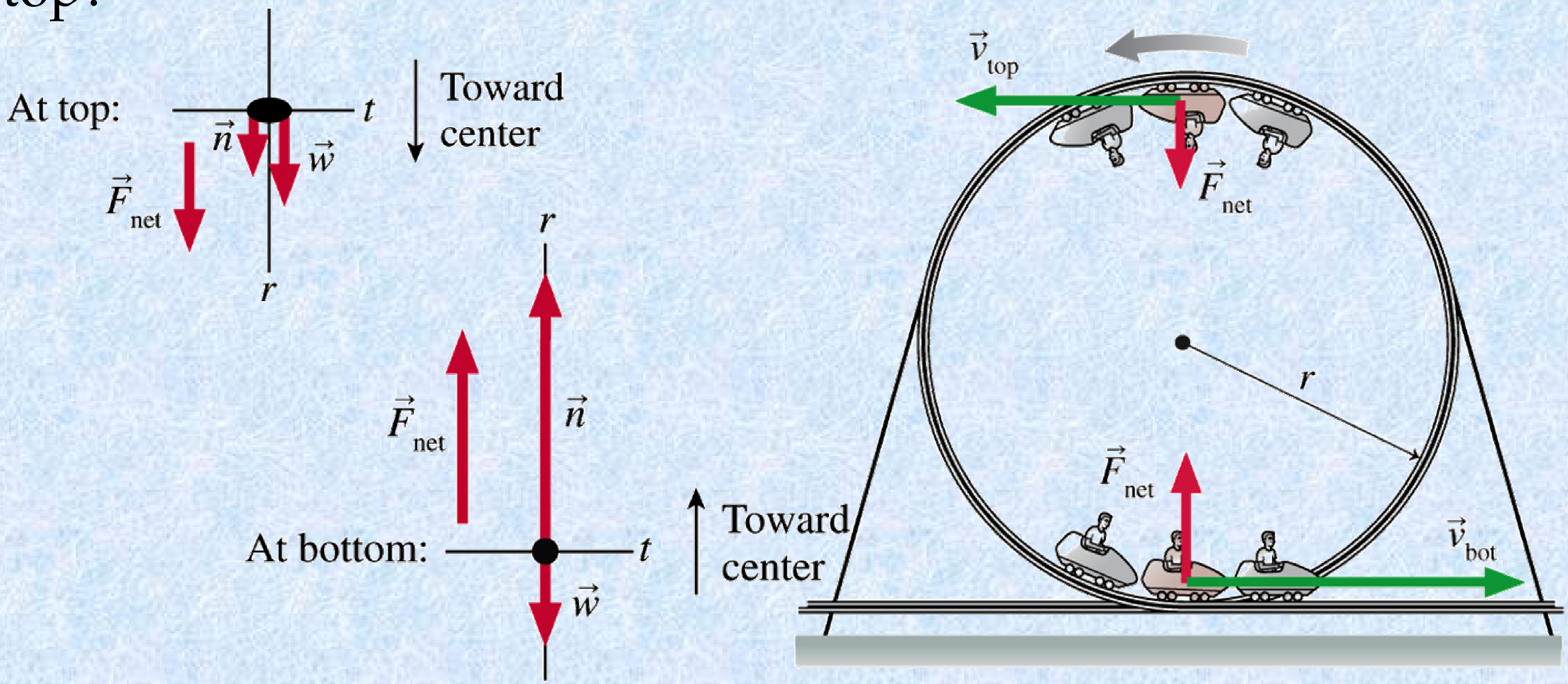
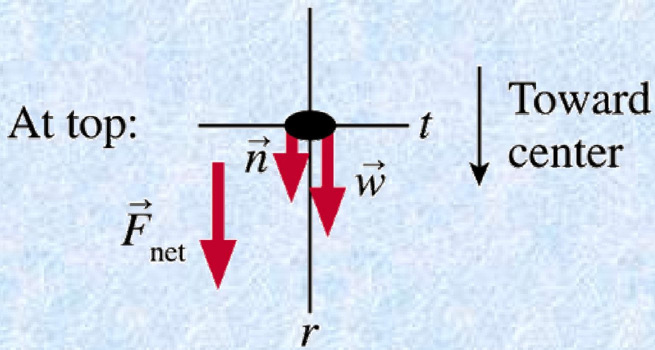
Chapter 7: Motion in a Circle

Example 5. A roller coaster goes around a vertical "loop-the-loop" of radius ***r***.

a) Why is one's ***apparent weight*** greater at the bottom than the top?



