**ChromaStar lab 7: The simple curve-of-growth (COG) of a spectral line**

Ian Short, Department of Astronomy & Physics, Saint Mary’s University

**LeveL:** Second year University

**Purpose:** To make simulated simple curves-of-growth (COGs) of spectral lines for atomic transitions of different strength for the Sun, to relate uncertainty in transition strength to uncertainty in derived abundance of the absorbing species, and to understand how to use a COG to measure spectroscopic abundance to determine the chemical composition of a star.

**Background:** The *simple* *curve-of-growth* (COG) of a spectral line is a plot of the log10 *Wλ(*log10 *<n*i*>)* relation for an individual spectral line, where *Wλ* is the *equivalent width* of the line (in mA) as determined from spectroscopy, and *<n*i*>* is the number density of the absorbing species (in cm-3) in the lower energy level, *i*, of the corresponding atomic line transition averaged over column depth in the star’s atmosphere (*note* that both the abscissa and the ordinate are logarithmic). The *shape* of the COG is important for understanding the different strength regimes of spectral lines, and for using the COG effectively. If the COG of a particular line is known in advance, then a measurement of *Wλ* can be used to determine *<n*i*>* graphically from the COG. Application of the *Boltzmann* *equation* can then lead to an estimate of the column depth average of the number density, *<N*k*>*, of the corresponding chemical species *k* (*ie.* of the corresponding ionization stage), and *then* application of the *Saha equation* can lead to an estimate of the column depth average of the number density of the corresponding chemical element *Z*, *<N*Z*>*. The situation is complicated in that for a given *<n*i*>* value, the value of *Wλ* also depends on the value of the line wavelength, *λ*o, and on the oscillator strength, *f*, of the transition *i→j*.

In stellar spectral modeling, what we can control as an *input* is *N*Z, and the standard way to specify this is on a normalized logarithmic scale using an “*A*­12 number”, where *A*12 *≡* log10*(N*Z*/N­*H*)* *+ 12*.

**Apparatus:**

The ChromaStar stellar atmospheric modelling WWW application: (www.ap.smu.ca/OpenStars/)

A spreadsheet application: (OpenOffice Calc (free!), MS Excel, …). You must be able to ‘Save’ (or ‘Export’ or ‘Print’) the file in a platform-independent format such as PDF – you might have to submit it electronically.

**Initial set-up:**

In ChromaStar, make sure the stellar input parameters are those of the Sun, and that all other parameters and options have their default values (best practice is to clear the browsing history with all optional data types checked and then ‘reload’ the ChromaStar page – that will re-set the parameters to their default values, including solar parameters for the star).

In the “Input:” section, open the optional input panel titled “Show/hide spectral line” – this is the input panel you will be working with.

*Tip*: Open the “Show/hide performance/realism” panel and check the right-most box – the one labeled “ONLY update spectral line”. This is not really necessary, but will allow your modeling to proceed more quickly – consider this if your device is taking more than a few seconds to compute each spectral line.

In the “Output:” section, under “Plots:”, check the box for “High resolution spectral line” – this displays two optional figures: A plot of the spectral line for which we will be studying the COG, and a diagram of the corresponding atomic energy levels and transition. *Note* that the value of *Wλ* is displayed in picometers (1 pm = 10 mA) in this plot, as well as in the textual output banner just above the graphical output.

In a spreadsheet application open a new document and save it with the filename “YourLastName-SCOGLab”. At the top of the sheet, enter a meaningful *title*, the *date*, and your *name*, and the *course name*. You *might* have to submit the spreadsheet electronically.

**Procedure:**

1. In ChromaStar, in the “spectral line” input panel, set up a line transition (*i→j*) for a line of *λ*o *=* 500 nm arising from the *ground state* (*ie.* *i=1*) of the *neutral* ionization stage (*ie. k=*I (Roman numeral “I”) (*note* that it does not matter for this lab what the actual element, *Z*, is!):

In this case, this just means making sure all the fields in this panel have their default values – that the “Line center wavelength (*λ*o)” is 500 nm, the “Stage I ground state ionization E (*χ*I)” and that for Stage II (*χ*II) are both 8.0 eV, the “Excitation E” of the lower level of the transition (*χ*i) is 0.0 eV, the “Stage I ground statistical weight (*g*I)”, that for Stage II (*g*II), and “Lower E level statistical weight (*g*i)” are all 1.0, the “Microturbulence (*ξ*T)” is 1.0 km s-1, the “Particle mass” is 12 amu, and that the “Log10 broadening enhancement (*γ*Extra)” is 0.0. (Some of these quantities are beyond the scope of this course, and you do not need to understand them to do this lab - these values will lead to a useful range of *Wλ* values.)