Chapter 7: Motion in a Circle



The apparent weight is the normal force. Thus, at the top…

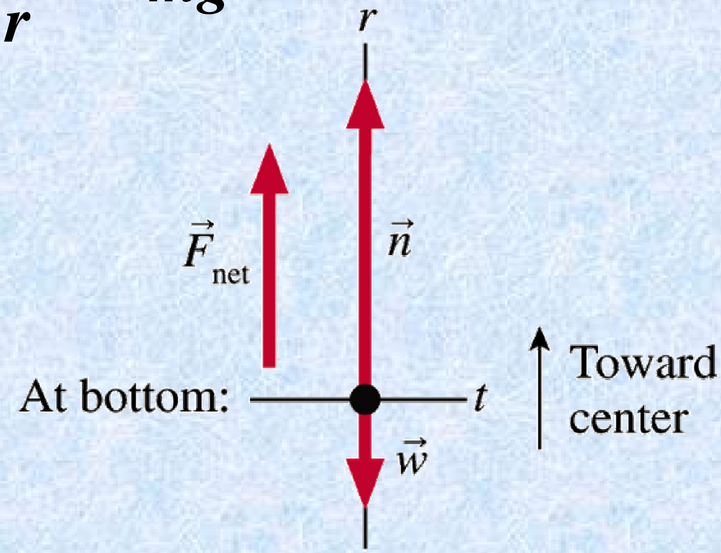
$$F_{net,top} = n_{top} + w = m\frac{v_{top}^2}{r}$$

$$n_{top} = m\frac{v_{top}^2}{r} - mg$$

and at the bottom…

$$F_{net,bot} = n_{bot} - w = m\frac{v_{bot}^2}{r}$$

$$n_{bot} = m\frac{v_{bot}^2}{r} + mg$$



$n_{bot} > n_{top}$ because $v_{bot} > v_{top}$ and
mg is added at the bottom but subtracted at the top.

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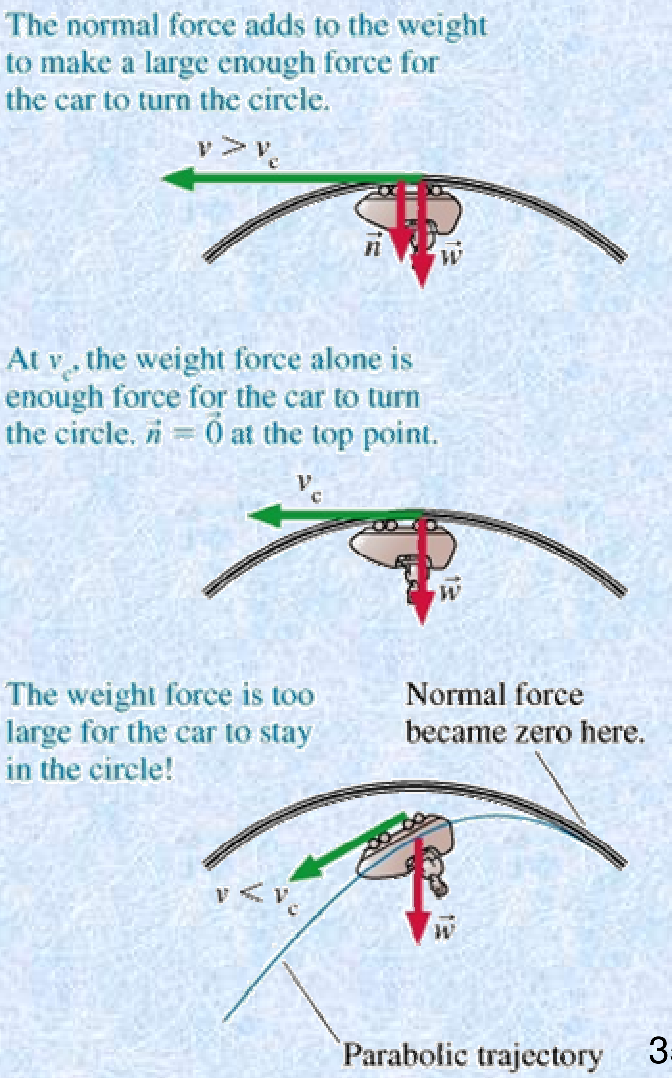
b) What is the minimum velocity (v_c) necessary for the car to stay on the track?

$$F_{net,top} = n_{top} + w = m\frac{v_{top}^2}{r}$$

For the velocity to be just enough to keep the car on the track, $n_{top} = 0$

$$\Rightarrow \quad \not{m}g = \not{m}\frac{v_c^2}{r} \quad \Rightarrow \quad v_c = \sqrt{rg}$$

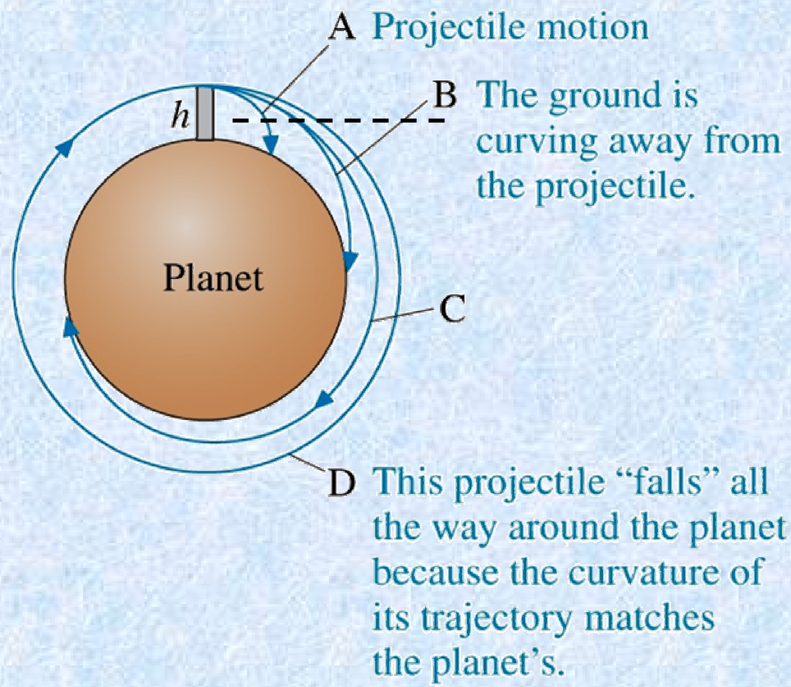
We often use the designation "critical" for quantities that are "just enough". Thus, the subscript "c" in v_c stands for "critical".



Chapter 7: Motion in a Circle

7.4 Circular orbits

A) Suppose you stand on a high tower, and jump with a horizontal velocity, v_0. This is projectile motion, as we have studied.



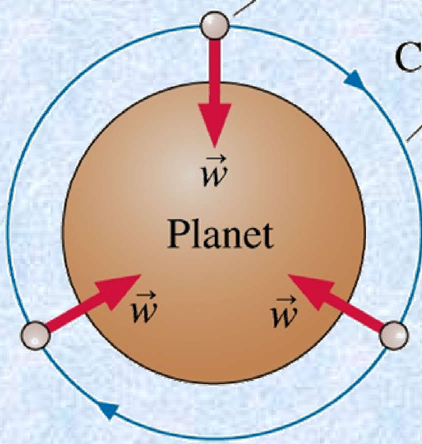
B) Now suppose v_0 is so high and your horizontal range so far that by the time you hit the ground, the curvature of the earth has noticeably lowered the ground from under you!

C) An even higher v_0 and you almost make it all the way around the earth before touching ground.

D) Now let v_0 be so high that the curvature of your path exactly matches the curvature of the earth! This is the definition of "being in orbit" and you are in "free-fall" around the earth!