1. In your spreadsheet, below the header information you’ve already entered, log the values of the relevant parameters that will remain *fixed*: The four stellar parameters from the “stellar” input panel (they should be the default solar values!), and all those parameters from the “spectral line” panel, *except* “*A*12 Number density” and “oscillator strength log10 *f*” (which you will be varying). This involves logging both the *name* of the parameter as it appears in the ChromaStar panel, and the corresponding *value*.
2. Find the input field for the *oscillator strength*,labeled “log10 *f*”, and change the value from the default (-1.0) to -4.5 (*f* is a measure of the *strength* of the transition *i→j* – the probability that it will absorb photons). *Caution*: If you “reload” the ChromaStar page for any reason during the procedure, the value of log10 *f* will revert to the default value, and you’ll have to set it again!
3. In your spreadsheet, below the fixed parameters you have logged, leave several blank rows and establish a data table that will have five columns. Give columns 2 and 3 the super-heading “log f = -4.5” and columns 4 and 5 the super-heading “log f = -4.0”. On the next row, give column 1 the heading “A12”, columns 2 and 4 “W (pm)” and columns 3 and 5 “log(W)”.
4. In the “spectral line” input panel set the value of the logarithmic abundance of the absorbing species, “*A*12 Number density” to 3.0. Click the “Model” button and model your first spectral line. In the graphical output section, check the appearance of the line in the “Spectral line profile” plot – at this low value of the abundance you should see a very weak line that’s almost non-existent. Log the input value of *A*12 in column 1 of your data table, and the output value of *Wλ* in column 2.
5. Repeat step 5) for a range of input *A*12 values from 3.0 to 9.0, with a sampling, *ΔA*12, of 0.25. This will lead to a substantial data table with 25 rows. Be sure to visually inspect the spectral line profile plot periodically - as *A*12 increases you should see the spectral line shape evolve from that of the *weak* line, through the *strong* line, and then into the *saturated* line (line with wings) regimes.
6. Program the spreadsheet to calculate corresponding log10 *Wλ* values in column 3. (You should only have to program the formula *once* for the first row, then the remaining rows can be filled in by “selecting” them all and “pasting” the formula – that’s the power of spreadsheets!)
7. Now *increase* the value of log10 *f* to -4.0 (while leaving all other spectral line parameters fixed!) and repeat steps 5) through 7) for the same set of input *A*12 values. This will complete all rows of columns 4 and 5.
8. Have the spreadsheet application make a “line plot” or a “scatter plot” (*ie.* symbols with no connecting lines) of log10 *Wλ (y*-axis) *vs* *A*12 *(x*-axis) and plot up the relation for *both* log10 *f* values. (This will involve “selecting” the entire data table and then opening the “Data” tab in the spreadsheet menus. Note that the default plot type is probably something inappropriate (like “bar chart”) and you’ll have to choose something that looks like a line plot or a scatter plot.) Make sure the plot is big enough for you to mark on it when doing the analysis below. This will be a bit tricky – the spreadsheet will want to plot up *Wλ vs* *A*12 as well because you have *Wλ* as well as log10 *Wλ* columns in your data table. You will have to edit the plot so as to remove the *Wλ vs* *A*12 relations – they are not useful, will distort the *y*-axis plot scaling, and are distracting clutter. Give the plot a meaningful *title*, and the axes the correct *labels* and add information in the plot, or nearby in the spreadsheet, indicating which plot symbols (or line) are for which log10 *f* value.
9. Print out your spreadsheet with the graph. You will need to mark on it in the analysis below, and hand it in with your submission.

**Analysis & Discussion:**

1. For spectral lines of *λ*o = 500 nm, what are the log10 *Wλ* ranges, in pm, for weak, strong, and saturated lines (three ranges), according to your COGs (the answer should be the same for both COGs!)?
2. Which of these three line strength regimes is best for spectroscopic abundance determination? Why?
3. a) Suppose a spectral line at *λ*o = 500 nm has a *measured* *Wλ* value in an observed spectrum of about 3 pm. What would be the inferred value of *A*12if a value of -4.5 were adopted for log10 *f*? What is the corresponding value of *<N*Z*/N*H*>*? Show your graphical analysis on the print-out of the plot, and the complete accompanying calculation. Your answer to 2) should guide your procedure here!

b) What would it be if a log10 *f* value of -4.0 were adopted?

1. From the results in 3), what is the *uncertainty*, *Δ<N*Z*/N*H*>*in inferred *<N*Z*/N*H*>* corresponding to an uncertainty, *Δ*log10 *f*, of 0.1 in the adopted *Δ*log10 *f* value for lines of this *Wλ* value? *Comment* on the value of having accurate *f* values for atomic transitions.
2. Assume that all of the element *Z* is in its neutral ionization stage, *N*I or singly ionized, *N*II stage (*ie*. *N = N*I *+ N*II) - not a bad approximation if the star’s atmosphere is relatively cool), and assume that the number density of Hydrogen in the solar atmosphere, *N*H, is 10­15 cm-3, and the number density of electrons, *N*­e, is 1012 cm-3. Using your value of *<N*Z*/N*H*>* for log10 *f* =-4.5. What is the column averaged *total* number density, *<N*I*>*, of the *neutral* stage, in cm-3? Assume that the partition functions, *Z*I and *Z*II, of the neutral (I) and singly ionized stage (II) may be approximated by the statistical weight of *their* ground states (*g*­I and *g*II, which you can find in the “spectral line” input panel), and that the *ionization temperature*, *T*ion, is equal to *T*eff,