What is the orbital speed of the projectile?

$$w = \not{m}g = \not{m}\frac{v_{orb}^2}{r} \Rightarrow v_{orb} = \sqrt{rg}$$



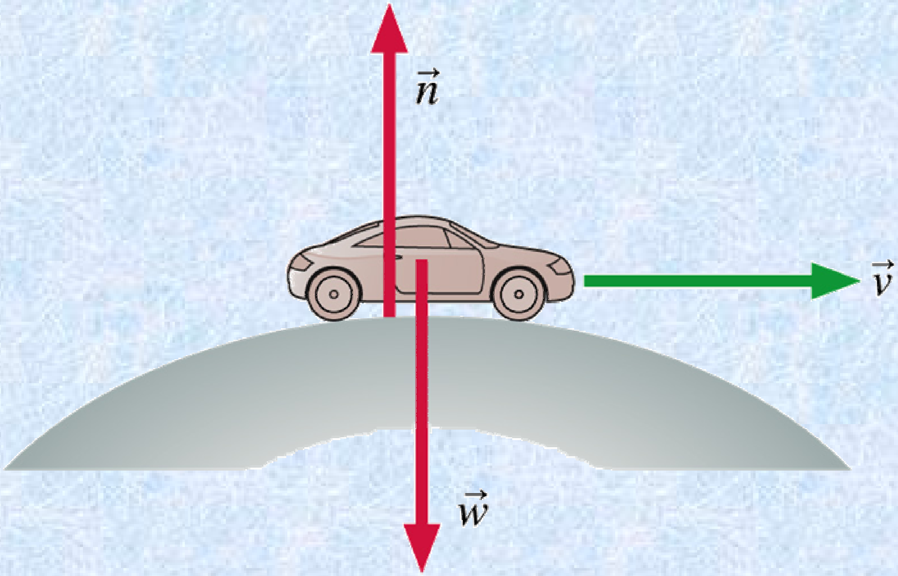
This is exactly the same as the "critical velocity" for the roller coaster car to stay on the track at the top of the loop-the-loop! (slide 35)

In fact, in both cases the object is in "free-fall" and feels "weightless".

The expression for v_{orb} is valid for "low orbits", *i.e.*, r ~ radius of the earth. For higher orbits, we must take into account that gravity falls off as the "inverse square" of the distance from the centre of the earth (Chapter 12). For r = 6.37 x 10^6 m, v_{orb} = 7,900 ms^{-1} = 28,400 kph (one orbit every 90 minutes).

Clicker question 7.6

A car is rolling over the top of a circular hill at speed v. At this instant:



a) $n < w$;

b) $n = w$;

c) $n > w$;

d) We can't tell unless we know v.

Clicker question 7.6

A car is rolling over the top of a circular hill at speed v. At this instant:

$$w - n = m\frac{v^2}{r}$$

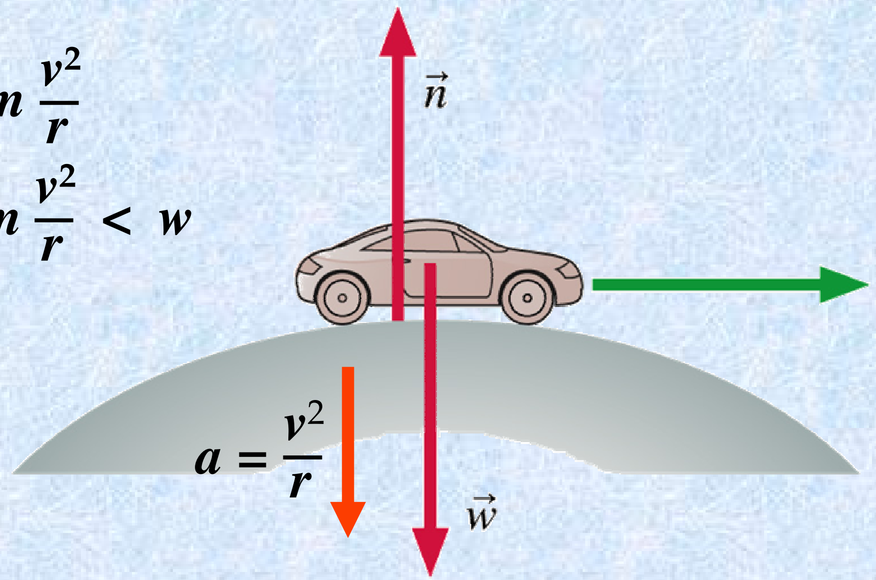
$$\Rightarrow \quad n = w - m\frac{v^2}{r} < w$$

a) $n < w$;

b) $n = w$;

c) $n > w$;

d) We can't tell unless we know v.



Clicker question 7.7

While in orbit around the Earth, an astronaut feels weightless because…

a) there is no gravity in space;

b) the astronaut is in free-fall;

c) a human's mass decreases the further he/she gets from the centre of the earth;

d) the lack of oxygen in space does funny things to an astronaut's perceptions.

Clicker question 7.7

While in orbit around the Earth, an astronaut feels weightless because…

a) there is no gravity in space;

b) the astronaut is in free-fall;

c) a human's mass decreases the further he/she gets from the centre of the earth;

d) the lack of oxygen in space does funny things to an astronaut's perceptions.

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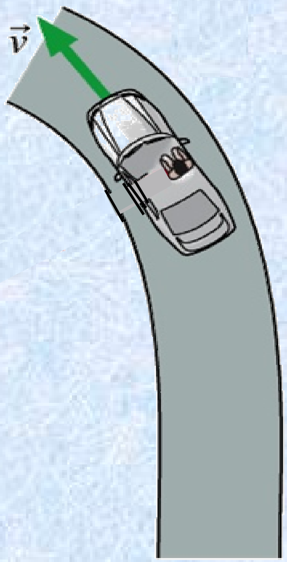
Clicker question 7.8

A car maintains a constant speed as it makes a left circular turn around a corner. The passenger feels the door pressing against him because…

a) of the centrifugal force of the turn;

b) the car door is providing the centripetal force necessary to force him to follow the circular path; or

c) he is accelerating outward and thus being thrown against the door.



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Clicker question 7.8

A car maintains a constant speed as it makes a left circular turn around a corner. The passenger feels the door pressing against him because…

a) of the centrifugal force of the turn;

b) the car door is providing the centripetal force necessary to force him to follow the circular path; or

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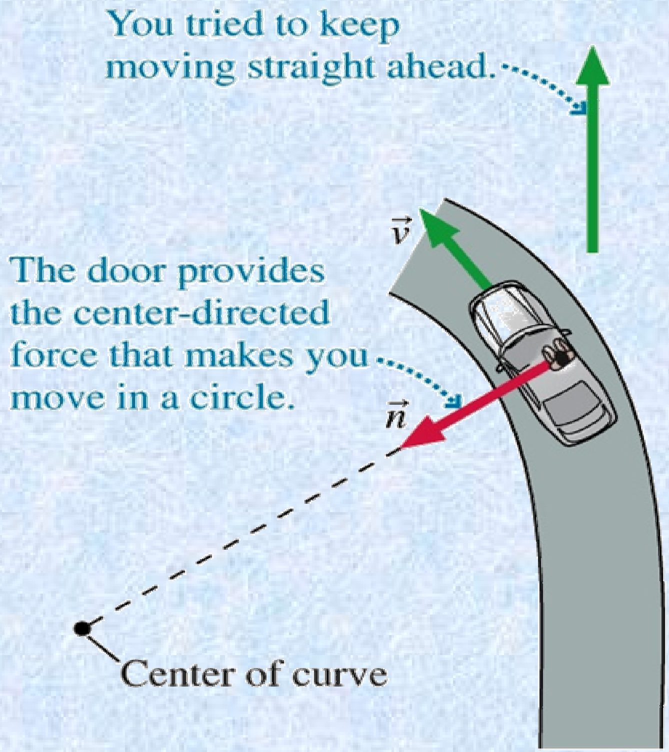


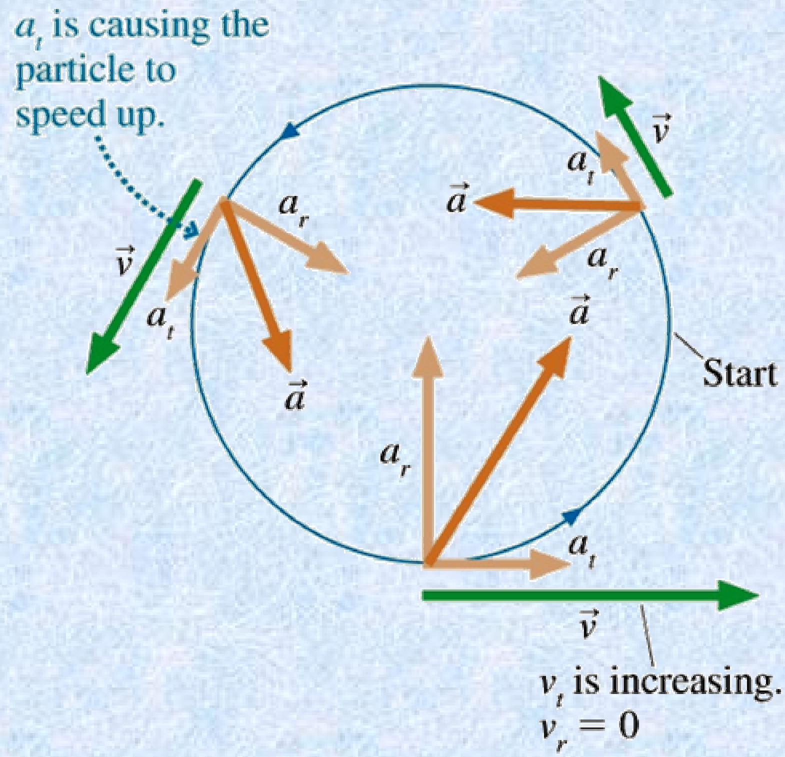
In this case, the centripetal force is the normal force.

7.6 Non-uniform circular motion

Consider an object moving in a circular path with a non-uniform speed. In this case, the acceleration has

- a radial (***r***)-component (⊥ to the velocity) that changes the particle's direction

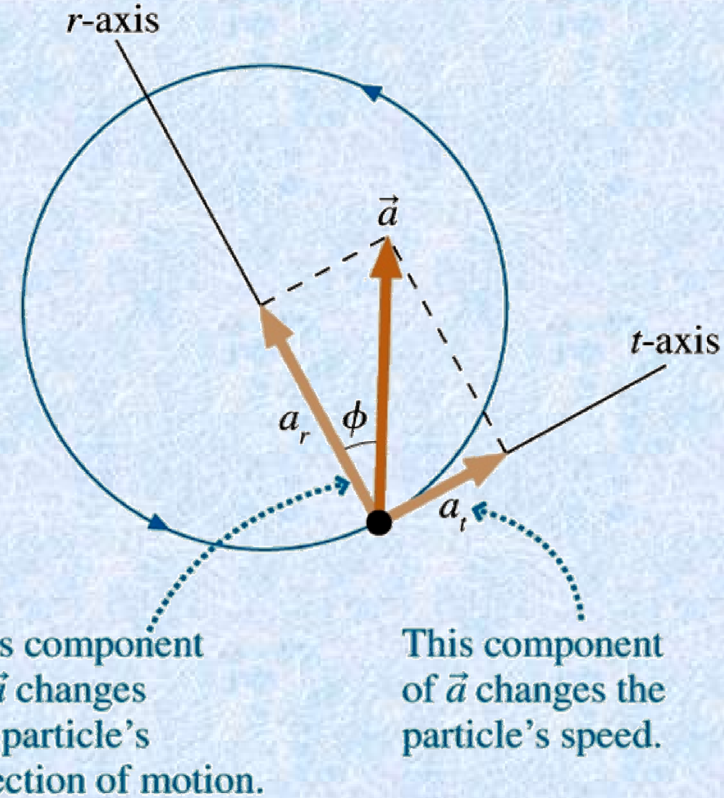
- a tangential (***t***)-component (|| to the velocity) that changes the particle's speed.



7.6 Non-uniform circular motion

Introduce a tangential acceleration component, a_t. Then the **angular acceleration**, α, is given by:

$$\alpha = \frac{d\omega}{dt} = \frac{d(v/r)}{dt} = \frac{1}{r}\frac{dv}{dt} = \frac{a_t}{r}$$



The kinematical equations are:

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\theta_f = \theta_i + \omega_i \Delta t + ½ \alpha \Delta t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

Note that the text does not use α, but keeps it as a_t/r.

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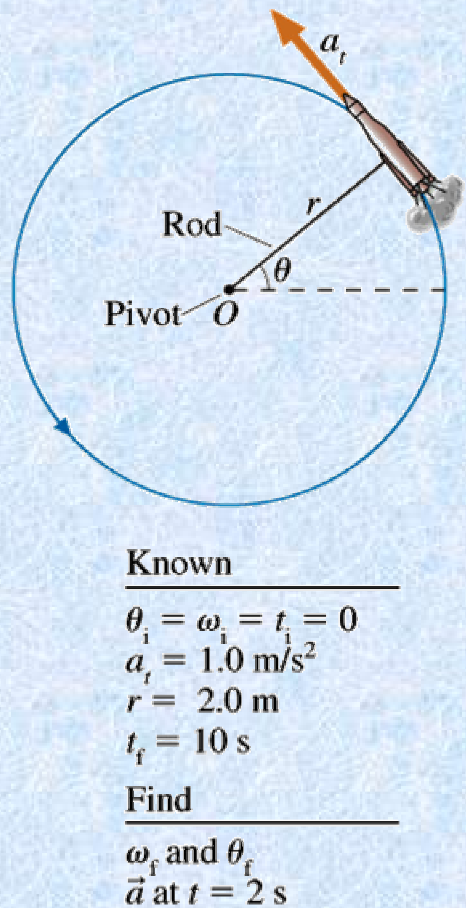
Example 6. **A toy rocket is attached to the end of a 2-m-long massless rod. The other end is fixed to a frictionless pivot causing the rocket to move in a horizontal circle. The rocket accelerates at 1 ms^{-2} for 10 s, then runs out of fuel.**



a) Find the angular speed, ω, after 10 s.

$$\alpha = \frac{a_t}{r} = \frac{1.0\ ms^{-2}}{2.0\ m} = 0.5\ \text{rad s}^{-2}$$

$$\omega_f = \omega_i + \alpha\Delta t = 0.0 + (0.5)(10.0) = 5.0\ \text{rad s}^{-1}$$

b) How many revolutions has the rocket made at 10 s?

$$\Delta\theta = \omega_i\Delta t + \tfrac{1}{2}\alpha\Delta t^2 = \tfrac{1}{2}(0.5)(10.0)^2 = 25\ \text{rad}$$

$$\Rightarrow \#\ \text{revolutions} = \Delta\theta / 2\pi = 4.0$$

c) Find $\vec{a}$ at $t = 2$ s.

notes 7.1

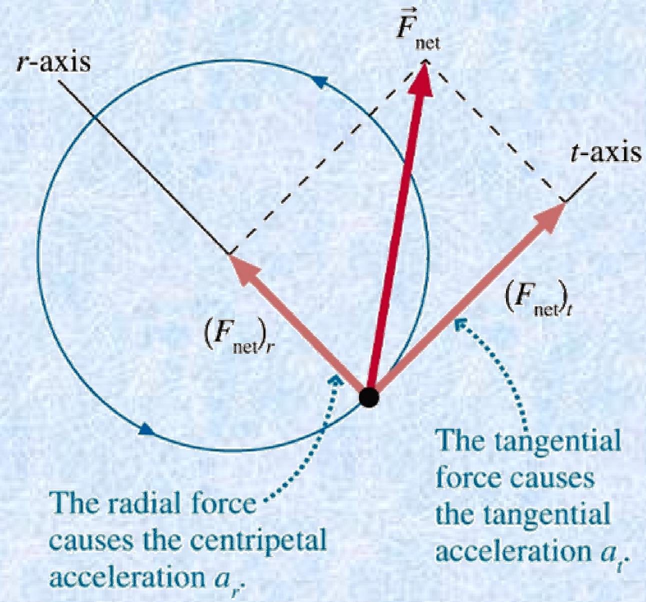
Chapter 7: Motion in a Circle

Dynamical equations:

$$r/\quad (F_{net})_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r$$

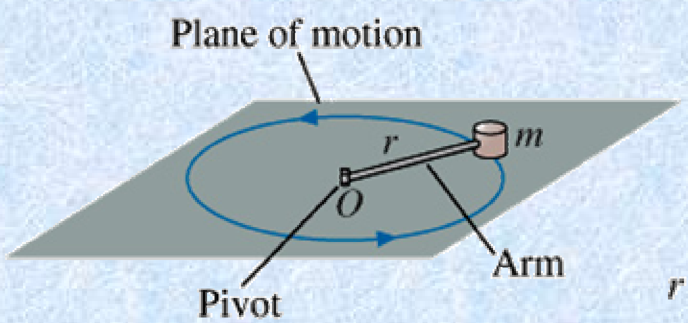
$$t/\quad (F_{net})_t = \sum F_t = ma_t = m\alpha r$$

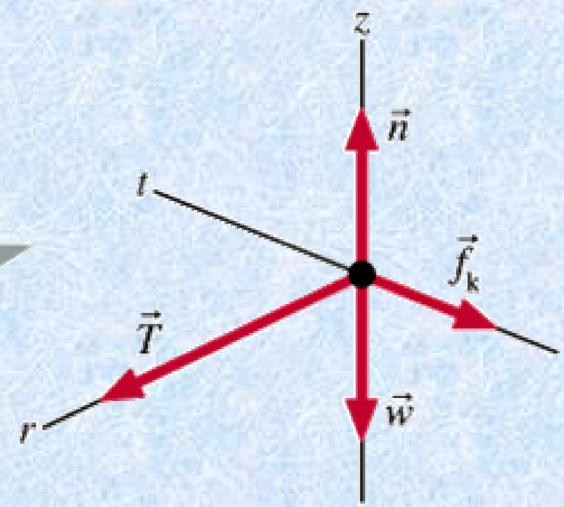
$$z/\quad (F_{net})_z = \sum F_z = 0$$



Example 6. **A motor spins a 2.0 kg block around on a 20-cm-long massless rod at 30 rad s^{-1} on a table with $\mu_k = 0.4$.**

a) If the pivot is frictionless, how long does it take for the block to come to rest?





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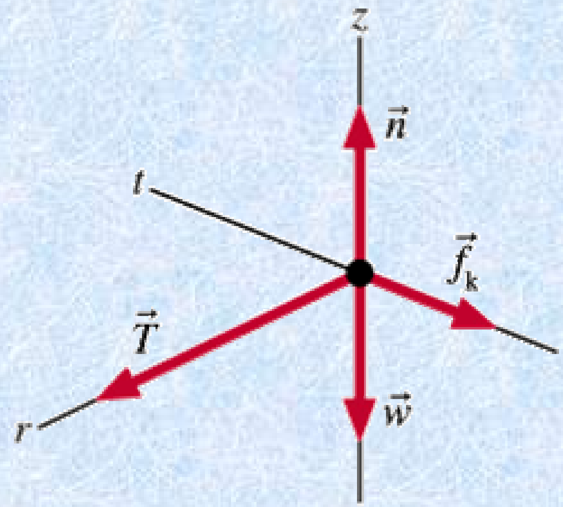
This is a rare example where all three directions are in play (although the ***r***-equation ends up not mattering).



$$r/\ \sum F_r = T = ma_r = m\omega^2 r$$

$$t/\ \sum F_t = -f_k = ma_t = m\alpha r$$

$$z/\ \sum F_z = n - w = 0$$

$$n = w = mg;\quad f_k = \mu_k n = \mu_k mg;\quad \alpha = -\frac{f_k}{mr} = -\frac{\mu_k \cancel{m} g}{\cancel{m} r}$$

$$\omega_f = \omega_i + \alpha\Delta t = 0$$

$$\Rightarrow \Delta t = -\frac{\omega_i}{\alpha} = \frac{r\omega_i}{\mu_k g} = \frac{(0.2)(30.0)}{(0.4)(9.8)} = \underline{1.53\ \text{s}}$$